MCR 3U0 **Semester 1, 2007 - 2008**

Grade 11 Functions (University Preparation)
Mastery Test #2 on Review Material

Mr. N. Nolfi
Victim: Mr. Solutions

Once again, your work is impressive Mr. S!!

KU	APP	TIPS	COM
13/13	15/15	13/13	21/21

1. Complete the following statements by filling in the blanks with logical answers that relate to what we have learned in the review unit of this course. **(8 KU)**

Many students find mathematics difficult because they see it as set of symbols and rules that are applied without justification and without understanding. To help resolve this problem, Mr. Nolfi explained that math is like a *dating service* because it's all about relationships. In addition, the *terminology* and *notation* of mathematics can be seen as an extension of natural languages whose purpose is to describe and explore physical and conceptual relationships.

In high school, students focus on the study of ***linear*** and ***quadratic*** relationships. ***Linear relationships*** model quantities that change at a constant rate. For example, if a car moves with a constant velocity, the relationship between ***distance travelled*** and ***time elapsed*** is ***linear***. Given a ***table of values***, it is possible to spot linear relationships because the ***first differences*** are always constant. ***Quadratic relationships***, on the other hand, model certain quantities that do not change at a constant rate. For example, if an object moves solely under the influence of gravity, then the relationship between position and ***time elapsed*** is ***quadratic***. Given a ***table of values***, it is possible to spot quadratic relationships because the ***first differences*** always change linearly and the ***second differences*** are constant.

Mr. Nolfi also explained that it is a ***very bad idea to memorize formulas blindly*** (i.e. without understanding). Instead, he suggested that we remember and understand some basic concepts that will help us to derive relationships quickly and easily whenever necessary. For example,

for equations of lines, we only need to remember slope = slope,
for the midpoint of a line segment, we only need to remember average of two numbers, $\frac{a+b}{2}$
and for the length of a line segment, we only need to remember the Pythagorean Theorem.

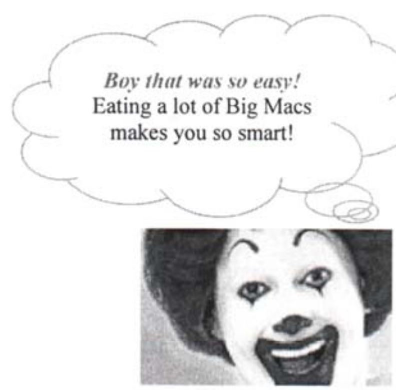
2. When Ronald M. was asked to factor the expression a^2+4a-5, he wrote the "solution" shown below. Is it correct? Explain. **(2 COM)**

$a^2+4a-5=(a-1)(a+5)$ → This is all that Ronald needed to show.

$\therefore a-1=0$ or $a+5=0$

$\therefore a=1$ or $a=-5$

He was asked to FACTOR an expression NOT to solve an equation.



KU	APP	TIPS	COM
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3. After solving a challenging math problem, Andrew E. finally gathered the courage to ask the curvaceous Polly Nomial to go to McDonald's with him. Miss Nomial, being very mathematically inclined and highly selective, agreed to go on a date with Andrew *only if* he could solve the equation $15q^2-30q-360=0$. Andrew's "solution" is shown below. Is it good enough to allow him to get the girl? If not, provide a *correct solution* along with a *graph* that clearly shows the *roots* of the *equation*. **(4 COM)**

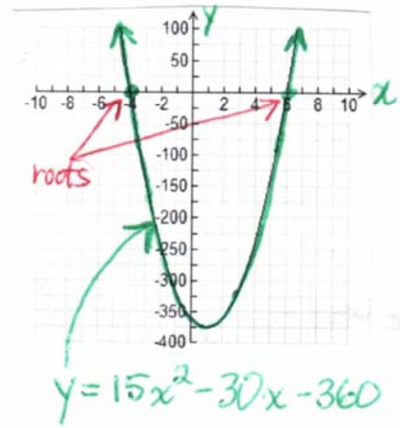
Please let me go on a date with you Polly. I'll take you to McDonald's for a special romantic dinner!

$15q^2-30q-360=0$
$\therefore 15(q^2-2q-24)=0$
$\therefore q^2-2q-24=0$

correct up to this point

$\therefore q^2-2q=24$
$\therefore q(q-2)=24$
$\therefore q=\dfrac{24}{q-2}$
$\therefore \dfrac{q}{1}=\dfrac{24}{q-2}$
$\therefore q=24$ or $q-2=1$
$\therefore q=24$ or $q=3$

$\therefore (q-6)(q+4)=0$
$\therefore q-6=0$ or $q+4=0$
$\therefore q=6$ or $q=-4$



4. Solve. Show all steps.

(a) Solve the following linear equation. **(5 APP, 2 COM)**

$-\frac{4}{5}(-2x-7)+\frac{3}{2}x=-7-\frac{11}{10}x$

$\therefore \frac{10}{1}\left[-\frac{4}{5}(-2x-7)+\frac{3}{2}x\right]=\frac{10}{1}\left[-7-\frac{11}{10}x\right]$
$\therefore \frac{\cancel{10}^{2}}{1}\left(-\frac{4}{\cancel{5}_{1}}\right)(-2x-7)+\frac{\cancel{10}^{5}}{1}\left(\frac{3}{\cancel{2}_{1}}\right)x=\frac{10}{1}\left(\frac{-7}{1}\right)-\frac{\cancel{10}^{1}}{1}\left(\frac{11}{\cancel{10}_{1}}\right)x$
$\therefore -8(-2x-7)+15x=-70-11x$
$\therefore 16x+56+15x=-70-11x$
$\therefore 31x+56=-70-11x$
$\therefore 31x+56+11x=-70-11x+11x$
$\therefore 42x+56=-70$
$\therefore 42x+56-56=-70-56$
$\therefore 42x=-126$
$\therefore \frac{42x}{42}=\frac{-126}{42}=-3$
$\therefore x=-3$

(b) Solve the following quadratic equation. **(5 APP, 2 COM)**

$2(6y-1)(y-1)=-6(y-1)(2y-5)+3y+25$

$\therefore 2(6y^2-7y+1)=-6(2y^2-7y+5)+3y+25$
$\therefore 12y^2-14y+2=-12y^2+42y-30+3y+25$
$\therefore 12y^2-14y+2=-12y^2+45y-5$
$\therefore 24y^2-59y+7=0$
$\therefore 24y^2-56y-3y+7=0$
$\therefore 8y(3y-7)-1(3y-7)=0$
$\therefore (3y-7)(8y-1)=0$
$\therefore 3y-7=0$ or $8y-1=0$
$\therefore y=\frac{7}{3}$ or $y=\frac{1}{8}$

$24\times 7=168$
$-56+(-3)=-59$
$-56(-3)=168$

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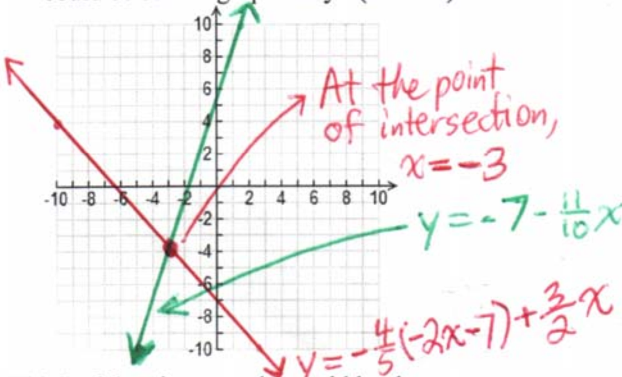
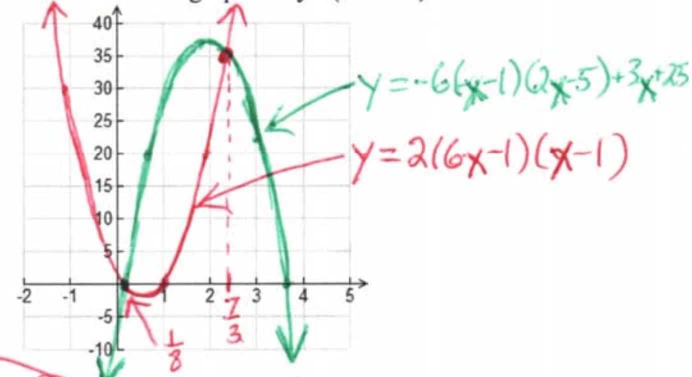
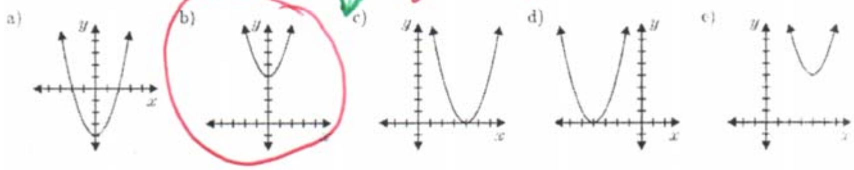
(c) Sketch a graph that shows how the equation in 4(a) could be solved graphically. **(2 TIPS)**



(d) Sketch a graph that shows how the equation in 4(b) could be solved graphically. **(2 TIPS)**



5. Which of the given graphs could be that of $y=x^2+4$? Explain. **(2 KU)**



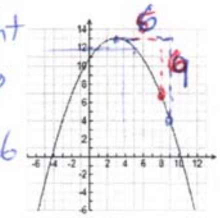
The correct graph is (b).
The graph of $y=x^2$
is translated up 4 units.

6. State an equation of the graph shown at the right. Explain. **(3 KU, 2 COM)**

Vertex: $(3,13)$ $\therefore h=3$ and $k=13$

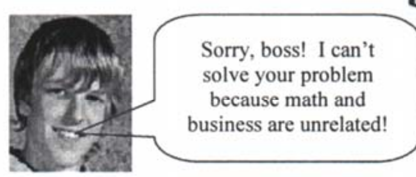
Also, $a=-\frac{1}{4}$ since the parabola opens downward and → 6 to the right from vertex, down 9 → $\frac{1}{4}$ of $6^2=36$ $\therefore a=-\frac{1}{4}$

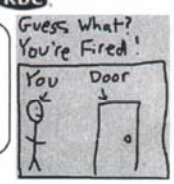
$\therefore y=-\frac{1}{4}(x-3)^2+13$ is an equation of the given parabola. //



7. RBC® has hired Gareth E. to calculate the number of insurance policies that should be sold to maximize profit. By carefully collecting and analyzing sales and marketing data, Gareth has determined that profit varies quadratically with the number of policies sold according to the function $P(x)=1500x-0.05x^2$. Note that $P(x)$ represents the profit obtained when x policies are sold. How many policies should be sold to maximize profit? **(4 TIPS, 3 COM)**

RBC Royal Bank





$P(x)=1500x-0.05x^2$

$\therefore P(x)=0.05x(30000-x)$

If we solve the equation $P(x)=0$, we obtain

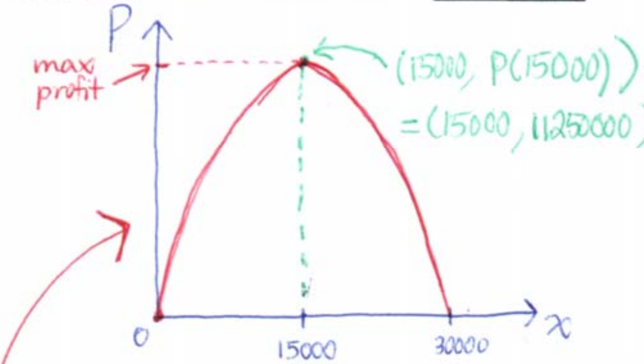
$0.05x(30000-x)=0$

$\therefore x=0$ or $x=30000$

$\therefore$ the x-co-ordinate of the vertex of the parabola is

$\frac{0+30000}{2}=15000.$

$\therefore$ 15000 policies must be sold to maximize profit. //



KU	APP	TIPS	COM
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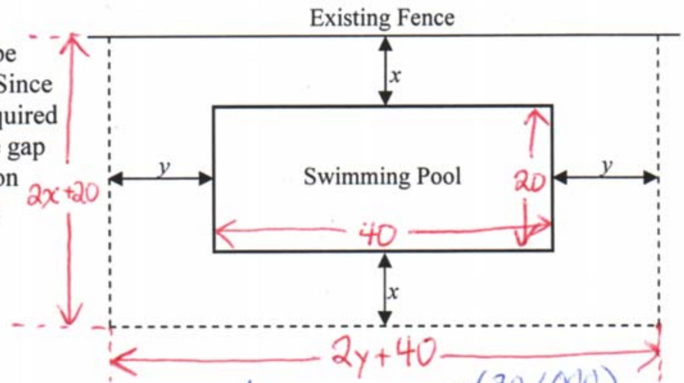
8. To help prevent drowning accidents, a protective fence is to be erected around a pool whose dimensions are 40 m by 20 m. Since there is an existing fence on one side, new fencing is only required around *three* sides of the pool (see diagram). In addition, the gap between the fence and the edge of the pool must be uniform on opposite sides of the pool (see diagram). If 200 m of fencing material is available, what is the maximum area that can be enclosed by the fencing? **(5 TIPS, 3 COM)**



Length of fence $= 2(2x+20) + 2y + 40 = 200$

$\therefore 4x + 2y + 80 = 200$

$\therefore 4x + 2y = 120$

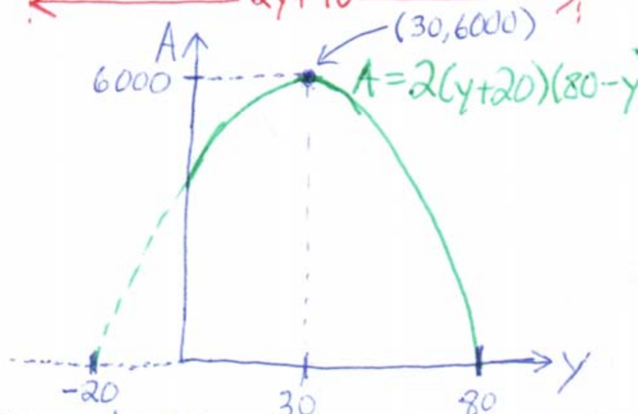
$\therefore 2x + y = 60$

Enclosed area $= A = (2x+20)(2y+40)$

Since $2x + y = 60$, $2x = 60 - y$

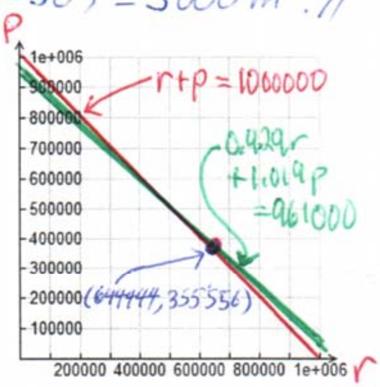
$\therefore A = (60 - y + 20)(2y + 40)$

$\therefore A = 2(y+20)(80-y)$



Since the zeros of this parabola are -20 and 80, the maximum occurs at the average of these values, $\frac{-20+80}{2} = 30$. Therefore, the maximum area that can be enclosed is $2(30+20)(80-30) = 5000\ m^2$. //

9. A certain gas station sells premium gasoline for \$1.019/L and regular gasoline for \$0.929/L. To suit the needs of price conscious customers, a "middle octane" gasoline, which is produced by mixing premium and regular gas, is also available. Suppose that 1000000 L of the "middle octane" gasoline is delivered to the gas station. If the "middle octane" gasoline sells for \$0.961/L, how many litres of regular and premium gas were used to produce the 1000000 L mixture? Use the grid at the right to show how this problem can be solved graphically. **(5 APP, 3 COM)**



Let r represent the number of litres of regular gasoline in the mixture and let p represent the number of litres of premium gasoline in the mixture. Then,

$r + p = 1000000$ ① (total # of litres = 1000000)

$0.929r + 1.019p = 0.961(1000000) = 961000$ ② (cost of regular + cost of premium = total cost)

From ①, $p = 1000000 - r$. Substituting in ②, we obtain

$0.929r + 1.019(1000000 - r) = 961000$

$\therefore -0.09r + 1019000 = 961000$

$\therefore -0.09r = -58000$

$\therefore r \doteq 644444$

$\therefore p \doteq 1000000 - 644444 = 355556$

There are about 644444 L of regular gas and about 355556 L of premium gas in the mixture. //

KU	APP	TIPS	COM
-0	-0	-0	-0