MCR3U0 – Final Culminating Activity

Victim: ____

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Description

Successful completion of this activity should help students to

- understand and apply the ***main ideas studied in this course***
- grasp how the ***main concepts*** of this course ***are related to one another***
- learn how to ***prepare for final exams*** in courses that involve ***problem solving***

This activity accounts for 10% of the final mark. It is to be completed in class over the course of the last few days of the semester.

Part I – Using Mathematical Terminology and Notation Correctly

1. You have learned in this course that not all equations are created equal. As shown in the following table, different equations serve different purposes.

(a) Give an example of each type of equation.

(b) Give a geometric (graphical) representation of each of the equations that you chose.

Equation that is Solved to find the Value(s) of the Unknown	*Equation that Describes a Relationship between two or more Quantities*	*An Identity*

2. Complete the following table.

Factor the given Expression	*Expand the given Expression*
$24x^3y^2 + 5x^2y^2 - 36xy^2$	$(2a-3b)(5a-7b)^2$
Simplify the given Expression	***Solve the given Equation***
$\dfrac{2x}{4x^2+6x} - \dfrac{3}{2x+3}$	$x^2 - 3x - 22 = 4(x-1)$

3. The following table contains a series of mathematical statements, some of which contain terminological and/or notational errors. Give a geometrical interpretation of each statement that does not contain any errors. Suggest corrections for the statements that do contain errors.

Statement(s)	***Give a geometrical interpretation of the correct statements. Suggest possible corrections of the statements that contain errors.***
$\sin = \frac{1}{2}$	
Solve $x^2 - 2x - 35$	
$\cos\theta = \sqrt{3}$ $\therefore$ N/A	
$a^2 + 4a - 5 = (a-1)(a+5)$ $\therefore a - 1 = 0$ or $a + 5 = 0$ $\therefore a = 1$ or $a = -5$	
$\begin{cases} 3x + 4y = 15 \\ 2x - 5y = 12 \end{cases}$	
$x^2y^3 - 5x^2y^3 + 3x^3 - 5a^2 - 7x^3$ $= -4x^2y^3 - 4x^3 - 5a^2$ However, $5a^2 - 7b^2$ is undefined.	

Part II – Understanding Mathematical Relationships from a Variety of Perspectives

1. Choose any ***quadratic*** function other than one that is found in our course notes. Then complete the following table.

Algebraic Perspective	***Verbal Perspective***	***Numerical Perspective***

Geometric Perspective	***Physical Perspective***

2. Choose any ***sinusoidal*** function other than one that is found in our course notes. Then complete the following table.

Algebraic Perspective	*Verbal Perspective*	*Numerical Perspective*

Geometric Perspective	*Physical Perspective*

3. We spent a great deal of time in this course discussing physical phenomena that can be modelled by linear, quadratic and sinusoidal functions. Unfortunately, we did not have enough time to cover another important family of functions known as ***exponential functions***. This question will help to introduce you to some of the fundamental properties of these functions.

(a) First, it is important to review some important properties of powers. Simplify each of the following (if possible). Write all answers using positive exponents only.

(i) $x^4\left(x^7\right)$ **(ii)** $\frac{y^6}{y^2}$ **(iii)** $\left[x^3\left(x^6\right)\left(x^{-11}\right)\right]^0$ **(iv)** $\frac{a^{-3}b^2}{a^4b^{-6}}$

(v) $x^4\left(y^7\right)$ **(vi)** x^4+x^7 **(vii)** $\left(a^{-3}b^2\right)^6$ **(viii)** $\frac{\left(m^{-3}n^{-2}\right)^{-1}}{\left(m^{-4}n^{-6}\right)^3}$

(b) In past courses, you have learned how to work with powers involving ***integer exponents*** (i.e. whole numbers, including zero and negative whole numbers). Have you wondered what would happen if a number were raised to a ***rational*** (fractional) exponent? For example, what ***meaning*** could we possibly give to a value such as $4^{\frac{1}{2}}$? The following arguments will help us to decide how to interpret powers such as these.

$4^{\frac{1}{2}}\left(4^{\frac{1}{2}}\right)=4^{\frac{1}{2}+\frac{1}{2}}=4^1=4$ $\quad 8^{\frac{1}{3}}\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)=8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}=8^1=8$ $\quad 256^{\frac{1}{4}}\left(256^{\frac{1}{4}}\right)\left(256^{\frac{1}{4}}\right)\left(256^{\frac{1}{4}}\right)=256^{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}=256$

$\therefore 4^{\frac{1}{2}}=\sqrt{4}=2$ $\quad \therefore 8^{\frac{1}{3}}=\sqrt[3]{8}=2$ $\quad \therefore 256^{\frac{1}{4}}=\sqrt[4]{256}=4$

From these examples, it's clear that we are forced to ***define*** $m^{\frac{1}{n}}$ in the following way:

$$m^{\frac{1}{n}}=\sqrt[n]{m}$$

In addition, by using the law $\left(x^a\right)^b = x^{ab}$, we can conclude that $m^{\frac{p}{n}} = \left(m^{\frac{1}{n}}\right)^p = \left(\sqrt[n]{m}\right)^p$. By using this fact and the definition on the previous page, evaluate each of the following:

(i) $169^{\frac{1}{2}}$ **(ii)** $64^{\frac{1}{3}}$ **(iii)** $125^{\frac{1}{3}}$ **(iv)** $16^{\frac{1}{4}}$

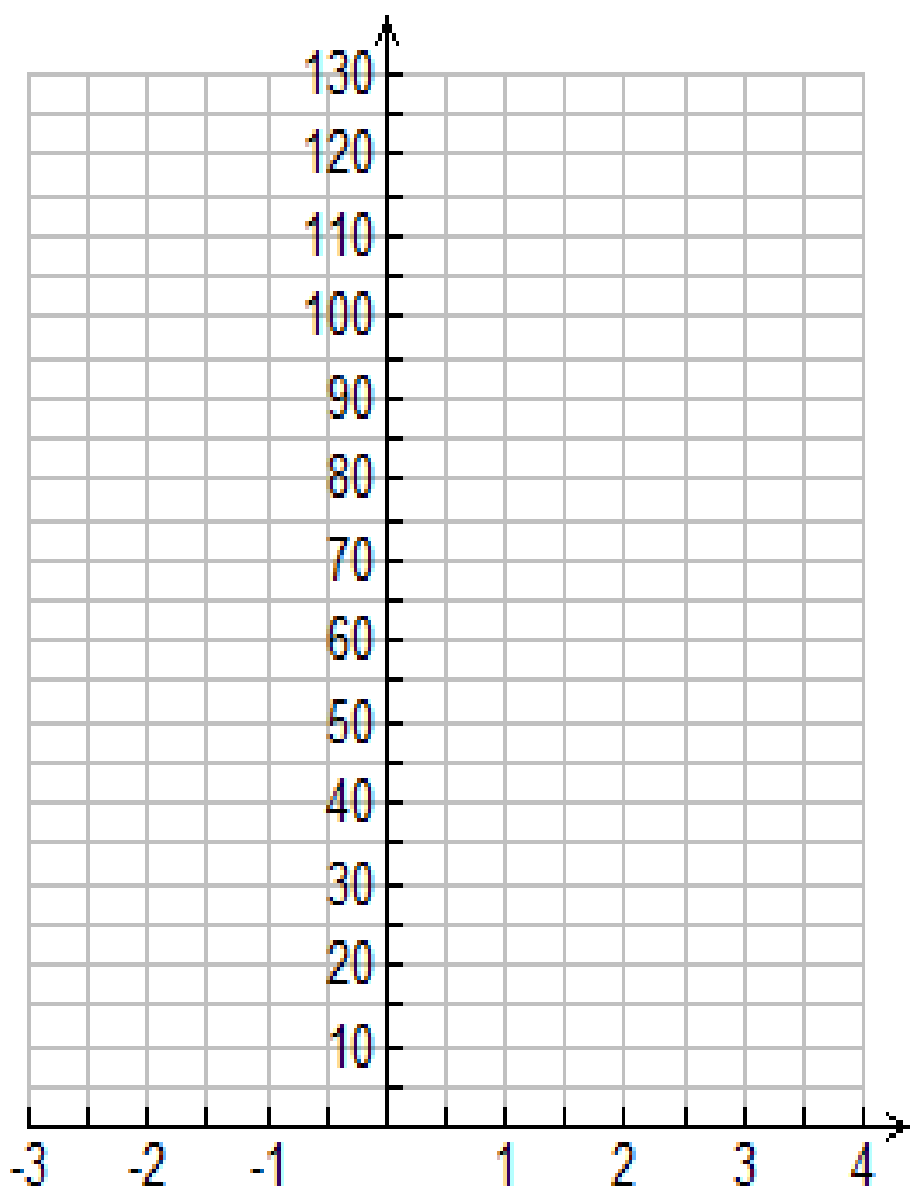
(v) $243^{\frac{1}{5}}$ **(vi)** $64^{\frac{2}{3}}$ **(vii)** $125^{\frac{4}{3}}$ **(viii)** $32^{\frac{11}{5}}$

(c) Use the law $\dfrac{x^a}{x^b} = x^{a-b}$ to explain why it is very reasonable to define $x^0 = 1$.

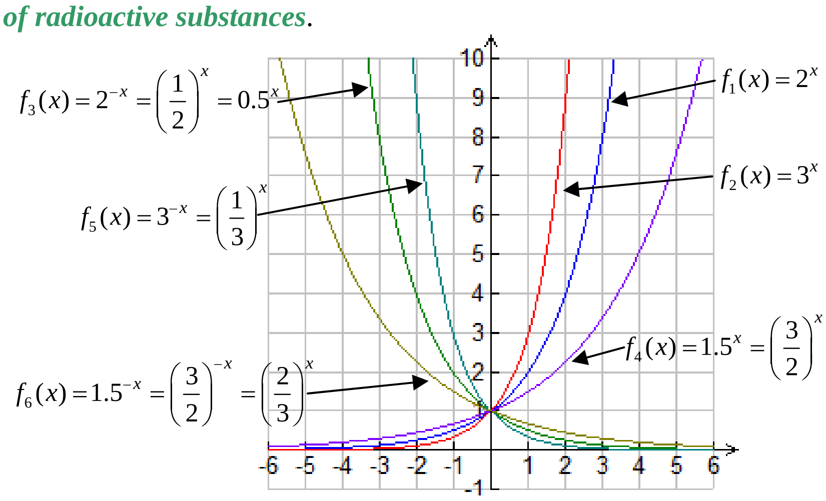
(d) Use the law $\dfrac{x^a}{x^b} = x^{a-b}$ to explain why it is very reasonable to define $x^{-n} = \dfrac{1}{x^n}$.

(e) Now we have enough information to plot graphs of exponential functions. Complete the table of values given below. Then use the same grid to sketch the graph of each exponential function.

x	$f(x) = 4^x$	$h(x) = 4^{-x}$
-3		
-2		
-1		
0		
$\frac{1}{2}$		
1		
$\frac{3}{2}$		
2		
$\frac{5}{2}$		
3		
$\frac{7}{2}$		



(f) As you should see from your graphs on the previous page and the graphs that follow, exponential functions have a very fast rate of growth or decay. This makes them useful in modelling processes such as ***population growth*** or ***decay of radioactive substances***.



x	y1(x) 2^x	y2(x) 3^x	y3(x) 0.5^x	y4(x) 1.5^x	y5(x) 3^(-x)	y6(x) 1.5^(-x)
-5	0.03125	0.004115	32	0.131687	243	7.59375
-4	0.0625	0.012346	16	0.197531	81	5.0625
-3	0.125	0.037037	8	0.296296	27	3.375
-2	0.25	0.111111	4	0.444444	9	2.25
-1	0.5	0.333333	2	0.666667	3	1.5
0	1	1	1	1	1	1
1	2	3	0.5	1.5	0.333333	0.666667
2	4	9	0.25	2.25	0.111111	0.444444
3	8	27	0.125	3.375	0.037037	0.296296
4	16	81	0.0625	5.0625	0.012346	0.197531
5	32	243	0.03125	7.59375	0.004115	0.131687
6	64	729	0.015625	11.3906	0.001372	0.087791
7	128	2187	0.007813	17.0859	0.000457	0.058528
8	256	6.56E+03	0.003906	25.6289	1.52E-04	0.039018
9	512	1.97E+04	0.001953	38.4434	5.10E-05	0.026012
10	1024	5.90E+04	0.000977	57.665	1.70E-05	0.017342

Complete the following:

(i) Exponential functions of the form $f(x)=a^x$ ($a>0$) all pass through the point with co-ordinates (0, 1). This happens because ____.

(ii) If $a>1$, the graph of $f(x)=a^x$ ________________ as x increases. On the other hand, if $0<a<1$, the graph of $f(x)=a^x$ ________________ as x increases.

(iii) The graph of $f(x)=a^{-x}$ is exactly the same as the graph of $g(x)=\left(\frac{1}{a}\right)^x$ because ________________________

____.

(iv) The graph of any exponential function of the form $f(x)=a^x$ ($a>0$) must stay above the x-axis for all values of x because ____.

In fact, the value of $f(x)=a^x$ can be made as close as we like to zero but it can never equal zero. Therefore, the x-axis is a horizontal ______________________ of the graph of $f(x)=a^x$.

(v) It does not make sense to define exponential functions for values of a less than or equal to zero because

____.

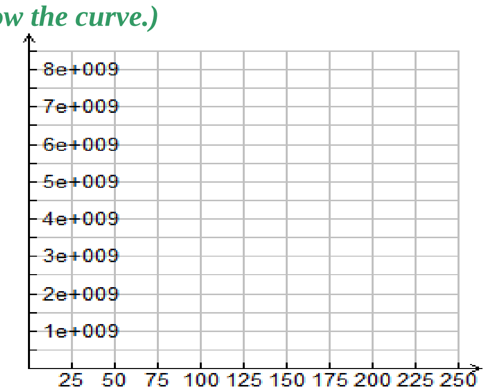
4. The table at the right lists ***world*** population figures from 1800 until the present.

Year	*Population*
1800	9.78×10^8
1850	1.262×10^9
1900	1.650×10^9
1950	2.521×10^9
2000	6.071×10^9
2008	6.641×10^9

(a) Use TI-Interactive or a graphing calculator to perform an ***exponential regression*** on the data given in the table. To simplify the equation of the function, set 1800 as the time t = 0 years. Write the equation that you obtain in the space provided below.

(b) Now use the provided grid to plot the data points in the table as well as the function that you found in part (a). Does an exponential function fit the data well?

(Be careful here! Do not join the data points as if you were completing a connect-the-dots picture! The regression is performed to find the curve that best fits the data. Unless the fit is exceptional, you should expect about half the points to lie above the curve and the other half to lie below the curve.)



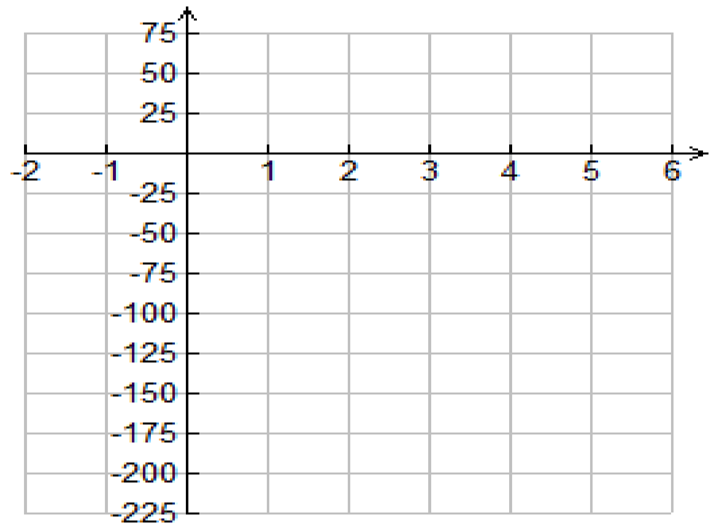
(c) Use the equation that you found in part (a) to predict the world population in the year 2100. On what assumption(s) is your prediction based?

5. Using $f(x) = 2^x$ as the base function, sketch the graph of $g(x) = -1.3\left(2^{1.1(x+1)}\right) + 27$.

(a) Use the following table to list the transformations that need to be applied to *f* to produce *g*.

Vertical	*Horizontal*
1. ______________________________	**1.** ______________________________
______________________________	______________________________
2. ______________________________	**2.** ______________________________
______________________________	______________________________

(b) Now use the provided grid to sketch the graph of *g*. (Show your work in the provided space.)



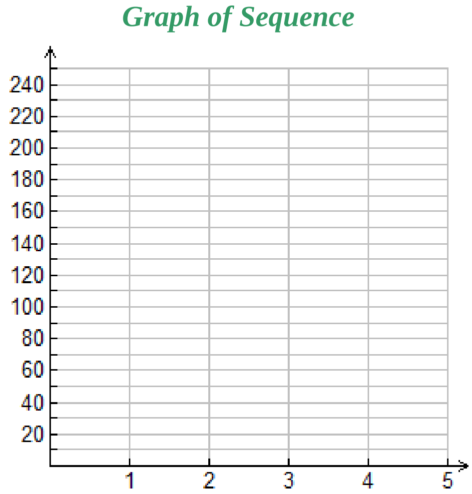
6. Consider the ***geometric sequence*** 4, 12, 36, 108, 324, …

(a) Write both a recursive and an explicit formula for the above sequence. Write the formulas in both term notation and function notation.

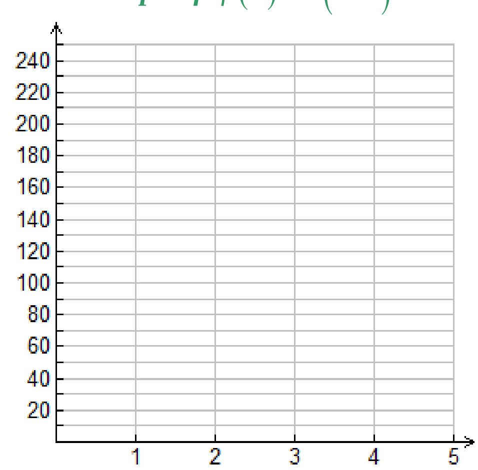
Recursive		*Explicit*	
Term Notation	*Function Notation*	*Term Notation*	*Function Notation*

(b) Sketch a graph of the sequence given above. (Remember, this is not a connect-the-dots exercise!) In addition, sketch the graph of the exponential function $f(x) = 4\left(3^x\right)$. How are the two graphs related to each other?

Graph of Sequence



Graph of $f(x) = 4\left(3^{x-1}\right)$



(c) Use the Internet to find an application of geometric sequences. Describe the application in the space provided below.