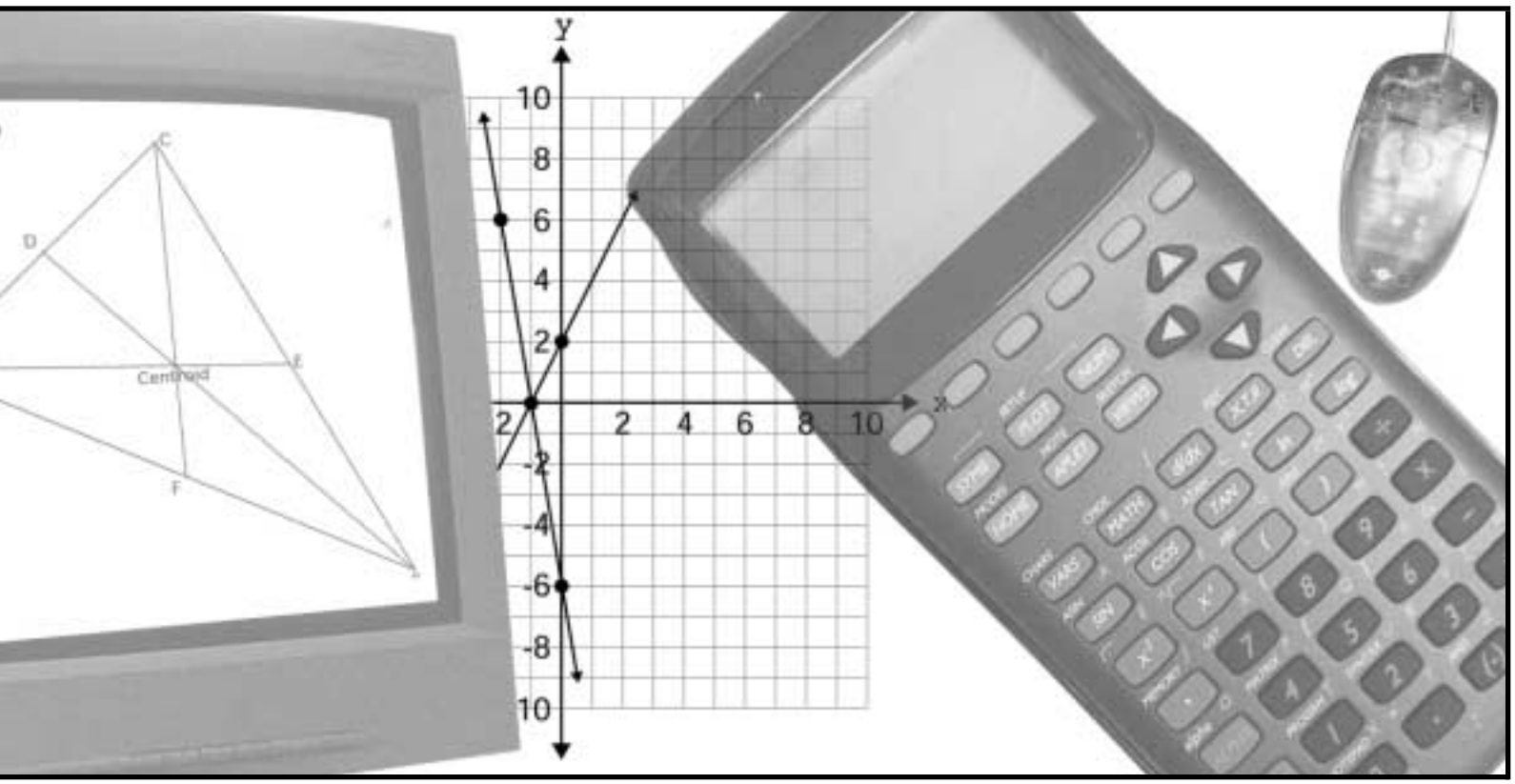
Grade 9 Assessment of Mathematics, 2000–2001



Tasks

EQAO
Education
Quality and
Accountability
Office

Directions to Students about Answering Tasks

1. For this part of the assessment, make sure you have the following items along with *Booklet 2*:
   - a pencil and an eraser or a pen
   - a scientific or a graphing calculator
   - a ruler and a protractor
2. Do all of your work (even your rough work) in *Booklet 2*.
3. You will have 40 min to do 3 tasks. Allow about 15 min for Tasks 1 and 2 and about 10 min for Task 3. Give yourself time to answer all of the questions.
4. Figures in this section are not drawn to scale.
5. The tasks are designed to allow you an opportunity to show what you know and what you can do. Provide as much information as you can to show your understanding. Your teacher may be marking some of your work. In addition, someone who does not know your work will mark all of it, including what your teacher has marked. So, you must provide clear, well-organized answers to illustrate your complete understanding and ability to communicate in mathematics.
6. Make sure you follow directions from the *Key Words and Phrases in Instructions* sheet. It is provided for you so you will know the kind of question that is being asked.

   For example, the question might ask you to "**Show your work**." Read the *Key Words and Phrases in Instructions* sheet. It says to record all calculations. If you use your calculator, you need to show what calculations you do. If you sketch a graph in the process of getting to your solution, show the sketch and label it. Use proper and correct mathematical conventions when you present your work.
7. When using a calculator, write down the numbers and operations that you carried out on the calculator.

   For example: Find the area of a circle with diameter of 7 cm.

   You need to write $A = \pi(3.5)^2$
   as well as the answer you got on your calculator.
8. There are always many different ways to solve a problem. Use your broad range of mathematical knowledge to present a complete and creative solution to each question.
9. You have **40 min** to work.
10. When you see the 

sign, you have completed the work for the day. Check your answers. Then wait quietly for directions from your teacher.

Key Words and Phrases in Instructions

Throughout the assessment, key words and phrases are used to identify the type of response required from you. The key words and their explanations are listed below. Refer to these explanations to ensure you are responding to the question that is asked.

Compare:
Tell what is the same and what is different.

Describe:
Tell about something in a step-by-step manner. Use words, numbers, graphs, diagrams, symbols, charts and/or pictures to do this.

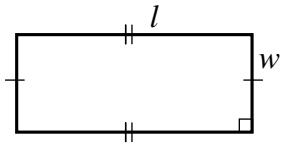
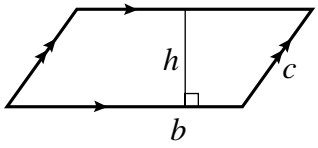
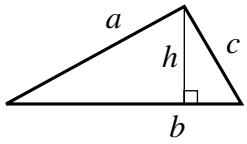
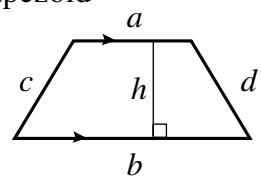
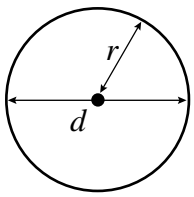
Explain:
Use words and symbols to make your solutions clear and understandable.

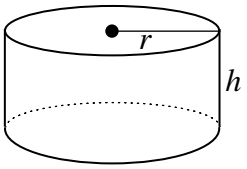
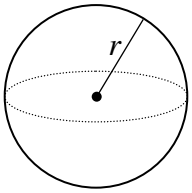
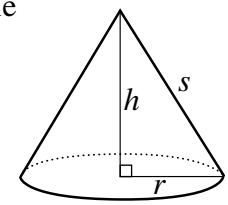
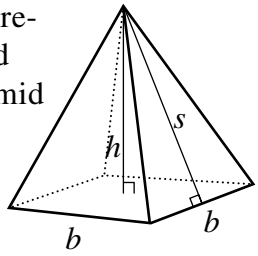
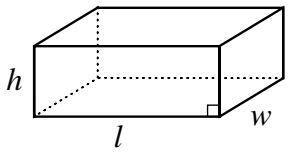
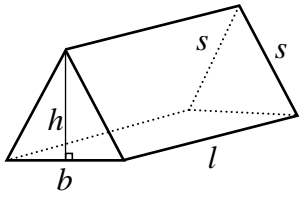
Give reasons for your answer:
Explain your reasoning in your own words. Give reasons and evidence to show your answer is correct or proper.

List:
Write down or identify in point form.

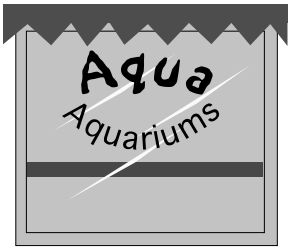
Show your work:
Record all calculations. Include all the steps you went through to get your answer. You may want to use words, numbers, graphs, diagrams, symbols, charts and/or pictures to explain your thinking.

Formula Sheet

Geometric Figure	Perimeter	Area/Surface Area
Rectangle 	$P = 2l + 2w$ $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ $P = 2b + 2c$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ **or** $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a + b)h}{2}$ **or** $A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ **or** $C = 2\pi r$	$A = \pi r^2$

Geometric Figure	Area/Surface Area	Volume
Cylinder 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $A_{total} = 2\pi r^2 + 2\pi rh$	$V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Cone 	$A_{cone} = \pi rs$ $A_{base} = \pi r^2$ $A_{total} = A_{cone} + A_{base}$	$V = \frac{1}{3}\pi r^2 h$
Square-based pyramid 	$A_{triangle} = \frac{1}{2} bs$ *(for each triangle)* $A_{base} = b^2$ $A_{total} = A_{4\ triangles} + A_{base}$	$V = \frac{1}{3} b^2 h$
Rectangular prism 	$A_{total} = wh + wh + lw + lw + lh + lh$ $A = 2(wh + lw + lh)$	$V = lwh$
Isosceles triangular prism 	$A_{triangle} = \frac{1}{2} bh$ *(for each triangle)* $A_{rectangles} = ls + lb + ls$ $A_{total} = A_{rectangles} + A_{2\ triangles}$	$V = \frac{1}{2} (bh)l$

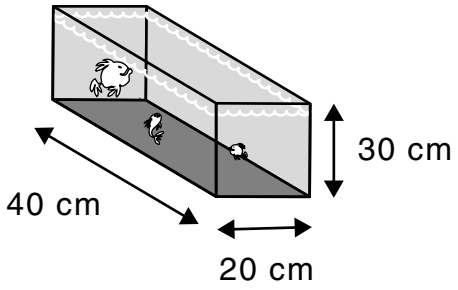
Task 1: Aquarium



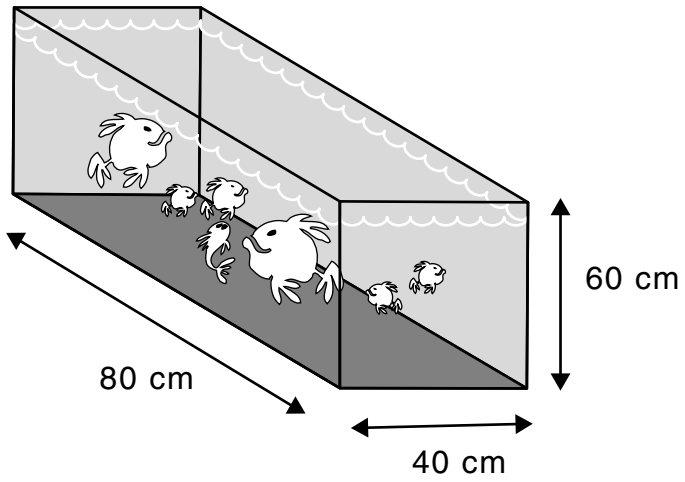
Aqua Aquariums sells aquariums in the shape of rectangular prisms. The aquariums are available in two sizes, small and large, with dimensions as shown. Each aquarium has glass sides and bottom, but no top.

NOTE: These aquariums are NOT drawn to scale.

Small



Large



a) Calculate the volume of each aquarium.

Small

Large

b) Calculate the total outside surface area of each aquarium.

HINT: The aquariums have no tops.

Small	Large

c) The cost of materials required to build the aquariums is \$0.002/cm^2 of **surface area**. Determine the cost of materials required to build each aquarium. **Show your work**.

Small	Large

d) The cost of the materials required to build the large aquarium is _____ times the cost of the materials required to build the small aquarium.

e) The selling price of the small aquarium is $24. The selling price of the large aquarium is $115.

Do the selling prices of the aquariums seem appropriate according to your calculations? **Give reasons for your answer.**

f) Mohammed went into the store to buy an aquarium. After comparing the small and large aquariums, he tells the owner, "The large aquarium should only cost two times as much as the small aquarium."

He gives the following reasons:

- The dimensions of the large aquarium are two times bigger than those of the small aquarium.
- It takes two times more material to build the larger aquarium.

Explain the **mathematical error** in Mohammed's reasons.

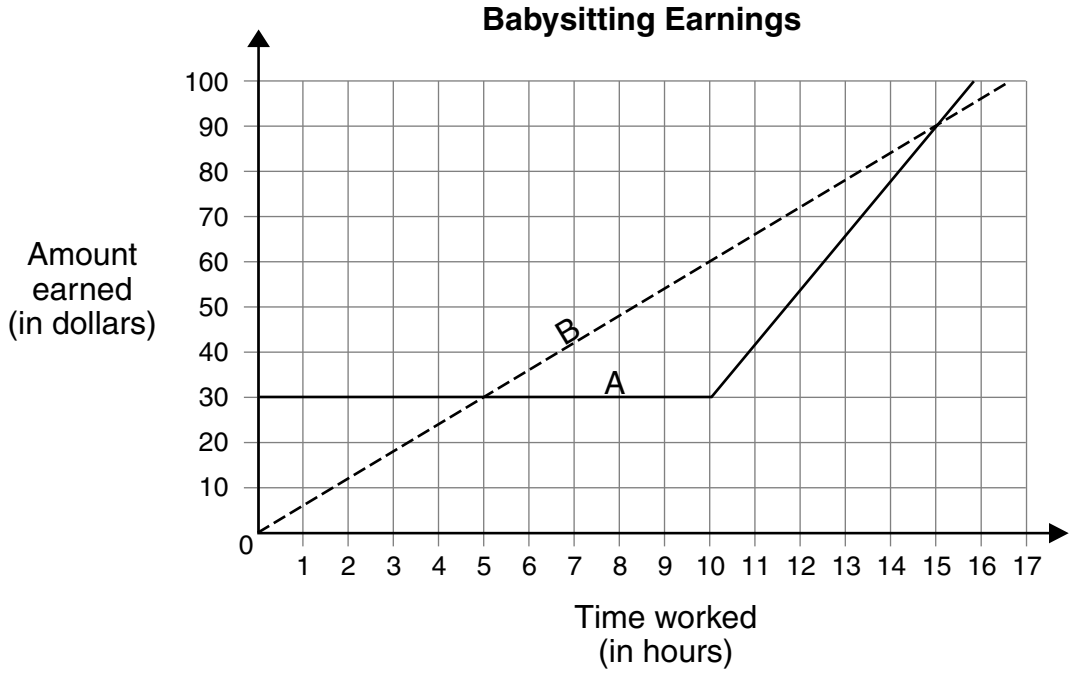
HINT:
To answer this question, refer to your previous answers for help.

Task 2: Babysitters' Club

Nadia and Lisa are comparing their **weekly earnings** from babysitting. The following graph shows their earnings compared to the number of hours they worked in the week.

a) Lisa says:
"If we both work **less than 5 hours** or **more than 15 hours**, I earn more than you do."

Label Lisa's line with her name. Write Nadia's name on the other line.



b) Describe what the graph shows about how each girl is paid for her week of work. Include specific mathematical details about hourly rates of pay.

c) Sana also offers babysitting in the home. She lives on the edge of town and travels by bus to the home where she babysits.

Sana charges a family a set fee of **$15.00 per week** to cover her bus pass **plus an additional $4.00 per hour**.

Draw the graph for Sana's earnings on the graph in question **a)**. Label your line.

d) Your neighbour needs a babysitter for 12 h this week.
How much would each of the three girls charge for this 12 h of babysitting?
Show your work or explain how you get each answer.

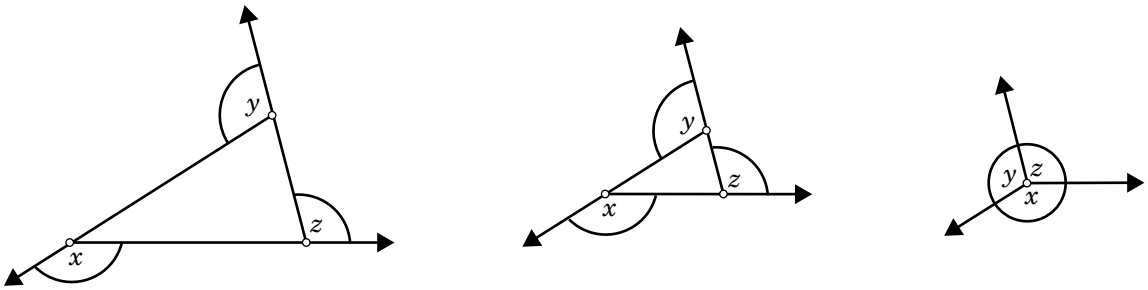
e) Several neighbours have inquired about babysitters. Some require a lot of hours of babysitting per week while others require very few hours. They have asked you which of the babysitters charges the least. What would your answer be?
Explain your reasoning. Be specific about the time intervals.

Task 3: Interior and Exterior Angles

Sandra is doing a dynamic geometry investigation of the **sum of the exterior angles** in polygons.
She uses dynamic geometry software. You don't have to use this software to do this task.

She constructs a triangle using rays instead of line segments.

She shrinks the triangle until it looks like three rays coming from a point, as shown below.



From this, she concludes:

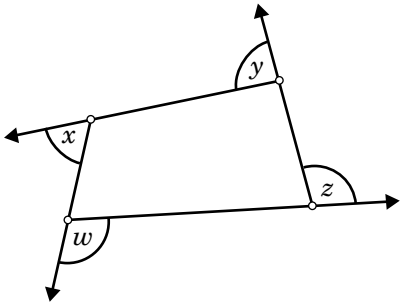
"The **sum of the exterior angles** of a triangle is 360°."

a) Refer to the diagrams above to **explain** why you agree or disagree with this conclusion.

b) Imagine Sandra starts with a **quadrilateral**.

Sketch two more diagrams like those in question **a)** to show what happens as Sandra shrinks the quadrilateral so that all four points get closer together.

Diagram I **Diagram II** **Diagram III**



c) What conjecture can you make about the **sum of the exterior angles of any quadrilateral**?

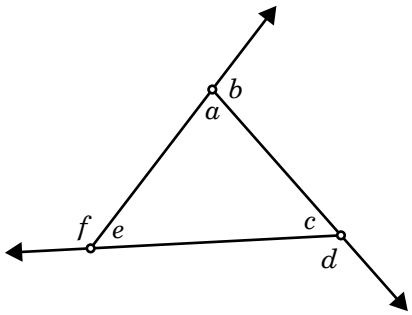
d) Make a conjecture or prediction about the **sum of the exterior angles of any polygon**.

e) Sandra's triangle shows three pairs of supplementary angles:

$a + b = 180°$

$c + d = 180°$

$e + f = 180°$



These three pairs of angles add up to 540°
(180° + 180° + 180° = 540°).

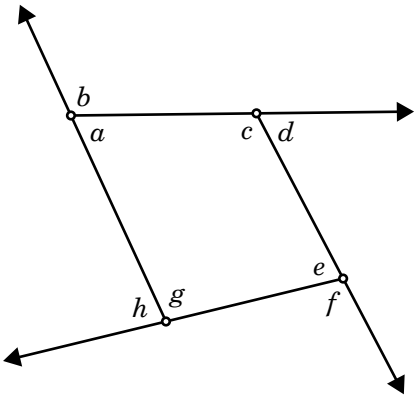
The **exterior** angles ($b + d + f$) add up to 360°.
She calculates that the **interior** angles must add up to 180° (540° – 360° = 180°).

She concludes:

"The sum of the **interior** angles of a triangle ($a + c + e$) is 180°."

Construct a similar argument **to explain why**:

"The sum of the **interior** angles of a quadrilateral is 360°."



Extended Response Coding Guide — Academic Program
Task #1 — Aquarium

b – blank: nothing at all is written for the solution

u – unrelated or unengaged: the student has written "I don't know" or a question mark; the student has simply rewritten the question exactly as posed; the student has offered unrelated comments or drawn pictures; the student has not engaged in the problem solution

Erasures – If it is rubbed out and readable, mark it. If it is rubbed out and not readable, give it a code of u.

Category	Parts	Codes	Descriptions
KU N	c), d)	1	• no correct answers
		2	• one correct answer, other answers are incorrect or missing [work may or may not be shown in **c)**]
		3	• two correct answers, other answer is incorrect or missing [work may or may not be shown in **c)**]
		4	• three correct answers with no supporting work in **c)**
		5	• three correct answers based on multiplying to find cost and dividing to find proportion [i.e., \$8.80 and \$35.20 in part **c)** and 4 in part **d)** with correct supporting work in **c)**] **Note:** answers need only be correct based on work in previous sections **Note:** correct units not necessary
KU M	a)	1	• no correct answers
		2	• one correct answer, the other incorrect or missing **or** correct volume formula used with error(s) in calculations
		3	• correct answers (i.e., 24 000 cm^3, 192 000 cm^3) **Note:** correct units not necessary
AP M	b)	1	• both answers incorrect and work shown is not appropriate to the context
		2	• inappropriate selection of formula (e.g., $2lw + 2lh + lw$) with correct substitution **or** • appropriate selection of formula with incorrect substitution
		3	• partially appropriate selection of formula with correct substitution (e.g., calculation does not include bottom of aquarium)
		4	• proper selection of formula for six sides with correct substitution (e.g., includes top of aquarium)
		5	• proper selection of formula for five sides with correct substitution (i.e., correct answers: 4400 cm^2 and 17 600 cm^2)
AP N	e)	1	• answers yes or no, no mention of previous calculations or selling price (e.g., "no it is not appropriate because I paid less for my aquarium")
		2	• answers yes or no, mentions previous calculations, no comparison to selling price (e.g., "no because in all my other calculations the large tank was 4 times bigger") **or** • answers yes or no, mentions selling price, no comparison to previous calculations
		3	• answers yes or no, indirect comparison of previous calculations to selling price (e.g., "No because the price is too high, it only costs \$6.40 to make the small one and \$25.60 to make the large one. They need to make a profit but this is too high.")
		4	• answers yes or no, direct comparison to previous calculations, but no proportional comparison made to selling price (e.g., "\$24 × \$8.80 = \$15.30, \$115 × \$35.20 = \$79.80 no this isn't fair, the companies are making too much profit.")
		5	• answers yes or no, direct proportional comparison between previous calculations and selling price (e.g., "It's appropriate because the cost of materials for the large aquarium is 4 times the cost of materials for the small aquarium and the selling price of the large aquarium is 4.79 times the selling price of the small aquarium. The rate is almost the same so the selling prices are appropriate.") **Note:** student work should be scored relative to answers in previous sections

Extended Response Coding Guide — Academic Program
Task #1 — Aquarium

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Category	Parts	Codes	Descriptions
PSN	f)	1	• no explanation provided (e.g., "Mohammed is wrong" **or** "Mohammed is right")
		2	• provides explanation with errors or inconsistencies (e.g., "Mohammed is right, the large aquarium should cost twice as much"; "In part **e)** I saw that the large aquarium was priced too high, so Mohammed is right, it should cost less.")
		3	• provides correct explanation limited to the context of this problem, which follows from answer given in part **b), c)** or **d)** (e.g., "I calculated that it takes 4 times as much material to make the large aquarium, so it should cost 4 times as much.")
		4	• provides correct explanation of the error in Mohammed's reasoning with reference to wider mathematical context (e.g., "The dimensions may be twice as large, but since the SA formula multiplies two dimensions together, this means the SA will be 4 times as large, so it will take 4 times as much material to make the large aquarium.") • **Note:** work is to be scored based on the answer in **d)**
CM	e), f) (presentation of thinking)	1	• communication of thinking is rarely clear and does not reveal processes (e.g., work shown and explanations given in **e)** and **f)** reveal little of the thinking process and are unclear)
		2	• communication is somewhat clear and reveals some processes (e.g., work shown and explanations given in **e)** and **f)** reveal some of the thinking process and are somewhat clear)
		3	• communication is clear and reveals processes (e.g., work shown and explanations given in **e)** and **f)** reveal the thinking process and are clear)
	a), b), c), e) (mathematical conventions)	1	• mathematical conventions are rarely used properly when required [e.g., does not include proper units (\$, cm^2, cm^3) in **a)**, **b)**, **c)** and **e)** where appropriate]
		2	• mathematical conventions are often used properly when required (e.g., includes units where appropriate and misuses equal signs consistently)
		3	• mathematical conventions [use of symbols (+, =, etc.), units (\$, cm^2, cm^3) and mathematical form] are used properly when required

Extended Response Coding Guide — Academic Program
Task #2 — Babysitters' Club

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Category	Parts	Codes	Descriptions
KU R	c)	1	• all points are incorrect
		2	• one correct point [e.g., the vertical intercept (0, 15) has been plotted]
		3	• at least two correct points have been plotted but line has not been drawn or is incorrect
		4	• line drawn correctly going through two or more correct points [e.g., straight line through (0, 15), (1, 19), (2, 23), …]
AP R	a)	1	• neither line is labelled correctly
		2	• one line is labelled correctly
		3	• both lines are labelled correctly
	b)	1	• none of the information stated is correct
		2	• stated information is correct or partially correct but contains no details about hourly rates of pay (e.g., "Nadia earns more than Lisa" or refers to slopes of A and B)
		3	• stated information is correct for both girls but hourly rates of pay are missing or incorrect (e.g., "Lisa is paid a flat rate for so many hours, then an hourly rate after that. Nadia earns the same hourly rate no matter how much she works.") **Note:** based on graph labels
		4	• identifies one appropriate babysitting rate (e.g., Nadia earns \$6/h **or** Lisa earns a flat rate of \$30 then she earns \$12/h after that) (**Note:** based on graph labels)
		5	• identifies both babysitting rates (e.g., "Lisa is paid a flat rate of \$30 for up to 10 hours, then she is paid \$12/h after that. Nadia earns \$6/h no matter how many hours she works.") **Note:** based on graph labels
	d)	1	• chooses inappropriate tool(s) to arrive at amounts that don't fit the problem
		2	• chooses appropriate tool(s) (e.g., calculations or graph as labelled) and arrives at one or two amounts that fit the problem
		3	• chooses appropriate tool (e.g., calculations or graph as labelled) and arrives at amount of pay for each babysitter that fits the problem (i.e., dollar amounts **close to** 54 for Lisa, 72 for Nadia and 63 for Sana)
PS R	e)	1	• reasoning illogical and inappropriate or no conclusions drawn (e.g., discussion of least cost does not follow from graph or calculations)
		2	• simple repetition of hourly pay rate details with no comparison or reference to graph (e.g., "Pick Sana because she charges only \$15 flat fee and \$4 an hour")
		3	• logical reasoning leads to appropriate and largely incomplete conclusion (e.g., least cost is stated for only one time interval)
		4	• logical reasoning leads to appropriate and somewhat incomplete conclusion (e.g., least cost is stated for only two time intervals, or the conclusion does not address precise time intervals: "For few hours Nadia charges the least. For many hours Sana charges the least. In the middle Lisa charges the least.")
		5	• logical reasoning leads to appropriate and complete conclusion that includes full details about time intervals (e.g., "hire Nadia for 5 h or less; hire Lisa for 5 h to 13 h; hire Sana for more than 13 h")
CM	b), d), e) (presentation of thinking)	1	• communication is unclear and does not reveal the thinking process
		2	• communication is somewhat clear and reveals some of the thinking process
		3	• communication is clear and reveals the thinking process
	b), d) (mathematical conventions)	1	• mathematical conventions are rarely used properly
		2	• mathematical conventions are sometimes used properly
		3	• mathematical conventions are used properly (e.g., proper units and equal signs)

Extended Response Coding Guide — Academic Program
Task #3 — Interior and Exterior Angles

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Category	Parts	Codes	Descriptions
KU M	c), d)	1	• one or more conjectures do not relate to the problem (e.g., sides are equal)
		2	• one or more conjectures do not relate correctly to the sum of exterior angles (e.g., sums to 180º)
		3	• one or more conjectures correctly relate to the problem in one part, other part is blank or incorrect
		4	• one or more conjectures correctly relate to the problem with conclusion that the sum is 360º
AP M	b)	1	• diagrams do not show appropriate fitting to the context
		2	• one diagram shows appropriate fitting to the context and the other diagram is missing or shows inappropriate fitting to the context
		3	• both diagrams show partially appropriate fitting to the context or • one diagram shows appropriate fitting to the context, the other shows inappropriate fitting to the context
		4	• one diagram shows appropriate fitting to the context and the other shows partially appropriate fitting to the context
		5	• both diagrams show appropriate fitting to the context
PS M	a)	1	• no conclusion stated, illogical reasoning given or • no conclusion stated, reasoning given that does not refer to the diagrams
		2	• conclusion stated with illogical reasoning that refers to the diagrams (e.g., "I agree because I measured them with my protractor") or • conclusion stated with reasoning that does not refer to the diagrams or • conclusion stated with no reasoning provided
		3	• conclusion stated and supported with logical but incomplete reasoning that refers to the diagrams (e.g., "I agree. In the last diagram the angles make a full circle, so they must add to 360º")
		4	• conclusion stated and supported with complete and logical reasoning that includes reference to the fact that the angles stay fixed as the diagrams shrinks (e.g., "I agree. As the diagram shrinks the marked angles, which are the exterior angles of the triangle stay the same. In the last diagram they make a full circle, so they must add to 360º")
	e)	1	• argument is illogical
		2	• argument does not connect to the given argument (e.g., "In a square each angle is 90º and there are four of them, so the total is 360º") • argument is partially logical and incomplete (e.g., each pair of angles add to 180º)
		3	• argument is logical and does not refer to diagram (e.g., four groups of 180º makes 720º, 360º)
		4	• argument is complete and logical (e.g., a + b + c + d + e + f + g + h = 720º but b + d + f + h = 360º so a + c + e + g = 720º – 360º = 360º)
CM	b), d), e) (presentation of thinking)	1	• communication is unclear and incomplete and does not reveal the thinking process
		2	• communication is partially clear and complete and reveals some of the thinking process
		3	• communication is clear and complete and reveals the thinking process
	b), d) (mathematical conventions)	1	• mathematical conventions are rarely used properly when required [e.g., does not include proper units (i.e., º) where appropriate, and most equations incomplete in e)]
		2	• mathematical conventions are sometimes used properly when required [e.g., sometimes includes proper units (i.e., º) where appropriate, and some equations are complete in e)]
		3	• mathematical conventions are used properly when required [e.g., includes proper units (i.e., º) where appropriate, and uses complete equations in e)]