MCR3U0 **Diagnostic Test** Name: Mr. Solutions

Part A: Fundamental Skills

1. *Evaluate* each expression. [15]

$-2+8\left[7-(2-5)^2\right]\div(-4)$

$=-2+8[7-(-3)^2]\div(-4)$

$=-2+8(7-9)\div(-4)$

$=-2+8(-2)\div(-4)$

$=-2+(-16)\div(-4)$

$=-2+4$

$=2$

5

$\left(\frac{2}{5}-1\frac{3}{4}\right)\div\left(\frac{-3}{2}\right)^2$

$=\left(\frac{2}{5}-\frac{7}{4}\right)\div\frac{9}{4}$

$=\left(\frac{8}{20}-\frac{35}{20}\right)\div\frac{9}{4}$

$=\frac{-27}{20}\div\frac{9}{4}$

$=\frac{-\cancel{27}^{\,3}}{\cancel{20}_{\,5}}\times\frac{\cancel{4}^{\,1}}{\cancel{9}_{\,1}}$

$=-\frac{3}{5}$

5

$\left(-4a^5b^3\right)^2\div\left(-2a^3b\right)^3$

$=16a^{10}b^6\div(-8a^9b^3)$

$=\frac{16a^{10}b^6}{-8a^9b^3}$

$=-2ab^3$

5

2. *Solve* each equation/system of equations. *Explain* the *graphical* (geometric) meaning of each solution. [16]

$-2(x+3)-4=5-3x$

$\therefore -2x-6-4=5-3x$

$\therefore -2x+3x=5+6+4$

$\therefore x=15$

4

$2x+5y+18=0$ (1)

$x+2y+6=0$ (2)

(2)×2, $2x+4y+12=0$ (3)

(1) − (3),

$y+6=0$

$\therefore y=-6$

Substituting in (2)

$x+2(-6)+6=0$

$\therefore x=6$

Therefore,

$x=6,\ y=-6$

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$2x+3=x^2-7x$

$\therefore x^2-7x-2x-3=0$

$\therefore x^2-9x-3=0$

$D=b^2-4ac=(+9)^2-4(1)(-3)$

$=81+12=93$

Since D is not a perfect square, the quadratic does not factor.

$\therefore x=\frac{9\pm\sqrt{(-9)^2-4(1)(-3)}}{2(1)}$

$\therefore x=\frac{9\pm\sqrt{93}}{2}$

4

Explanation: The lines $y=-2(x+3)-4$ and $y=5-3x$ intersect at a point whose x-co-ordinate is 15

1

Explanation: The lines $2x+5y+18=0$ and $x+2y+6=0$ intersect at the point $(6,-6)$

1

Explanation: The line $y=2x+3$ and the parabola $y=x^2-7x$ intersect at the points with x-co-ordinates $\frac{9+\sqrt{93}}{2}$ and $\frac{9-\sqrt{93}}{2}$

1

3. *Factor* each expression fully. [7]

$-2a^2 + 50$

$= -2(a^2 - 25)$

$= -2(a+5)(a-5)$

2

$x^2 - x - 6$

$= (x-3)(x+2)$

Rough Work:

$(-3)(2) = -6$

$-3 + 2 = -1$

2

$5a^2 + 7a - 6$

$= 5a^2 + 10a - 3a - 6$

$= 5a(a+2) - 3(a+2)$

$= (a+2)(5a-3)$

Rough:

$5(-6) = -30$

$10(-3) = -30$

$10 + (-3) = 7$

3

4. ***Sketch*** the graph of each relation. [13]

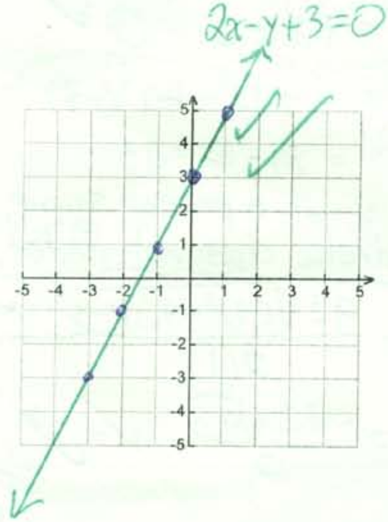
$2x - y + 3 = 0$

State slope and y-intercept.

$y = 2x + 3$

$\therefore$ slope $= m = 2$

and y-intercept $= b = 3$



slope $= 2 = \frac{2}{1} = \frac{\Delta y}{\Delta x}$

1 right, 2 up <u>OR</u>

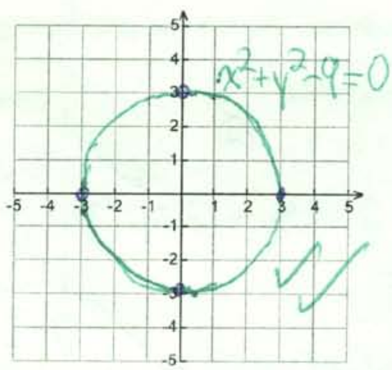
1 left, 2 down

4

$x^2 + y^2 - 9 = 0$

$\therefore x^2 + y^2 = 9$

Circle, radius = 3, centre (0, 0)



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$y = -2x^2 - 4x - 1$

State the co-ordinates of the vertex.

$y = -2x^2 - 4x - 1$

$= -2(x^2 + 2x) - 1$

$= -2(x^2 + 2x + 1^2 - 1^2) - 1$

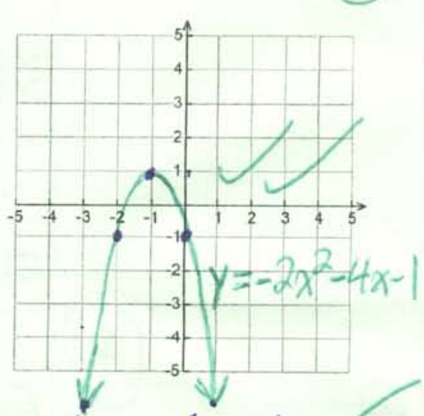
$= -2[(x+1)^2 - 1] - 1$

$= -2(x+1)^2 + 2 - 1$

$= -2(x+1)^2 + 1$

$\therefore y = -2(x+1)^2 + 1$

6

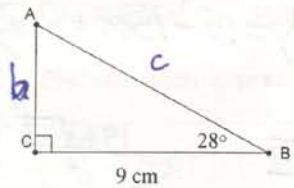


Vertex: $(-1, 1)$

opens downward

vertical stretch factor: 2

5. **Solve each triangle.[9]**



$\tan 28^\circ = \frac{b}{9}$

$\therefore b = 9\tan 28^\circ \doteq 4.79$

$c^2 = b^2 + 9^2$

$\therefore c^2 = (9\tan 28^\circ)^2 + 81$

$\therefore c = \sqrt{(9\tan 28^\circ)^2 + 81}$

$\doteq 10.19$

$\angle A = 90^\circ - 28^\circ = 62^\circ$

$\therefore b \doteq 4.79,\ c \doteq 10.19,\ \angle A = 62^\circ$

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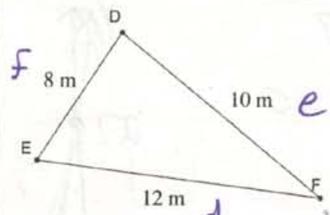

$d^2 = e^2 + f^2 - 2ef\cos D$

$\therefore \cos D = \frac{e^2+f^2-d^2}{2ef}$

$\therefore \cos D = \frac{10^2+8^2-12^2}{2(10)(8)}$

$\therefore \angle D = \cos^{-1}\left(\frac{10^2+8^2-12^2}{2(10)(8)}\right)$

$\doteq 82.8^\circ$

Similarly, $\cos F = \frac{d^2+e^2-f^2}{2de}$

$\therefore \angle F = \cos^{-1}\left(\frac{12^2+10^2-8^2}{2(12)(10)}\right)$

$\doteq 41.4^\circ$

$\therefore \angle E \doteq 180^\circ - 82.8^\circ - 41.4^\circ$

$= 55.8^\circ$

$\therefore \angle D \doteq 82.8^\circ,\ \angle E \doteq 55.8^\circ,\ \angle F \doteq 41.4^\circ$

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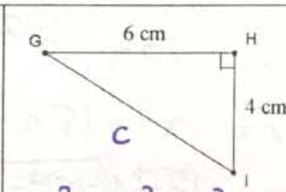


$c^2 = 6^2 + 4^2 = 36 + 16 = 52$

$\therefore c = \sqrt{52} \doteq 7.21$

$\tan \angle G = \frac{4}{6} = \frac{2}{3}$

$\therefore \angle G = \tan^{-1}\left(\frac{2}{3}\right) \doteq 33.7^\circ$

$\therefore \angle I = 90^\circ - \angle G$

$\doteq 90^\circ - 33.7^\circ$

$= 56.3^\circ$

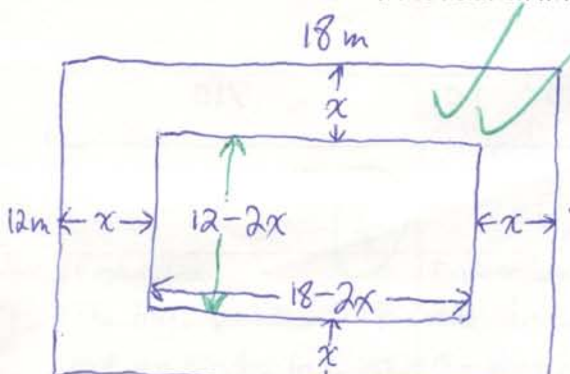
$\therefore c \doteq 7.21,\ \angle G \doteq 33.7^\circ,\ \angle I \doteq 56.3^\circ$

3

Part B: Conceptual Skills.

1. A landscaper wishes to plant a border of tulips **within** a rectangular garden having dimensions 18 m by 12 m. To obtain the desired appearance, the area of the tulip border should be half of the area of the entire garden. How wide should the border be? **[10]**

   - Include a fully labelled sketch of the garden including the tulip border.
   - Write an equation that models this problem.
   - State what quantity each variable represents.
   - Include a sketch of the relation with axes properly labelled.



Let x represent the width of the border

Total area $= (18\text{m})(12\text{m}) = 216\text{ m}^2$

Area of border $= 216 - (12-2x)(18-2x)$

$= 216 - (216 - 60x + 4x^2)$

$= 60x - 4x^2$ (continued →)

$0 < x < 12$ → The width of the border must be greater than 0 but less than the width of the entire garden area of border $= \frac{1}{2}$ of total area

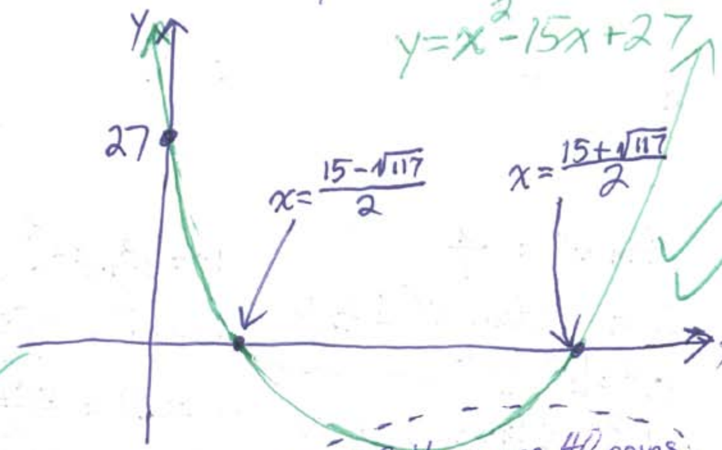
$\therefore\ 60x - 4x^2 = 108$ (divide both sides by 4)

$\therefore\ 15x - x^2 = 27$

$\therefore\ x^2 - 15x + 27 = 0$

$\therefore\ x = \dfrac{15 \pm \sqrt{(-15)^2 - 4(1)(27)}}{2(1)}$

$= \dfrac{15 \pm \sqrt{117}}{2}$



inadmissable

$\therefore\ \cancel{x \doteq 12.9}$ or $x \doteq 2.1$

Since $0 < x < 12$, $x \doteq 2.1$

The border should have a width of about 2.1 m.

If there are 40 coins altogether,

2. A jar contains several dimes and quarters having a total value of \$8.50. How many dimes and how many quarters are in the jar? **[5]**

Let d represent the number of dimes. Then, the number of quarters must be $40 - d$.

Now, (value of dimes) + (value of quarters) = 8.5

$\therefore\ 0.1d + 0.25(40 - d) = 8.5$

$\therefore\ 0.1d + 10 - 0.25d = 8.5$

$\therefore\ -0.15d = -1.5$

$\therefore\ d = 10$ and $40 - d = 30$

5

There are 30 quarters and 10 dimes.

K 60/60	~~/65~~	T	15/15	C	10/10

Recommendation: