MCR 3U0	Grade 11 Functions (University Preparation) Unit 5 – Quiz 1 – Trigonometric Ratios	Semester 1, 2010 - 2011

Mr. N. Nolfi

Victim: Mr. Solutions   Inspiring work Mr. S.!!

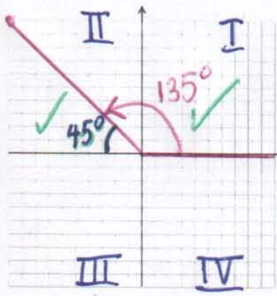
KU	TIPS	COM
24/24	10/10	14/14

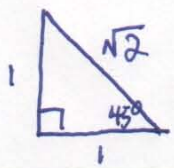
1. Use ***special triangles*** and ***related first quadrant (acute) angles*** to evaluate each of the following. In addition, use the provided grid to show the angle of rotation along with the related acute angle. DO NOT USE A CALCULATOR UNLESS YOU WOULD LIKE A MARK OF ZERO! **(16 KU)**

(a) $\cos 135°$

$= -\cos 45°$

$= -\frac{1}{\sqrt{2}}$





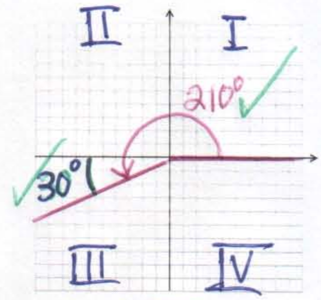
Quad. II

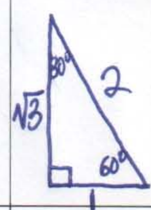
$\therefore \cos 135° < 0$

(b) $\tan 210°$

$= \tan 30°$

$= \frac{1}{\sqrt{3}}$





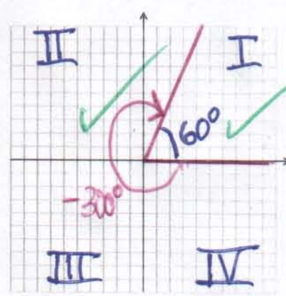
Quad. III

$\therefore \tan 210° > 0$

(c) $\csc(-300°)$

$= \csc 60°$

$= \frac{2}{\sqrt{3}}$



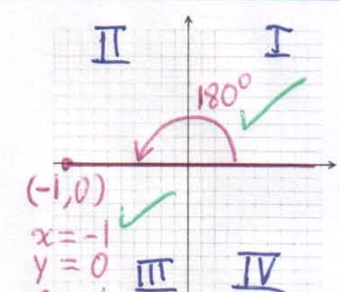
Quad. I

$\therefore \csc(-300°) > 0$

(d) $\cot 180°$

$= \frac{x}{y}$

$= \frac{-1}{0}$, which is undefined

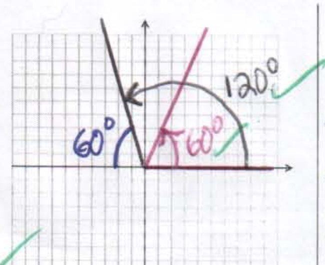


2. Solve for θ. In each case, $0 \le \theta \le 360°$. DO NOT USE A CALCULATOR FOR PART (a)! **(8 KU)**

(a) $\csc\theta = \frac{2}{\sqrt{3}}$

$\therefore$ related 1st quadrant angle is $60°$

$\therefore \theta = 60°$ or $\theta = 120°$ (see diagram)



(b) $\tan\theta = -0.88345632$

$\therefore \theta = \tan^{-1}(-0.88345632)$

$\therefore \theta \doteq -41.6°$ (calculator)

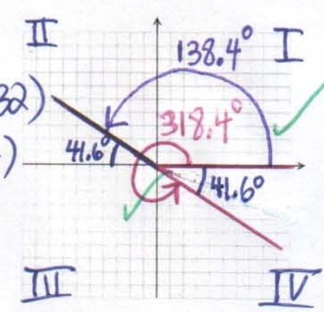
But $0 \le \theta \le 360°$

$\therefore \theta \doteq 138.4°$

OR

$\theta \doteq 318.4°$

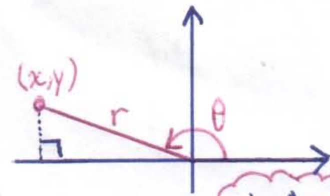
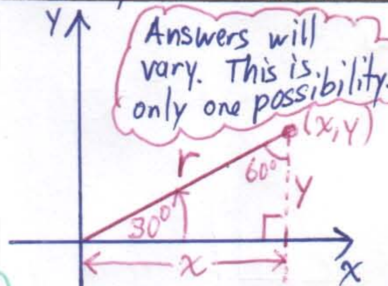
(see diagram)



$\tan\theta < 0$

$\therefore \theta$ must lie in quadrant II or IV

3. Explain why each of the following statements is true. Use diagrams to illustrate your answer. **(6 TIPS)**

Statement	*Explanation*
(a) $\sin\theta$ ***is defined*** for all values of θ while $\csc\theta$ ***is not defined*** for some values of θ. addition last addition first	$\sin\theta = \frac{y}{r}$ Since r represents the length of the terminal arm, $r>0$. Therefore, $\frac{y}{r}$ must always be defined since $r \neq 0$. On the other hand $\csc\theta = \frac{r}{y}$. Since y represents the y-co-ordinate of the end-point of the terminal arm, $y=0$ whenever the terminal arm lies on the x-axis. Therefore, $\csc\theta = \frac{r}{y}$ is undefined whenever the terminal arm lies on x-axis.  $\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$
(b) $\cos 30^0 + \cos 30^0 \neq \cos\left(30^0 + 30^0\right)$ DO NOT EVALUATE $\cos 30^0$ or $\cos 60^0$! Use a diagram to show why the left-hand side cannot possibly equal the right-hand side. Note: We should expect L.H.S. ≠ R.H.S. because operations are performed in a different order.	 $\cos 30^\circ + \cos 30^\circ = \frac{x}{r} + \frac{x}{r} = \frac{2x}{r}$ $\cos(30^\circ + 30^\circ) = \cos 60^\circ = \frac{y}{r}$ Clearly, $x > y$ $\therefore \frac{2x}{r} > \frac{y}{r}$ $\therefore \cos 30^\circ + \cos 30^\circ > \cos(30^\circ + 30^\circ)$

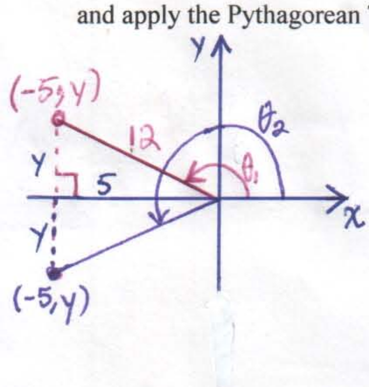
4. The following is a sample of the work of Stu P. Iddumbass, a mathematics student who pays little attention to Mr. Nolfi. Instead, he turns on the mental "auto-pilot" and manipulates symbols without understanding their meaning. Explain what is wrong with this student's work. **(4 COM)**

$$\frac{\cancel{\tan} x}{\cancel{\tan} 4x} = \frac{\cancel{x}}{4\cancel{x}} = \frac{1}{4}$$

Mr. Iddumbass is treating "tan" as if it were a number multiplying a value. This is clearly NOT the case! The expression "tan x" means that the "tan" function is applied to the argument "x." It represents one value and the "tan" cannot be separated from the "x." A simple counterexample shows that $\frac{\tan x}{\tan 4x} \neq \frac{1}{4}$.

e.g. let $x = 30^\circ$ $\therefore \frac{\tan x}{\tan 4x} = \frac{\tan 30^\circ}{\tan(4 \times 30^\circ)} = \frac{\tan 30^\circ}{\tan 120^\circ} = \frac{\left(\frac{1}{\sqrt{3}}\right)}{(-\sqrt{3})} = -\frac{1}{3}$

5. If $\cos\theta = -\frac{5}{12}$ and $0 \le \theta \le 360°$, evaluate $\sin\theta$ and $\tan\theta$. DO NOT USE A CALCULATOR!! (Hint: Draw a diagram and apply the Pythagorean Theorem.) **(4 TIPS)**



$\cos\theta = \frac{-5}{12}$

$\therefore x = -5,\ y = ?,\ r = 12$

By the Pythagorean Theorem,

$x^2 + y^2 = r^2$

$\therefore (-5)^2 + y^2 = 12^2$

$\therefore 25 + y^2 = 144$

$\therefore y^2 = 144 - 25 = 119$

$\therefore y = \pm\sqrt{119}$

If θ lies in quadrant II, $\sin\theta = \frac{\sqrt{119}}{12}$, $\tan\theta = \frac{\sqrt{119}}{-5}$

If θ lies in quadrant III, $\sin\theta = \frac{-\sqrt{119}}{12}$, $\tan\theta = \frac{\sqrt{119}}{5}$