MCR3U0 Semester 1, 2010 - 2011

Grade 11 Functions (University Preparation)
Unit 3 – Quadratics – Major Test

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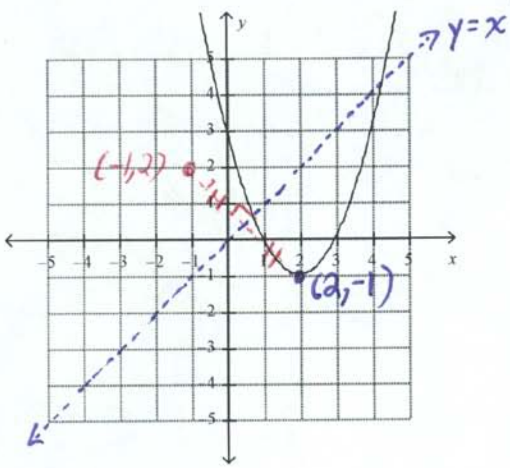
Name: Mr. Solutions

KU	APP	TIPS	COM
/20	/16	/18	/10

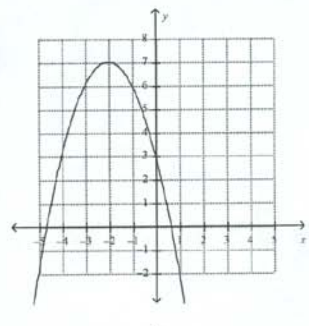
***Multiple Choice* (6 KU)**

For questions 1 to 6, select the best answer. Write the letter of your choice in the provided blank space.

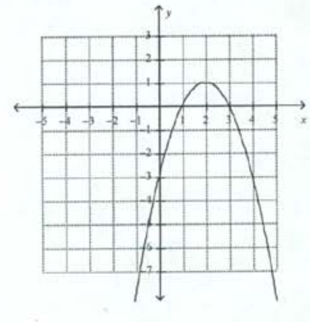
1. d Which of the following is the graph of the ***inverse*** of the quadratic function whose graph is shown at the left?



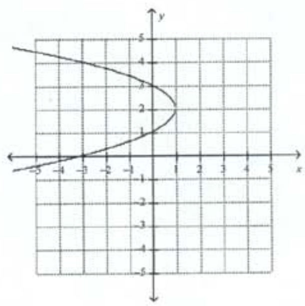
(a)



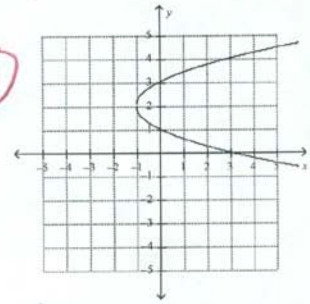
(b)



(c)



(d)



2. b In which of the following is the relation between x and y ***quadratic***?

(a)

x	y	
−2	23	
−1	14	−9
0	5	−9
1	−4	−9
2	−13	−9

linear

(b)

x	y	1st	2nd
−2	9	–	–
−1	6	−3	–
0	5	−1	2
1	6	1	2
2	9	3	2

quadratic (2nd differences constant)

(c)

x	$y = x^3$
−2	−8
−1	−1
0	0
1	1
2	8

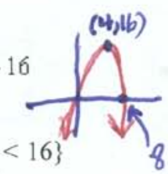
cubic

(d)

x	y	
−2	−6	
−1	−2	+4
0	2	+4
1	6	+4
2	10	+4

linear

3. c Kelly has 16 m of fencing to be used to enclose her flower garden. The function $f(x) = -(x-4)^2 + 16$ models the ***area*** of the garden, where x is the ***length of fencing*** in metres. What is the ***domain*** of f?



(a) $\{x \in \mathbb{R} : 1 \leq x \leq 4\}$ (b) $\{x \in \mathbb{R} : -4 \leq x \leq 4\}$ (c) $\{x \in \mathbb{R} : 0 < x < 8\}$ (d) $\{x \in \mathbb{R} : 0 < x < 16\}$

4. c Given, $f(x) = -(x+4)^2 - 7$ what are the domain and range of the ***inverse***?

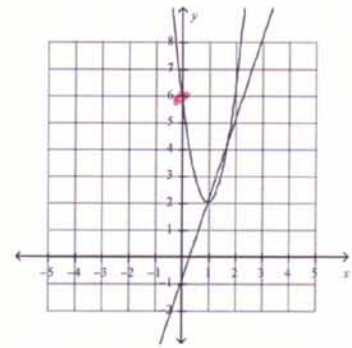
(a) $D = \{x \in \mathbb{R}\}$, $R = \{y \in \mathbb{R}\}$

(b) $D = \{x \in \mathbb{R}\}$, $R = \{y \in \mathbb{R} \mid y \geq -7\}$

(c) $D = \{x \in \mathbb{R} \mid x \leq -7\}$, $R = \{y \in \mathbb{R}\}$

(d) $D = \{x \in \mathbb{R}\}$, $R = \{y \in \mathbb{R} \mid y \leq -7\}$

5. a Which of the following systems of functions models the points of intersection shown in the graph?



(a) $f(x) = 4x^2 - 8x + 6$
$g(x) = 3x - 1$

(b) $f(x) = 2x^2 - 4x + 3$
$g(x) = 3x - 1$
can be eliminated because y-int of f is 3

~~(c)~~ $f(x) = -4x^2 + 8x - 6$
$g(x) = 3x - 1$

~~(d)~~ $f(x) = -2x^2 + 8x - 3$
$g(x) = 3x - 1$
can be eliminated since parabola opens upward

6. d How many ***points of intersection*** do the functions $f(x) = 9x^2 + 12.5x + 8.9$ and $g(x) = -6x - 3.5$ have?

(a) More than two (b) 2 (c) 1 (d) 0

$9x^2 + 12.5x + 8.9 = -6x - 3.5 \Rightarrow 9x^2 + 18.5x + 12.4 = 0$

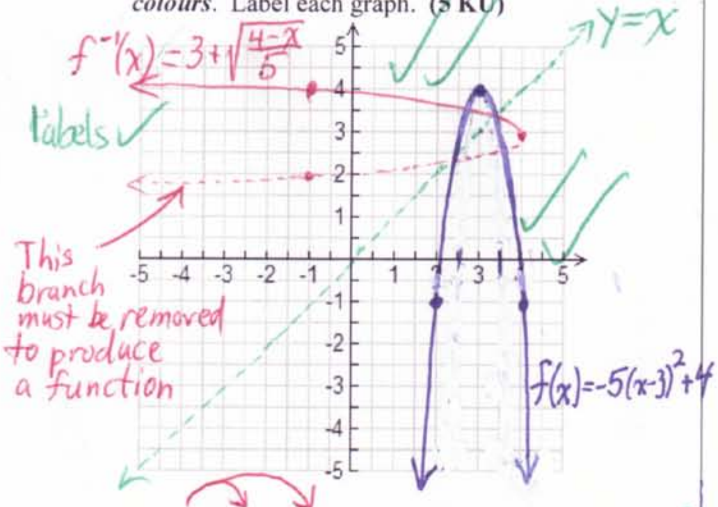
$b^2 - 4ac = (18.5)^2 - 4(9)(12.4) = -104.15 < 0$

Full Solutions

Write complete solutions for each of the following problems.

7. Given the quadratic function $f(x) = -5(x-3)^2 + 4$, do the following: **(10 KU altogether)**

(a) On the provided grid, accurately sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ ***using two different colours***. Label each graph. **(5 KU)**



(b) Determine an equation of $y = f^{-1}(x)$. **(5 KU)**

Interchange x and y:

$x = -5(y-3)^2 + 4$

$\therefore x - 4 = -5(y-3)^2$

$\therefore \frac{x-4}{-5} = (y-3)^2$ multiply numerator and denominator by -1

$\therefore (y-3)^2 = \frac{4-x}{5}$

$\therefore y - 3 = \pm\sqrt{\frac{4-x}{5}}$

$\therefore y = 3 \pm \sqrt{\frac{4-x}{5}}$

$\therefore y = 3 + \sqrt{\frac{4-x}{5}}$ or $y = 3 - \sqrt{\frac{4-x}{5}}$

To ensure that the inverse relation is a function, we must choose only one of these.

$\therefore f^{-1}(x) = 3 + \sqrt{\frac{4-x}{5}}$ $(x \leq 4)$

(Note: It is customary to choose the upper branch of the "sideways parabola" obtained upon reflection in the line $y = x$.)

8. Write $4\sqrt{5}\left(7\sqrt{15} - \sqrt{10}\right) - 12\sqrt{3}$ in ***simplest form***. **(4 KU)**

$= 28\sqrt{75} - 4\sqrt{50} - 12\sqrt{3}$

$= 28\sqrt{25 \times 3} - 4\sqrt{25 \times 2} - 12\sqrt{3}$

$= 28\sqrt{25}\sqrt{3} - 4\sqrt{25}\sqrt{2} - 12\sqrt{3}$

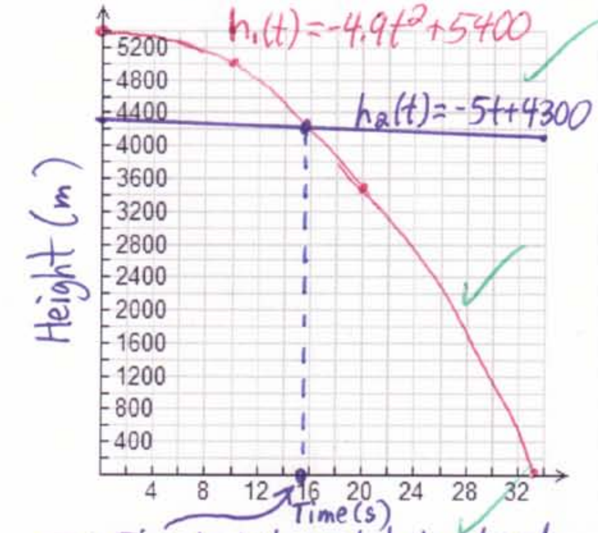
$= 28(5\sqrt{3}) - 4(5\sqrt{2}) - 12\sqrt{3}$

$= 140\sqrt{3} - 12\sqrt{3} - 20\sqrt{2}$

$= 128\sqrt{3} - 20\sqrt{2}$

9. Pallavi parachutes out of airplanes to fight fires. After jumping from an airplane, her height above the ground in metres is modelled by the quadratic function $h_1(t) = -4.9t^2 + 5400$, where t is the time in seconds since she jumped. After she releases her parachute, her height above the ground in metres is given by the linear function $h_2(t) = -5t + 4300$, where t is also the time in seconds since she jumped. **(8 APP)**

(a) Represent the situation graphically. ***Do not forget to label the axes and the graphs!***



→ Time at which parachute is released

(b) Use your graphs to ***estimate*** the time at which Pallavi released her parachute. Show your estimate on the above grid.

- Just under 16 seconds after jumping

(c) How long after jumping did Pallavi release her parachute? Use an ***algebraic method*** to determine this.

When Pallavi releases her parachute, $h_1(t) = h_2(t)$.

$\therefore -4.9t^2 + 5400 = -5t + 4300$

$\therefore -4.9t^2 + 5t + 1100 = 0$

$\therefore t = \dfrac{-5 \pm \sqrt{5^2 - 4(-4.9)(1100)}}{2(-4.9)}$

$\therefore t \doteq -14.5$ or $t \doteq 15.5$

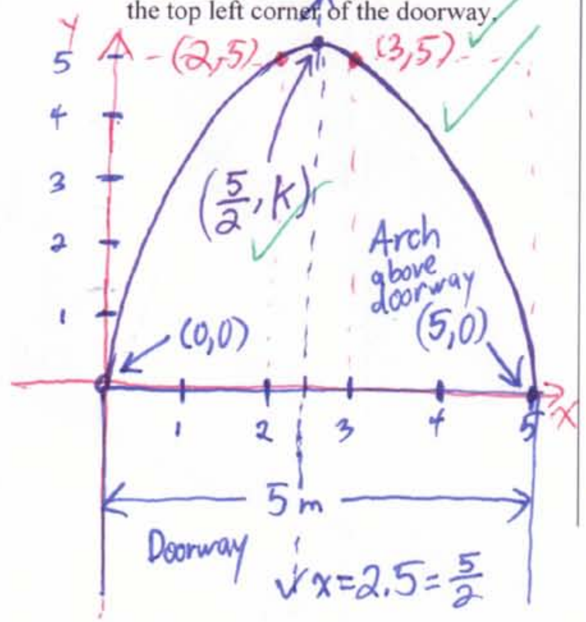
Since $t \geq 0$, $t \doteq 15.5$

Pallavi released her parachute about 15.5 seconds after jumping.

(My estimate was very good!)

10. Nabeel is making a ***parabolic*** arch at the top of his garage doors. The doorway is 5 m wide and the height of the arch, 2 m from the edge of the doorway, is 5 m. **(8 APP)**

(a) Sketch a diagram of the doorway and the parabolic arch. Include a co-ordinate system with the origin placed at the top left corner of the doorway.



(b) Determine an equation that describes the shape of the parabolic arch. Show your work!

Since the zeros are 0 and 5, the x-co-ordinate of the vertex must be $\frac{5}{2}$.

$\therefore$ the equation must take the form

$$y = a\left(x - \tfrac{5}{2}\right)^2 + k$$

$\because$ the points (5,0) and (3,5) lie on the curve, they must satisfy the equation

$\therefore \frac{25}{4}a + k = 0$ ①

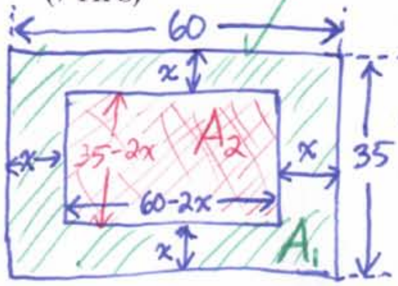
and $\frac{1}{4}a + k = 5$ ②

① − ② $\Rightarrow \frac{24}{4}a = -5 \Rightarrow 6a = -5 \Rightarrow a = \frac{-5}{6}$

Sub. in ② $\Rightarrow \frac{1}{4}\left(\frac{-5}{6}\right) + k = 5 \Rightarrow k = 5 + \frac{5}{24} = \frac{125}{24}$

$\boxed{\therefore \text{equ'n is } y = -\frac{5}{6}\left(x - \frac{5}{2}\right)^2 + \frac{125}{24}}$

11. Gagandeep, Aesha, Akaansha, Rose and Cicily have decided to construct a gigantic rectangular playpen for Eslam, Ashutosh and Atal. To prevent Eslam, Ashutosh and Atal from escaping, the girls have decided to surround the playpen with a mine field of uniform width. The playpen is to be built within a rectangular region having dimensions 60 metres by 35 metres. Calculate the dimensions of the playpen if its area must be ***double*** that of the mine field. **(7 TIPS)**



A_2 = area of playpen = $(35-2x)(60-2x)$

A_1 = area of mine field = total area $- A_2 = 2100-(35-2x)(60-2x)$

playpen area = 2 × (mine field area)

$\therefore (35-2x)(60-2x) = 2[2100-(35-2x)(60-2x)]$

$\therefore (35-2x)(60-2x) = 4200 - 2(35-2x)(60-2x)$

$\therefore 3(35-2x)(60-2x) = 4200$ (Divide B.S. by 3)

$\therefore 2100 - 190x + 4x^2 = 1400$

$\therefore 4x^2 - 190x + 700 = 0$

$\therefore 2x^2 - 95x + 350 = 0$

$\therefore x = \dfrac{95 \pm \sqrt{(-95)^2 - 4(2)(350)}}{2(2)}$

$\therefore x \doteq 4.03$ or $x \doteq 43.47$

inadmissable because $0 < x < 17.5$

//// (green): mine field (A_1)
XXX (red): playpen (A_2)

$0 < x < \frac{35}{2}$

$\therefore 0 < x < 17.5$

The dimensions of the playpen are approximately 51.94 m by 26.94 m

12. Consider the quadratic function $f(x) = -3x^2 - kx + 11$, where k represents any real number. **(11 TIPS)**

(a) Express the co-ordinates of the ***vertex*** of this quadratic as ***functions*** of k.

in terms of K

$f(x) = -3x^2 - kx + 11$

$= -3(x^2 + \frac{k}{3}x) + 11$

$= -3\left[x^2 + \frac{k}{3}x + \left(\frac{k}{6}\right)^2 - \left(\frac{k}{6}\right)^2\right] + 11$

$= -3\left[\left(x+\frac{k}{6}\right)^2 - \frac{k^2}{36}\right] + 11$

$= -3\left(x+\frac{k}{6}\right)^2 + \frac{3k^2}{36} + 11$

$= -3\left(x+\frac{k}{6}\right)^2 + 11 + \frac{k^2}{12}$

The co-ordinates of the vertex are

$\left(-\frac{k}{6},\ 11 + \frac{k^2}{12}\right)$

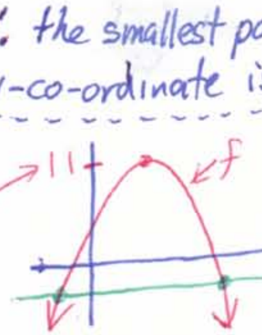
(b) Determine the smallest possible y-co-ordinate of the vertex of this quadratic function.

Since $\frac{k^2}{12} \geq 0$ for all $k \in \mathbb{R}$,

$11 + \frac{k^2}{12} \geq 11$ for all $k \in \mathbb{R}$

This term is non-negative for all values of k

$\therefore$ the smallest possible y-co-ordinate is 11



can't be smaller than 11

(c) For what value(s) of k will the function $g(x) = -2$ have exactly two points of intersection with $f(x)$?

$-3x^2 - kx + 11 = -2$

$\therefore -3x^2 - kx + 13 = 0$

For two points of intersection,

$b^2 - 4ac > 0$

$\therefore (-k)^2 - 4(-3)(13) > 0$

$\therefore k^2 + 156 > 0$

But this is true for all values of K. Therefore, f and g always have 2 points of intersection, regardless of the value of K.