Part II Answers – MCR 3U0 Culminating Activity

1. See page 2, unit 0.
2. See page 34, unit 2

3(a) (i) x^{11} (ii) y^4 (iii) 1 (iv) $\frac{b^8}{a^7}$ (v) cannot be simplified (vi) cannot be simplified (vii) $\frac{b^{12}}{a^{18}}$ (viii) $m^{15}n^{20}$

3(b) (i) $169^{\frac{1}{2}} = \sqrt{169} = 13$ (ii) $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$ (iii) $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$
(iv) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$ (v) $243^{\frac{1}{5}} = \sqrt[5]{243} = 3$
(vi) $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$ (vii) $125^{\frac{4}{3}} = (\sqrt[3]{125})^4 = 5^4 = 625$
(viii) $32^{\frac{11}{5}} = (\sqrt[5]{32})^{11} = 2^{11} = 2048$

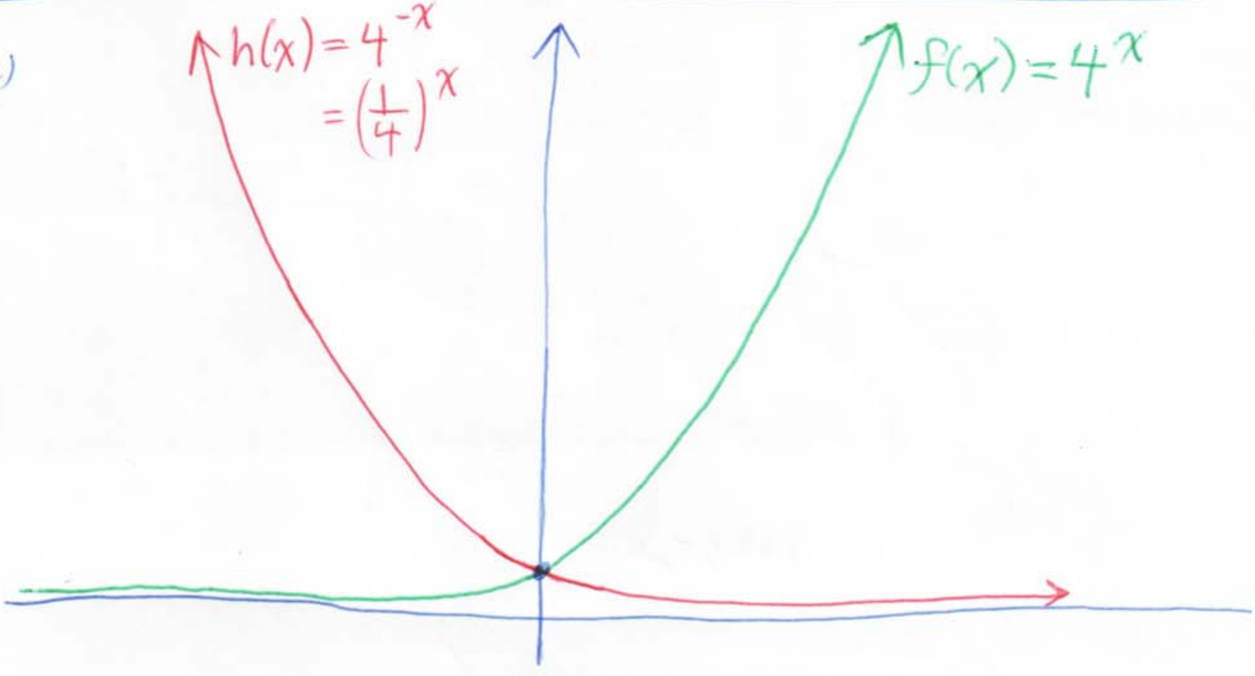
3(c) $\frac{x^a}{x^a} = 1$ But $\frac{x^a}{x^a} = x^{a-a} = x^0$

$\therefore x^0 = 1$

3(d) $\frac{x^0}{x^n} = \frac{1}{x^n}$ (using 4(b)) But $\frac{x^0}{x^n} = x^{0-n} = x^{-n}$

$\therefore x^{-n} = \frac{1}{x^n}$

3(e)



3f) (i) $f(x)=a^x$ passes through $(0,1)$ because $f(0)=a^0=1$

(ii) If $a>1$, the graph of $f(x)=a^x$ increases as x increases

(iii) If $0<a<1$, " " " " decreases " " "

(iv) Graph of $f(x)=a^x$ stays above x-axis because a^x is positive no matter what x is (as long as $a>0$).... Therefore, the x-axis is a horizontal asymptote

(v) For values of $a<0$, a^x is undefined for many values of x.

e.g. $(-2)^{\frac{1}{2}} = \sqrt{-2}$, which is undefined

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4. Use TI-Interactive

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5. $f(x)=2^x$, $g(x) = -1.3\left(2^{1.1(x+1)}\right)+27$

$= -1.3f\left(1.1(x+1)\right)+27$

Vertical
1. Stretch by factor of 1.3 and reflect in x-axis
2. Shift up 27 units

Horizontal
1. Compress by factor of $1.1^{-1} = \frac{1}{1.1}$
2. Shift 1 unit left

$$(x,y) \longrightarrow \left(\tfrac{1}{1.1}x-1,\ -1.3y+27\right)$$

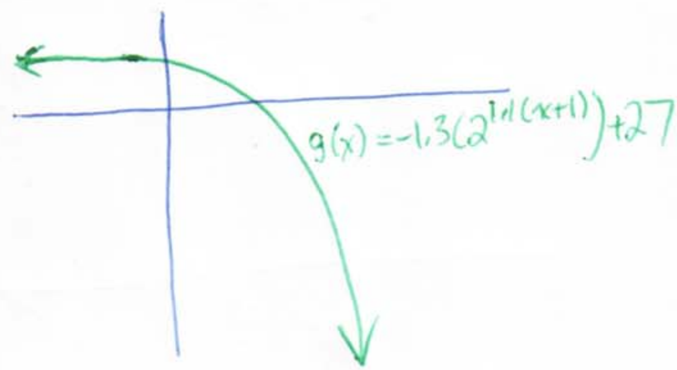
$f(x)=2^x$ Rounded to 2 decimal places

$(-1,\frac{1}{2}) \longrightarrow (-1.90,\ 26.35)$

$(0,1) \longrightarrow (-1,\ 25.7)$

$(1,2) \longrightarrow (-0.09,\ 24.4)$

$(2,4) \longrightarrow (0.81,\ 21.8)$



6. $4, 12, 36, 108, 324, \ldots$

(a) Recursive

$$\begin{cases} t_1 = 4 \\ t_n = 3t_{n-1}, \; n \in \mathbb{N}, \; n \geq 2 \end{cases}$$

OR

$$\begin{cases} f(1) = 4 \\ f(n) = 3f(n-1), \; n \in \mathbb{N}, \; n \geq 2 \end{cases}$$

Explicit

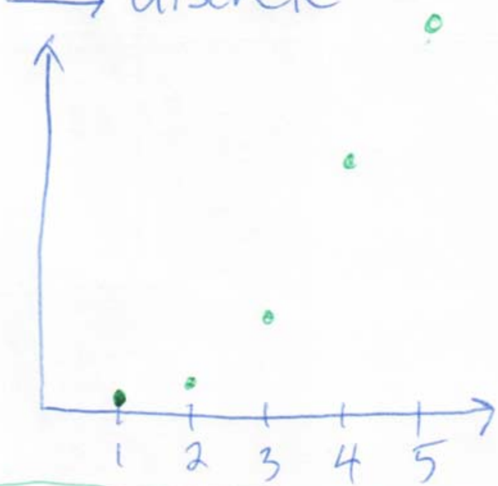
$$t_n = 4(3^{n-1})$$

OR

$$f(n) = 4(3^{n-1})$$

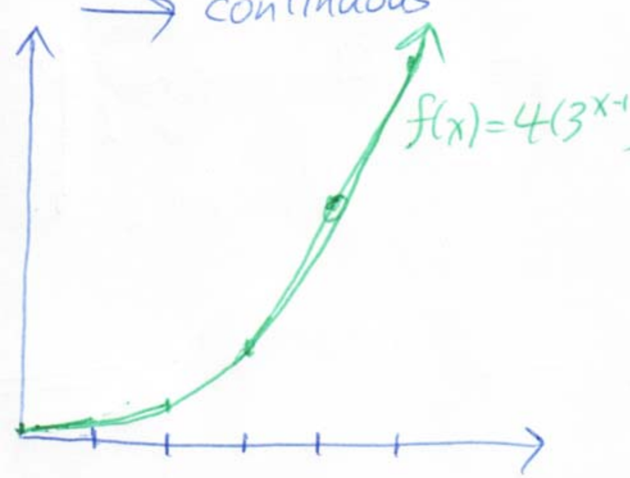
(b) Graph of Sequence

→ discrete



Graph of $f(x) = 4(3^{x-1})$

→ continuous



(c) Answers will vary

Not important for final exam.