MCR 3U0 Unit 2 Test Rational Functions **Date:** _____

Name: Mr. Solutions

K 24/24	A 9/9	T 9/9	C 10/10

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. b A diver standing on a platform 8 metres above a pool jumps off the platform. The function $h(t) = -5t^2 + t + 8$ describes the diver's height in metres, t seconds after jumping. The diver's velocity is described by the function $v(t) = -10t + 1$. Which of the following is the simplified form of the expression $h(t)v(t)$?

   a. $h(t)v(t) = -5t^2 - 9t + 9$ **b.** $h(t)v(t) = 50t^3 - 15t^2 - 79t + 8$

   c. $h(t)v(t) = 50t^3 - 10t^2 + 8$ d. $h(t)v(t) = 50t^3 - 15t^2 - 79t$

   $(-10t+1)(-5t^2+t+8)$

   $= 50t^3 - 10t^2 - 80t - 5t^2 + t + 8$

2. c Which pair of functions is equivalent?

   a. $f(x) = 10x^2 + 9x + 8$, $g(x) = 8x^2 + 10x + 9$

   b. $f(x) = x^2 - 3x + 5$, $g(x) = x^2 + 3x - 5$

   c. $f(x) = 18x - 24$, $g(x) = 6(3x - 4)$

   d. $f(x) = (5x - 7) + (-2x + 1) + 4x$, $g(x) = (x + 4) - (-4x + 2) + 2x$

3. b Simplify $(11v^2 - 6vw - 3w^2) - (-7v^2 + vw + 13w^2)$.

   a. $2(vw)^2 - 7vw$ **b.** $18v^2 - 7vw - 16w^2$ c. $18v^2 - 5vw + 10w^2$ d. $4v^2 - 5vw + 10w^2$

4. d A rectangular dining room has an area that is described by the function $A(x) = 3x^2 + 17x + 20$. If the width of the room is $x + 5$, what is the length?

   a. $(3x + 5)$ b. $(3x + 10)$ c. $(3x + 15)$ **d.** $(3x + 4)$

5. d Simplify $\dfrac{18t^4 - 24t^2 - 6t}{18t^2} =$ $\dfrac{6t(3t^3 - 4t - 1)}{3 \cdot 18t^2}$

   a. $t^2 - 4t - 1$ b. $-23t^2 - 6t$ c. $\dfrac{t^3 - 4t - 1}{t}$ **d.** $\dfrac{3t^3 - 4t - 1}{3t}$

6. d What are the restrictions on the variable for the rational expression $\dfrac{d^2 + 10d + 25}{5d^2 - 25d}$?

   a. $d \neq -5, d \neq 0, d \neq 5$ b. $d \neq 5$ c. $d \neq 0$ **d.** $d \neq 0, d \neq 5$

7. Simplify $(10q^2 + 8r^2 - 18) + (15q^2 - 32 + 9r^2)$. **(K2)**

   $= 10q^2 + 15q^2 + 8r^2 + 9r^2 - 18 - 32$

   $= 25q^2 + 17r^2 - 50$

8. Simplify $\dfrac{7x^2 - 28y^2}{x^2 + 8xy + 12y^2}$ and state any restrictions on the variables. **(K4)**

   $\dfrac{7x^2 - 28y^2}{x^2 + 8xy + 12y^2} = \dfrac{7(x^2 - 4y^2)}{(x + 6y)(x + 2y)}$

   $= \dfrac{7(x - 2y)(x + 2y)}{(x + 6y)(x + 2y)}$

   $= \dfrac{7(x - 2y)}{x + 6y}$, $x \neq -6y$, $x \neq -2y$

9. Simplify $\frac{4-x}{3x^2-4x-4} \div \frac{5x-20}{6x^2-17x+10}$ and state any restrictions on the variables. **(K6)**

$$\frac{4-x}{3x^2-4x-4} \div \frac{5x-20}{6x^2-17x+10}$$

$$= \frac{-(x-4)}{(3x+2)(x-2)} \times \frac{(x-2)(6x-5)}{5(x-4)}$$

$$= \frac{-(6x-5)}{5(3x+2)}$$

$$= \frac{5-6x}{5(3x+2)}$$

$x \neq -\frac{2}{3}, \frac{5}{6}, 2, 4$

10. Simplify $\frac{5k}{k^2-k-6} + \frac{4}{k^2+4k+4}$ and state any restrictions on the variables. **(K6)**

$$\frac{5k}{k^2-k-6} + \frac{4}{k^2+4k+4}$$

$$= \frac{5k(k+2)}{(k+2)(k-3)(k+2)} + \frac{4(k-3)}{(k+2)^2(k-3)}$$

$$= \frac{5k^2+10k}{(k+2)^2(k-3)} + \frac{4k-12}{(k+2)^2(k-3)}$$

$$= \frac{5k^2+14k-12}{(k+2)^2(k-3)}$$

$k \neq -2, 3$

Application

11. Michelle owns a company that sells gift wrap and cards. The function $R(x) = -9x^2 + 60x + 1450$ models her company's revenue, for x weeks. The function $E(x) = 2x^2 - 10x + 25$ models the company's production expenses.

a) Write the simplified form of the profit function, $P(x) = R(x) - E(x)$. **(A3)**

$$= -9x^2+60x+1450-(2x^2-10x+25)$$

$$= -9x^2-2x^2+60x+10x+1450-25$$

$$= -11x^2+70x+1425$$

b) How much profit will the company make after 4 weeks? **(A2)**

$$P(4) = -11(4)^2+70(4)+1425$$

$$= -11(16)+280+1425$$

$$= 1529$$

After four weeks, the company will have a profit of about $1500.

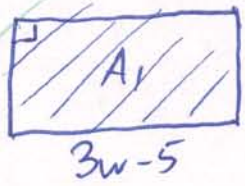
12. The length of a rectangle is 5 less than triple the width. Determine how the area ***changes*** if the length of the rectangle is increased by 1 and the width is decreased by 4. Show your work. **(A4)**

Let A_1 represent the area of the first rectangle and A_2 represent the area of the second rectangle.

Then $A_1 = w(3w-5)$ $A_2 = (w-4)(3w-4)$

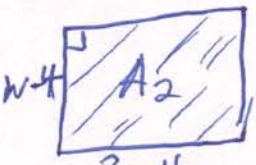
The change in area, ΔA, is found as follows:

Before



$$\Delta A = A_2 - A_1$$
$$= (w-4)(3w-4) - w(3w-5)$$
$$= 3w^2 - 16w + 16 - 3w^2 + 5w$$
$$= -11w + 16$$

After



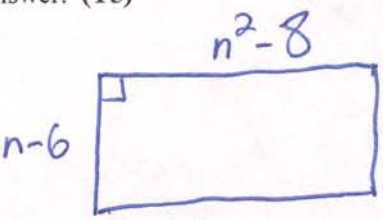
3w-5+1

Thinking

13. The area of a rectangle is described by the function $A(n) = n^3 - 6n^2 - 8n + 48$. What is a possible perimeter of the rectangle? Is this the only possible perimeter? Describe how you found your answer. **(T5)**

$$A(n) = n^3 - 6n^2 - 8n + 48$$
$$= n^2(n-6) - 8(n-6)$$
$$= \underbrace{(n-6)}_{\text{width}}\underbrace{(n^2-8)}_{\text{length}}$$

* factoring by grouping *

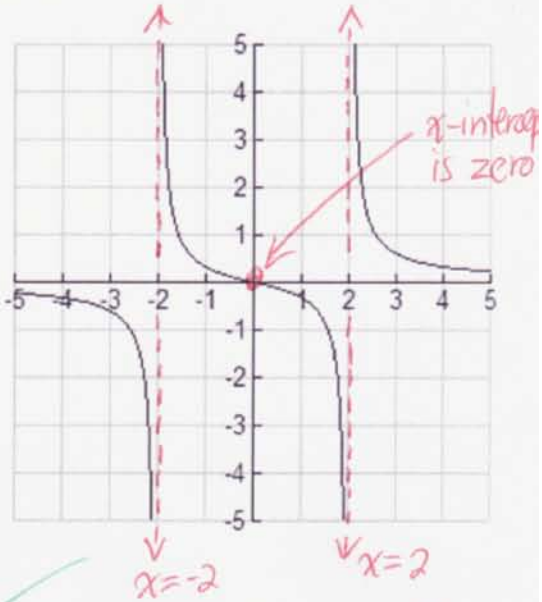


By writing A(n) in factored form, we see that the dimensions of the rectangle COULD BE $(n-6) \times (n^2-8)$. In this case, the perimeter, P, is given by the following:

$$P = 2(n-6) + 2(n^2-8) = 2n^2 + 2n - 28$$

This is NOT the only possible perimeter because a rectangle of a given area has infinitely many different dimensions.

14. State a possible equation of the rational function whose graph is shown at the right. Justify your answer. **(T4)**



This function appears to have

(a) a vertical asymptote with equation $x=-2$

(b) a vertical asymptote with equation $x=2$

(c) an x-intercept of zero

The function $f(x) = \dfrac{x}{(x+2)(x-2)}$ possesses all three of these features. Therefore, a possible equation is

$$f(x) = \frac{x}{(x+2)(x-2)}$$

Note: To ensure that the function has the vertical asymptotes stated above, the numerator CANNOT have the factors $x+2$ or $x-2$. If this were the case, the graph would have holes instead of asymptotes.