MCR 3U0 | **Semester 1, 2007 - 2008**

Grade 11 Functions (University Preparation)
Unit 1 – Major Test – Functions, Relations and Transformations

Mr. N. Nolfi

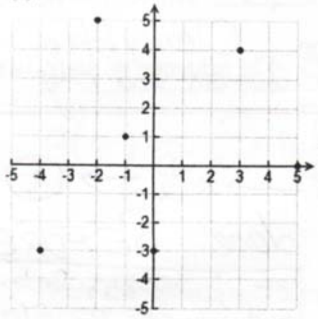
Victim: Mr. Solutions

As usual, your work is brilliant Mr. S.!!

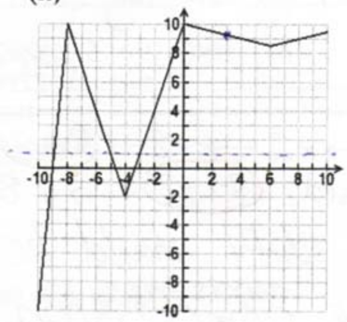
KU	APP	TIPS	COM
10 /10	18 /18	12 /12	14 /14

1. Study each graph carefully and then answer the questions found immediately below the graphs. **(10 KU)**

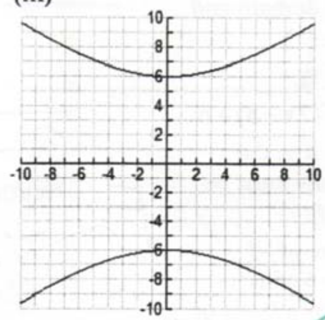
(i)



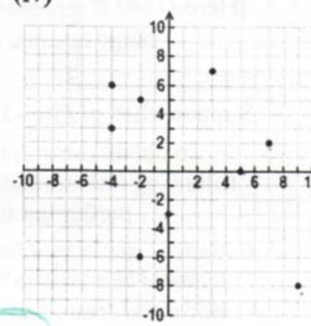
(ii)



(iii)



(iv)



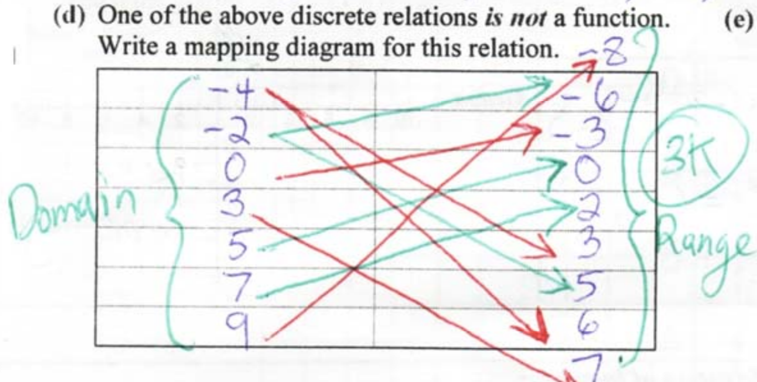
(a) Which of the above relations are continuous? (ii), (iii) 1K

(b) Which of the above relations are functions? (i), (ii) (0, –3) 1K

(c) One of the above relations is a discrete function. Write the function as a set of ordered pairs.

{(–4, –3), (–2, 5), (–1, 1), (3, 4), (5, 0)} 2K

(d) One of the above discrete relations *is not* a function. Write a mapping diagram for this relation.



(e) Suppose that the continuous function given above is called *f*. Evaluate each of the following.

any of these answers accepted

$f(0) =$ 10	$f(a) = 1 \quad \therefore a =$ –3, –5 or –9
$f(3) =$ 9	$f(-b) = 0 \quad \therefore b =$ 9, and approx 4.8, 3.2
$f(-4) =$ –2	$f(3-5) =$ $f(-2) = 4$

3K

2. State whether each of the following is true or false. Provide an explanation to support each response. **(4 TIPS, 4 COM)**

Statement	*True or False?*	*Explanation*
For all functions *f* and all real numbers *u* and *c*, $f(u+c) = f(u) + f(c)$	F	For example, let $f(x)=\sqrt{x}$ Then $f(16+9)=\sqrt{16+9}=\sqrt{25}=5$ but $f(16)+f(9)=\sqrt{16}+\sqrt{9}=4+3=7$ $\therefore$ L.S. $\neq$ R.S.
The equation $\frac{x^2}{16}+\frac{y^2}{9}=1$ describes a function.	F	This equation describes an ellipse (not a circle) which is not a function. For every value of x such that $-4<x<4$, there are two values of y that satisfy the equation

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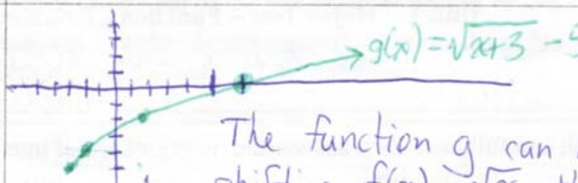
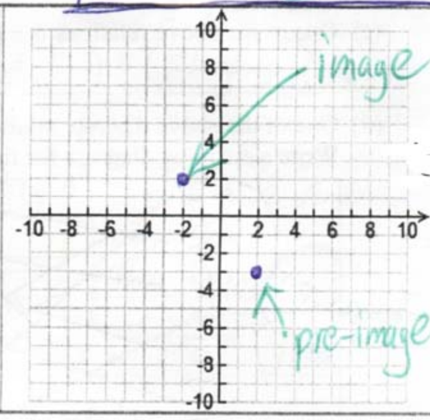
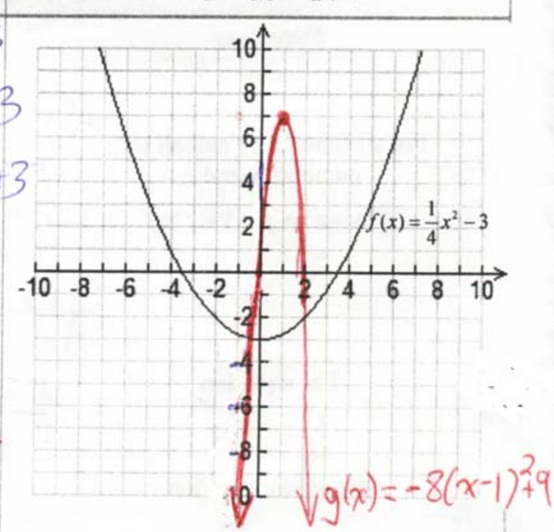
Statement	***True or False?***	***Explanation***
For the function $g(x)=\sqrt{x+3}-5$, $D=\{x\in\mathbb{R}: x\geq -3\}$ and $R=\{y\in\mathbb{R}: y\geq -5\}$. (Here D and R represent domain and range respectively.)	T	 The function g can be obtained by shifting $f(x)=\sqrt{x}$ three units left and five units down. Clearly then, the given domain and range are correct.
Suppose that $g(x)=-3f(2x-8)+6$. To obtain the graph of g, the following transformations must be performed to f: • Vertical stretch by a factor of -3 followed by a shift up by 6 units ✓ • Horizontal compression by a factor of $1/2$ followed by a shift 8 units right. ✗	F	$x \longrightarrow \times 2 \longrightarrow -8 \longrightarrow 2x-8$ Reversed: $2x-8 \longrightarrow +8 \longrightarrow \times\frac{1}{2} \longrightarrow x$ The shift should be done BEFORE the compression. $\boxed{\text{or if } 2x-8 \text{ is factored and written as } 2(x-4), \text{ compress by } \tfrac{1}{2} \text{ first, then shift right } \underline{4} \text{ units}}$

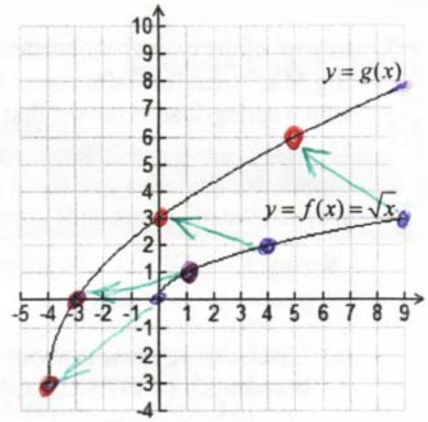
3. Complete the following table. **(4 APP)**

Pre-image	$(2,-3)$	***Trans-formation in Mapping Notation***	$(x,y)\longrightarrow$ $(-3x+4, -y-1)$	***Graph***	
Trans-formation	***Horizontal*** 1. Stretch by a factor of -3. 2. Translate 4 units right ***Vertical*** 1. Reflect in the x-axis. 2. Translate down 1 unit.	***Image***	$(-3(2)+4, -(-3)-1)$ $=(-2,2)$		

4. Complete the following table. **(4 APP, 2 COM)**

Equation of Pre-image Function	***Transformation***	***Equation of Image Function***	***Graph of y = g(x)***
$f(x)=\frac{1}{4}x^2-3$	***Verbal*** Horizontal Compress by a factor of $\frac{1}{4}$ followed by a shift right by 1 unit Vertical Stretch by a factor of -2 followed by a shift up 3 units	$g(x)=-2f(4(x-1))+3$ $=-2[\frac{1}{4}(4(x-1))^2-3]+3$ $=-2[\frac{1}{4}(16(x-1)^2)-3]+3$ $=-2[4(x-1)^2-3]+3$ $=-8(x-1)^2+6+3$	
	Function Notation $g(x)=-2f(4(x-1))+3$	$\boxed{\therefore g(x)=-8(x-1)^2+9}$	
	Mapping Notation $(x,y)\longrightarrow$ $(\frac{1}{4}x+1, -2y+3)$		

5. The graph of $y = g(x)$ is a transformation of the graph of $y = f(x) = \sqrt{x}$



(a) Using both ***function notation*** and ***mapping notation***, state how $y = f(x) = \sqrt{x}$ can be transformed into $y = g(x)$. **(2 APP)**

$g(x) =$ 3 $f($ $x+4$ $)$ -3

$(x, y) \rightarrow ($ $x-4$, $3y-3$ $)$

$(0,0) \rightarrow (-4,-3)$
$(1,1) \rightarrow (-3,0)$
$(4,2) \rightarrow (0,3)$
$(9,3) \rightarrow (5,6)$

(b) State an equation of g. **(2 APP)**

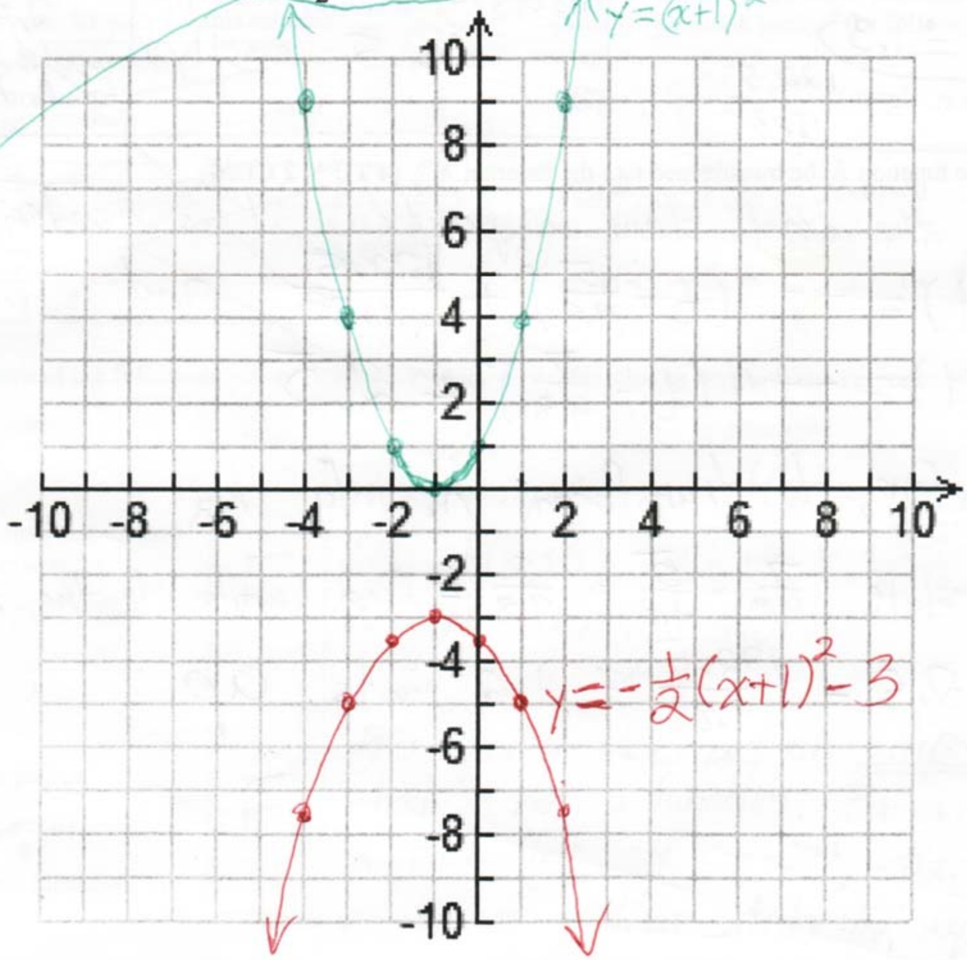
$g(x) = 3\sqrt{x+4} - 3$

(c) State the domain and range of g. **(2 APP, 2 COM)**

$D = \{x \in \mathbb{R} \mid x \geq -4\}$
$R = \{y \in \mathbb{R} \mid y \geq -3\}$

6. Sketch the graphs of $y = (x+1)^2$ and $y = -\frac{1}{2}(x+1)^2 - 3$ on the same set of axes. **(4 APP, 2 COM)**

Vertex: $(-1,-3)$
Stretch vertically by $-\frac{1}{2}$



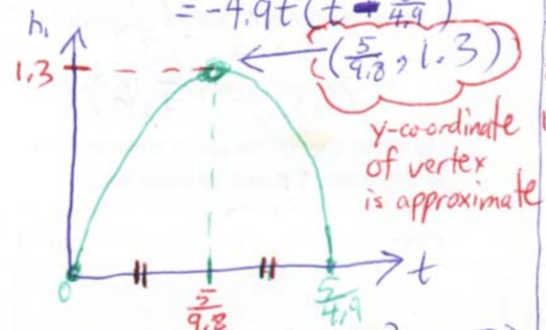
7. Instead of the typical chocolates, candies and potato chips, Mr. Nolfi decided to give Andrew a special Big Mac® treat for Halloween. Unbeknown to Andrew, however, Mr. Nolfi rigged the Big Mac® box with a spring-loaded device that would cause the burger to leap out of the box once the lid was raised.

Mr. Nolfi tested two different springs to see which would have better performance. The results of his experiment are listed below. In each case, the function gives the height of the hamburger above the ground in metres, *t* seconds after the lid is raised.

Spring 1	***Spring 2***
$h_1(t) = -4.9t^2 + 5t$	$h_2(t) = -4.9t^2 + 7t$

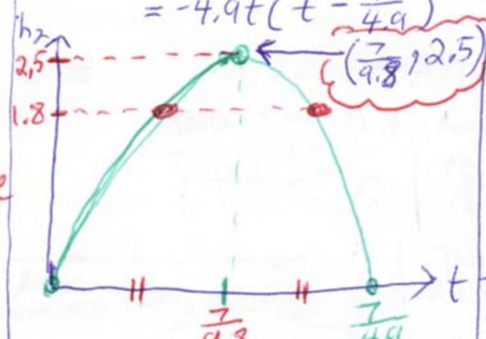
(a) Given that Andrew is about 1.8 m tall, which spring would give him a better chance of "catching" the Big Mac with his mouth? **(4 TIPS, 2 COM)**

$h_1(t) = -4.9t^2 + 5t$
$= -4.9t(t - \frac{5}{4.9})$



$h_1(\frac{5}{9.8}) = -4.9(\frac{5}{9.8})^2 + 5(\frac{5}{9.8})$
$\doteq 1.3$
exact value is $\frac{122.5}{9.8^2}$

$h_2(t) = -4.9t^2 + 7t$
$= -4.9t(t - \frac{7}{4.9})$



$h_2(\frac{7}{9.8}) = -4.9(\frac{7}{9.8})^2 + 7(\frac{7}{9.8})$
$= 2.5$ (exactly)

Conclusion
Using the first spring, the burger achieves a maximum height of about 1.3m, which is too low for Andrew. Using the second spring, however, the maximum height is 2.5m, which would give Andrew two chances to catch the burger.

(b) How can the function h_1 be transformed into the function h_2? **(4 TIPS, 2 COM)**

From the work done in part (a), it is evident that

$h_1(t) = -4.9(t - \frac{5}{9.8})^2 + \frac{122.5}{9.8^2}$ and

$h_2(t) = -4.9(t - \frac{7}{9.8})^2 + 2.5$

Therefore, to transform h_1 into h_2,

translate $\frac{7}{9.8} - \frac{5}{9.8} = \frac{2}{9.8} \doteq 0.2$ units to the right

and $2.5 - \frac{122.5}{9.8^2} \doteq 1.2$ units up