Superb work Mr. S.!!

MCR 3U0 **Unit 1 Test Introductions to Functions** Date: _Oct. 6, 2010_

Name: _Mr. Solutions_

| K | 16 /16 | A | 20 /20 | T | 13 /13 | C | 10 /10 |

Multiple Choice (Knowledge – One Mark Each)

*Identify the choice that **best** completes the statement or answers the question.*

1. __b__ Which relation *is* a function?

   a. $\{(0, 1), (3, 2), (5, -3), (0, 2)\}$

   b. $\{(-3, -2), (-1, 3), (0, -2), (3, 4)\}$

   c. $\{(-4, -7), (-9, 5), (4, -2), (-9, 0)\}$

   d. $\{(-7, -7), (-2, 5), (-1, 6), (-2, -5)\}$

2. __b__ Which of the following relations is *not* a function?

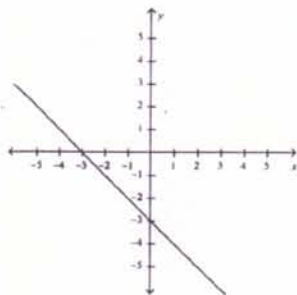   a. The relation between height and time if a tree grows 4 cm/yr

   b. The relation between students' heights and weights

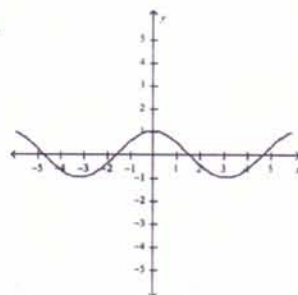   c. The relation between distance and time if a car travels 85 km/h

   d. The relation between money earned and time if interest for a bank account is 5% per month
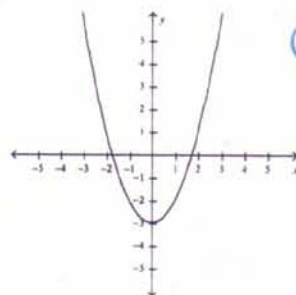
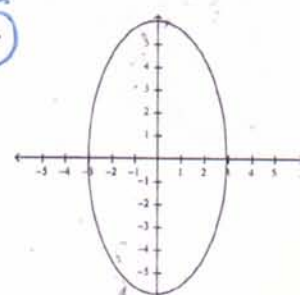3. __d__ Which is *not* the graph of a function?

   a.   b.   c.   d.

4. __a__ Given $f(x) = -14 - 3x$, evaluate $f(-4)$.

   a. -2 b. 2 c. -30 d. 68

   $f(-4) = -14 - 3(-4)$
   $= -14 + 12 = -2$

5. __a__ Tanya thought of a number. She doubled the number and subtracted 15 from the result. She then multiplied the difference by the original number. Which function represents Tanya's number?

   a. $f(x) = x(2x - 15)$ b. $f(x) = x(15 - 2x)$ c. $f(x) = x(x^2 - 15)$ d. $f(x) = x(15 - x^2)$

6. __d__ What are the domain and range of the function $f(x) = \sqrt{x - 7} + 3$?

   a. Domain $= \mathbb{R}$
      Range $= \mathbb{R}$

   b. Domain $= \{x \in \mathbb{R} : x \geq 7\}$
      Range $= \{y \in \mathbb{R} : y \leq 3\}$

   c. Domain $= \{x \in \mathbb{R} : x \leq 7\}$
      Range $= \{y \in \mathbb{R} : y \leq 3\}$

   d. Domain $= \{x \in \mathbb{R} : x \geq 7\}$
      Range $= \{y \in \mathbb{R} : y \geq 3\}$

7. __b__ A pre-paid cellular phone plan charges \$35 for activation and \$0.03 per minute. The relation of cost, in dollars, to time, in minutes, is described by the function $C(t)=35+0.03t$. What is the inverse of the function?

   a. $C^{-1}(t)=\dfrac{1}{35+0.03t}$ b. $C^{-1}(t)=\dfrac{t-35}{0.03}$ c. $C^{-1}(t)=\dfrac{1}{35}+\dfrac{1}{0.03t}$ d. $C(t)=-(35+0.03t)$

8. __C__ Which of the following transformations is required to graph $y=|4x-8|$ from its parent function?

   a. Compress the graph of $f(x)=|x|$ horizontally by the factor $\dfrac{1}{4}$, followed by a translation 8 units right.

   b. Stretch the graph of $f(x)=|x|$ horizontally by the factor 4, followed by a translation 8 units left.

   c. Compress the graph of $f(x)=|x|$ horizontally by the factor $\dfrac{1}{4}$, followed by a translation 2 units right.

   d. Stretch the graph of $f(x)=|x|$ horizontally by the factor 4, followed by a translation 2 units left.

9. __a__ In the graph shown, the parent function is the dotted graph. The solid graph has the equation of the form $y=f(kx)$ Which of the following is the equation for the solid graph?

   a. $f(x)=\left|\dfrac{1}{3}x\right|$

   b. $f(x)=|3x|$

   c. $f(x)=\dfrac{3}{x}$

   d. $f(x)=\left(\dfrac{1}{3}x\right)^{2}$
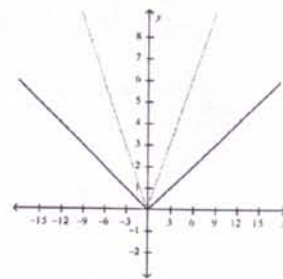
10. __b__ Which of the following transformations is required to graph $y=\sqrt{-(x-6)}$ from its parent function?

   a. Reflect the graph of $f(x)=\sqrt{x}$ in the x-axis, then translate it 6 units to the left.
   b. Reflect the graph of $f(x)=\sqrt{x}$ in the y-axis, then translate it 6 units to the right.
   c. Reflect the graph of $f(x)=\sqrt{x}$ in the y-axis, then translate it 6 units to the left.
   d. Reflect the graph of $f(x)=\sqrt{x}$ in the x-axis, then translate it 6 units to the right.

11. __C__ The point $(2,-6)$ lies on the graph of $y=f(x)$. What are the co-ordinates of the image of $(2,-6)$ on the graph of $y=-f(-3x)$?

   a. $(-24,6)$ b. $\left(-\dfrac{2}{3},-6\right)$ c. $\left(-\dfrac{2}{3},6\right)$ d. $(-2,18)$

Knowledge (Written Responses)

12. Consider the function $f(x)=-3(x-7)^{2}-2$.
    Determine $f(3s)$. (/2)

$$f(3s)=-3(3s-7)^{2}-2$$
$$=-3(9s^{2}-42s+49)-2$$
$$=-27s^{2}+126s-149$$

13. For $h(t)=-5t-7$ determine $h^{-1}(4)$.   (/3)

   Let $y=-5t-7$
   Interchange y and t:
   $$t=-5y-7$$
   $$\therefore y=\dfrac{t+7}{-5}=-\dfrac{1}{5}(t+7)$$
   $$\therefore h^{-1}(t)=-\dfrac{1}{5}(t+7)$$
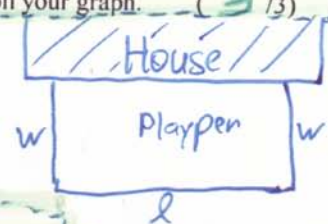 $$\therefore h^{-1}(4)=-\dfrac{1}{5}(4+7)=\dfrac{-11}{5}$$

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Application

14. Ashutosh has 110 m of fencing to make a rectangular playpen for Eslam. The wall of Ashutosh's house will serve as one side of the playpen, so he will use fencing for only three sides of the rectangular area.

a) Express the area of the playpen as a function of its *length*.   (**3** /3)

b) Sketch a graph that shows the area as a function of *length*.   (**2** /2)

c) Determine the domain and range of the area function.   (**2** /2)

d) Determine the dimensions of the playpen that give a total area of 120 m². Show this on your graph.   (**3** /3)

e) Determine the dimensions that give the *maximum area*. (**3** /3)

(13)

(a) $A = \ell w = \ell(55 - \frac{1}{2}\ell) = 55\ell - \frac{1}{2}\ell^2$

$\therefore A(\ell) = 55\ell - \frac{1}{2}\ell^2$ ✓ (3)

(b) See graph at right

(c) $D = \{\ell \in \mathbb{R} \mid 0 < \ell < 110\}$ ✓ $R = \{A \in \mathbb{R} \mid 0 < A \le 1512.5\}$

$\ell + 2w = 110$
$\therefore 2w = 110 - \ell$
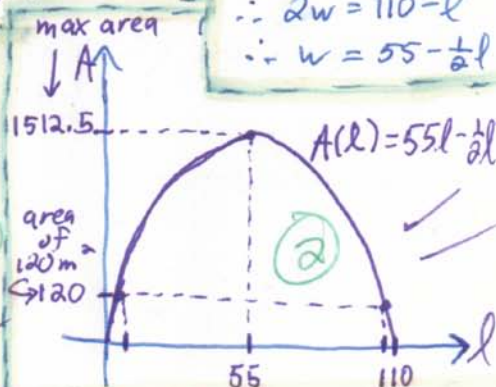$\therefore w = 55 - \frac{1}{2}\ell$

(d) $A(\ell) = 120$

$\therefore -\frac{1}{2}\ell^2 + 55\ell - 120 = 0$ ✓

$\therefore \ell^2 - 110\ell + 240 = 0$

$\therefore \ell = \frac{110 \pm \sqrt{110^2 - 4(1)(240)}}{2}$

$\therefore \ell \doteq 107.8$ or $\{\ell \doteq 2.2$ ✓
$w \doteq 1.1$ $\}$ $\{w \doteq 53.9$

(e) From the graph it's clear that for maximum area, $\ell = 55$. Therefore,

$w = 55 - \frac{1}{2}\ell$
$= 55 - \frac{1}{2}(55)$
$= 27.5$ ✓

max area
$\downarrow A$
1512.5
area of 120 m² → 120
55   110
$A(\ell) = 55\ell - \frac{1}{2}\ell^2$

House
w Playpen w
ℓ

15. Complete the steps outlined in the table to sketch the graph of the function $g(x) = \frac{9}{4}\sqrt{-2x+14} - 5$.   (**10** /10)

Equation of Parent Function	Transformations of $y = f(x)$ Required to Obtain $y = g(x)$	Graphs of $y = f(x)$ and $y = g(x)$
	Verbal Description $g(x) = \frac{9}{4}\sqrt{-2(x-7)} - 5$	
	Horizontal	
	1. Compress by $\frac{1}{2}$, reflect in y-axis ✓	
	2. Translate 7 units right ✓	
	Vertical	
	1. Stretch by factor $\frac{9}{4}$ ✓	
$f(x) = \sqrt{x}$	2. Translate 5 units down	
	Function Notation Description $g(x) = \frac{9}{4}f(-2x+14) - 5$ ✓	
	Mapping Notation Description $(x,y) \rightarrow (-\frac{1}{2}x+7, \frac{9}{4}y - 5)$ ✓	

f g

$(0,0) \rightarrow (7, -5)$

$(1,1) \rightarrow (\frac{13}{2}, \frac{-11}{4})$

$(4,2) \rightarrow (5, -\frac{1}{2})$

$(9,3) \rightarrow (\frac{5}{2}, \frac{7}{4})$

$(16,4) \rightarrow (-1, 4)$

3 ✓

Thinking

16. Transformations are applied to the function $f(x) = \dfrac{-3}{2x}$ to produce the function g. Write the equation of $y = g(x)$ after each of the following transformations of f: (/5)

a) horizontal compression by the factor $\frac{1}{2}$: $\quad g(x) = f(2x) = \dfrac{-3}{2(2x)} = \dfrac{-3}{4x}$ ✓

b) vertical stretch by the factor 3: $\quad g(x) = 3f(x) = 3\left(\dfrac{-3}{2x}\right) = \dfrac{-9}{2x}$ ✓

c) reflection in the x-axis: $\quad g(x) = -f(x) = -\left(\dfrac{-3}{2x}\right) = \dfrac{3}{2x}$ ✓

d) translation 10 units right and 1 unit up: $\quad g(x) = f(x-10) + 1 = \dfrac{-3}{2(x-10)} + 1$ ✓
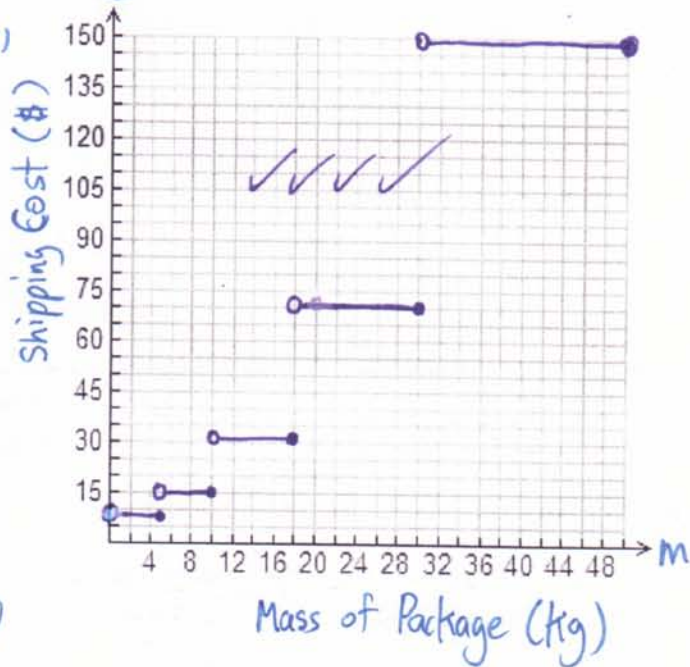
17. A shipping company uses the mass of a package to determine the shipping charges. For packages with a mass up to and including 5 kg, the cost is $8.50. For packages with a mass greater than 5 kg but less than or equal to 10 kg, the cost is $14.95. For packages with a mass greater than 10 kg but less than or equal to 18 kg, the cost is $31.50. For packages with a mass greater than 18 kg but less than or equal to 30 kg, the cost is $70.85. Finally, for packages weighing more than 30 kg but less than or equal to 50 kg, the cost is $149.95.

a) Graph the situation on a coordinate grid. Label your graph fully. (4 /4)

b) State the domain and range of the relation. (2 /2)

c) Explain why this relation must be a function (2 /2) C

(4)

(b) $D = \{m \in \mathbb{R} \mid 0 < m \leq 50\}$ ✓

$R = \{8.5, 14.95, 31.5, 70.85, 149.95\}$ ✓

(c) For any mass between 0 kg and 50 kg, there is a unique shipping cost. ✓ (i.e., there cannot be two or more different prices for the same mass)

Shipping Cost ($) vs Mass of Package (kg)

Labels → communication

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