MCR 3U0 Semester 1, 2010 – 2011

Grade 11 Functions (University Preparation)
Unit 4 – Exponential Functions – Quiz on 4.1 – 4.4

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Victim: Mr. Solution   Inspiring work Mr. S.!!

KU	APP	TIPS	COM
10/10	10/10	5/5	10/10

1. ***Evaluate*** each expression. **(10 KU)**

(a) $\dfrac{\left(32^{0.4}\right)\left(32^{\frac{1}{5}}\right)}{32^{\frac{4}{5}}}$

$= \dfrac{32^{0.4+0.2}}{32^{0.8}}$

$= \dfrac{32^{0.6}}{32^{0.8}}$

$= 32^{-0.2}$

$= \dfrac{1}{32^{\frac{1}{5}}}$

$= \dfrac{1}{\sqrt[5]{32}}$

$= \dfrac{1}{2}$

OR $\dfrac{\left(32^{\frac{2}{5}}\right)\left(32^{\frac{1}{5}}\right)}{32^{\frac{4}{5}}} = \dfrac{32^{\frac{3}{5}}}{32^{\frac{4}{5}}} = 32^{-\frac{1}{5}} = \dfrac{1}{\sqrt[5]{32}}$

(b) $\left(-5m^3n^{-2}\right)^{-2}\sqrt{169m^6n^8}$, where $m = -4$ and $n = 2$

Should simplify to $|m^3|$ since $\sqrt{\ }$ means "positive square root of"

$= (-5)^{-2}m^{-6}n^{4}\sqrt{169}\sqrt{m^6}\sqrt{n^8}$

$= \dfrac{1}{(-5)^2}(m^{-6})(n^4)(13)(m^3)(n^4)$

$= \dfrac{13}{25}m^{-3}n^{8}$

$= \dfrac{13}{25}(-4)^{-3}(2)^{8}$

$= \dfrac{13}{25}\left(\dfrac{1}{(-4)^3}\right)(256)$

$= \dfrac{13}{25}\left(\dfrac{1}{\cancel{-64}_{1}}\right)\left(\dfrac{\cancel{256}^{4}}{1}\right) = -\dfrac{52}{25}$ * See note above.

Note Technically the answer should be positive because $\sqrt{m^6}$ should be positive.

2. ***Simplify*** each expression. **(10 APP)**

(a) $\dfrac{\left(\frac{1}{3}x^{-5}y^{-3}\right)^{-4}}{\left(4x^{-4}y^{-1}\right)^{3}}$

$= \dfrac{\left(\frac{1}{3}\right)^{-4}x^{20}y^{12}}{4^3x^{-12}y^{-3}}$

$= \dfrac{3^4x^{20}y^{12}}{4^3x^{-12}y^{-3}}$

$= \dfrac{81}{64}x^{20-(-12)}y^{12-(-3)}$

$= \dfrac{81}{64}x^{32}y^{15}$

Note: $\left(\frac{1}{3}\right)^{-4} = \dfrac{1}{\left(\frac{1}{3}\right)^4} = \dfrac{1}{\left(\frac{1}{81}\right)} = \dfrac{1}{1}\times\dfrac{81}{1} = 81$

(b) $\left(\dfrac{\left(x^{12}\right)^{0.25}\left(216x^{9}\right)}{(3x)^{6}\left(x^{18}\right)^{0.5}}\right)^{-\frac{1}{3}}$

$= \left[\dfrac{x^3(216x^9)}{3^6x^6x^9}\right]^{-\frac{1}{3}}$

$= \left[\dfrac{216x^{12}}{729x^{15}}\right]^{-\frac{1}{3}}$

$= \dfrac{216^{-\frac{1}{3}}x^{-4}}{729^{-\frac{1}{3}}x^{-5}}$

$= \dfrac{\left(\frac{1}{\sqrt[3]{216}}\right)}{\left(\frac{1}{\sqrt[3]{729}}\right)}x^{-4-(-5)} = \dfrac{\left(\frac{1}{6}\right)}{\left(\frac{1}{9}\right)}x^{1}$

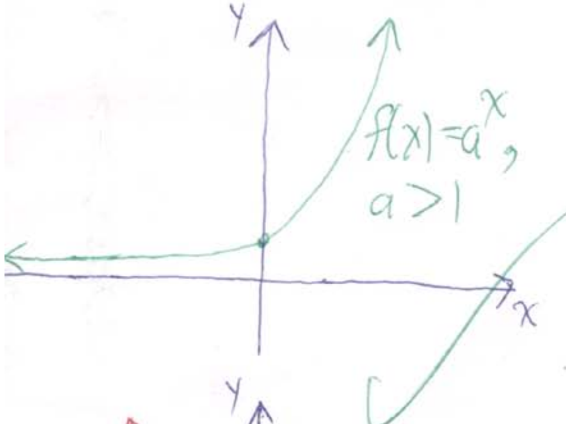
$= \dfrac{1}{\cancel{6}_{2}}\left(\dfrac{\cancel{9}^{3}}{1}\right)x = \dfrac{3}{2}x$

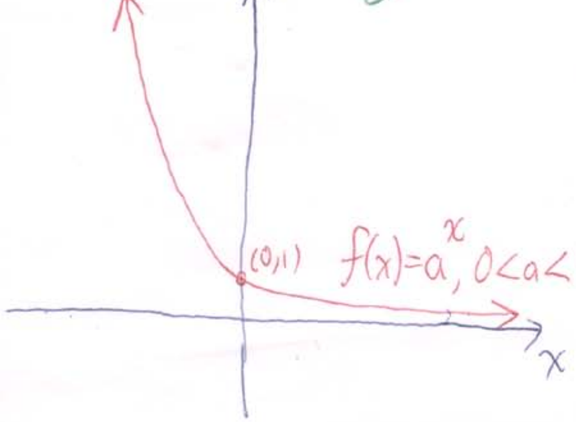
3. Consider the function $f(x)=x^{\frac{a}{b}}$, where a and b represent ***positive integer constants***. Explain why f is **defined for all real values of x if b is odd** but it is **undefined for certain values of x if b is even**. Give examples to support your answer. **(5 COM)**

x	$f(x)=x^{\frac{1}{2}}$	$g(x)=x^{\frac{1}{3}}$
-27	undefined	-3
-8	undefined	-2
-1	undefined	-1
0	0	0
4	2	$\doteq 1.5874$
8	$\doteq 2.8284$	2
9	3	$\doteq 2.08$

Whenever b is even, $f(x)=x^{\frac{a}{b}}$ is a square root, fourth root, sixth root, etc. All such functions are UNDEFINED for negative values of x. For example, $f(x)=x^{\frac{1}{2}}=\sqrt{x}$ is undefined for all negative values of x. The function $g(x)=x^{\frac{1}{3}}=\sqrt[3]{x}$, however, is defined for all values of x.

4. Consider the exponential function $f(x)=a^x$ where a represents the base of the exponential function. Which values of a result in exponential growth and which values of a result in exponential decay? Explain. In addition, sketch graphs to illustrate your answer. **(5 TIPS)**





As shown in the graph to the left, exponential growth occurs if $a>1$. This happens because for any base greater than 1, a^x must increase if x increases. For example, $2^1=2$ but $2^3=8>2$.

Conversely if $0<a<1$, then exponential decay occurs. For bases between 0 and 1, a^x decreases if x increases. For instance, $\left(\frac{1}{2}\right)^1=\frac{1}{2}$ but $\left(\frac{1}{2}\right)^3=\frac{1}{8}<\frac{1}{2}$.