MCR 3U0 **Semester 1, 2010 - 2011**

Grade 11 Functions (University Preparation)
Unit 5 – Major Test – Trigonometric Ratios

Mr. N. Nolfi

Victim: Mr. Solutions

You solutions are elegant Mr. S.!

KU	APP	TIPS	COM
19/19	25/20	15/15	10/10

Multiple Choice **(10 KU)**

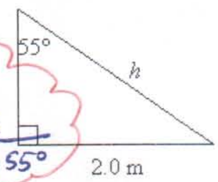
Identify the choice that best completes the statement or answers the question.

1. __b__ If $\cot\theta = 0.75$, determine the value of θ to the nearest degree ($0 \le \theta \le 360°$).

a. 53°, 217° b. 53°, 233°
c. 37°, 217° d. 37°, 233°

$\cot\theta = 0.75$
$\therefore \tan\theta = \frac{1}{0.75}$
$\therefore \theta = \tan^{-1}\left(\frac{1}{0.75}\right)$

2. __c__ Determine the length of the hypotenuse of the triangle to the nearest tenth of a metre.



a. 2.9 m b. 2.1 m
c. 2.4 m d. 3.1 m

$\frac{h}{2} = \csc 55°$
$\therefore h = 2\csc 55° \doteq \frac{2}{\sin 55°}$

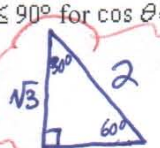
3. __b__ Determine the *exact value* of $\tan^2 45° - \cos 30°$.

a. $1 + \frac{\sqrt{3}}{2}$ b. $\frac{2-\sqrt{3}}{2}$
c. $2 - \frac{\sqrt{3}}{2}$ d. $1 - \sqrt{3}$

$\tan^2 45° - \cos 30°$
$= 1^2 - \frac{\sqrt{3}}{2}$
$= \frac{2-\sqrt{3}}{2}$

4. __a__ Using the appropriate special triangle, determine θ if $0° \le \theta \le 90°$ for $\cos\theta = \frac{\sqrt{3}}{2}$.

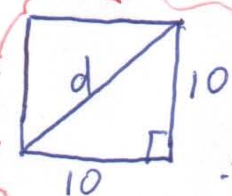
a. 30° b. 45°
c. 60° d. 90°



$\frac{\sqrt{3}}{2} = \frac{adj.}{hyp.} = \cos 30°$

5. __c__ A square has a side length of 10 cm. If a diagonal is drawn, what is the length of the diagonal? Give the *exact value*, not an approximation.

a. $4\sqrt{3}$ cm b. $8\sqrt{3}$ cm
c. $10\sqrt{2}$ cm d. $5\sqrt{2}$ cm



$d^2 = 10^2 + 10^2$
$\therefore d^2 = 200$
$\therefore d = \sqrt{200} = 10\sqrt{2}$

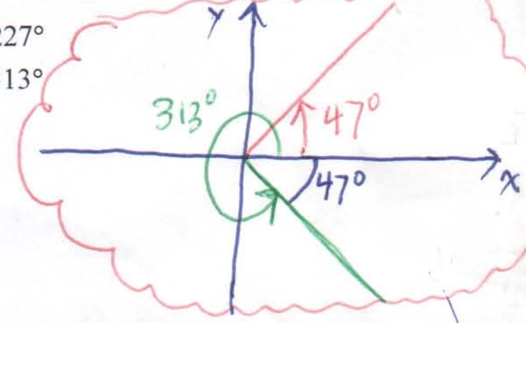
6. __b__ Determine the exact value of $\cos 45° \times \frac{\tan 30°}{\sin 60°}$

a. $\frac{2\sqrt{2}}{3}$ b. $\frac{\sqrt{2}}{3}$
c. $\frac{3\sqrt{2}}{2}$ d. $\frac{\sqrt{3}}{3}$

$\frac{1}{\sqrt{2}} \times \frac{\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}}$
$= \frac{2}{\sqrt{18}} = \frac{2}{3\sqrt{2}} = \frac{2\sqrt{2}}{(3\sqrt{2})\sqrt{2}}$
$= \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$

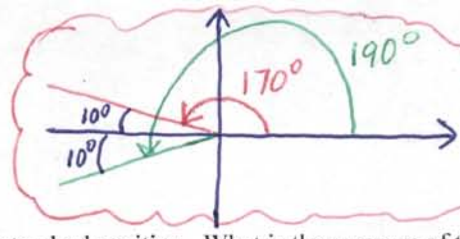
7. __d__ If $\cos\theta = 0.6798$, determine the value of θ to the nearest degree ($0 \le \theta \le 360°$).

a. 47°, 133° b. 47°, 227°
c. 47°, 317° d. 47°, 313°

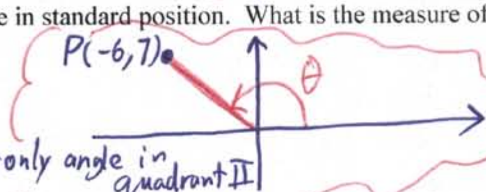


8. __d__ Which of the following has the same value as $\cos 170°$?

a. $\cos 280°$ b. $\cos 350°$
c. $\cos 10°$ d. $\cos 190°$



9. __d__ The point $P(-6, 7)$ lies on the terminal arm of an angle in standard position. What is the measure of the principal angle (i.e. $0 \le \theta \le 360°$) θ to the nearest degree?



a. 49° b. 229°
c. 311° d. 131° ← only angle in quadrant II

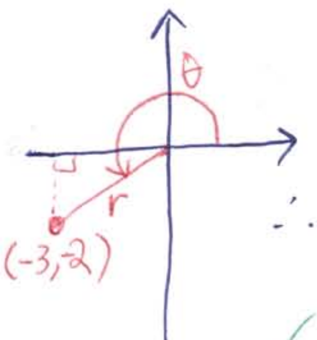
10. __d__ Which of the following values can be used as a ***counterexample*** to demonstrate that the equation $\sin^2\alpha \sec\alpha = \sin\alpha \sec\alpha$ is ***not*** an identity?

a. $\alpha = 0°$ b. $\alpha = 360°$
c. $\alpha = 180°$ d. $\alpha = 235°$

$\sin 0° = \sin 180° = \sin 360° = 0$
$\therefore \alpha = 0°, 180°, 360°$ all satisfy the equation

11. Angle θ is a principal angle (i.e. $0 \le \theta \le 360°$) that lies in quadrant 3 such that $\tan\theta = \frac{2}{3}$. **(4 KU)**

a) State the other five trigonometric ratios *as fractions*.



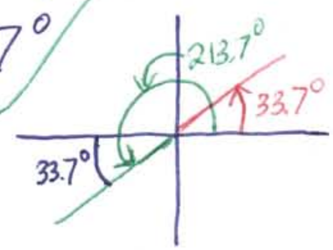
$\tan\theta = \frac{2}{3} = \frac{y}{x}$
and θ is in quadrant III
$\therefore x = -3, y = -2$

$r^2 = x^2 + y^2$
$\therefore r^2 = (-3)^2 + 2^2$
$\therefore r^2 = 9 + 4 = 13$
$\therefore r = \sqrt{13}$

$\therefore \sin\theta = \frac{y}{r} = -\frac{2}{\sqrt{13}}$
$\cos\theta = \frac{x}{r} = -\frac{3}{\sqrt{13}}$
$\csc\theta = -\frac{\sqrt{13}}{2}$
$\sec\theta = -\frac{\sqrt{13}}{3}$
$\cot\theta = \frac{3}{2}$

b) Determine the value of θ to the nearest degree.

$\tan\theta = \frac{2}{3}$
$\therefore \theta = \tan^{-1}\left(\frac{2}{3}\right)$
$\therefore \theta \doteq 33.7°$ (Quadrant I)
Since θ lies in quadrant III,
$\theta \doteq 180° + 33.7°$
$= 213.7°$



12. Determine the exact value of $\cos 60°(1 - \sin 30°) + 4\tan 45°(\tan 60° - \sin 45°)$. Express your answer as a single fraction. **(5 KU)**

$\cos 60°(1 - \sin 30°) + 4\tan 45°(\tan 60° - \sin 45°)$
$= \frac{1}{2}\left(1 - \frac{1}{2}\right) + 4(1)\left(\sqrt{3} - \frac{1}{\sqrt{2}}\right)$
$= \frac{1}{2}\left(\frac{1}{2}\right) + 4\left(\frac{\sqrt{6} - 1}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$
$= \frac{1}{4} + \left(\frac{4\sqrt{2}(\sqrt{6} - 1)}{2}\right)\left(\frac{2}{2}\right)$
$= \frac{1}{4} + \frac{8\sqrt{2}(\sqrt{6} - 1)}{4}$
$= \frac{1 + 8\sqrt{12} - 8\sqrt{2}}{4}$
$= \frac{1 + 8(2\sqrt{3}) - 8\sqrt{2}}{4}$
$= \frac{1 + 16\sqrt{3} - 8\sqrt{2}}{4}$

13. Prove that each of the following equations is an identity: **(10 APP)**

(a) $1 = \dfrac{(\sin^4 x - \cos^4 x)}{\tan x \sin x \cos x - \cos^2 x}$

$$L.S. = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\left(\frac{\sin x}{\cancel{\cos x}}\right)\left(\frac{\sin x}{1}\right)\left(\frac{\cancel{\cos x}}{1}\right) - \cos^2 x}$$

$$= \frac{(\sin^2 x - \cos^2 x)(1)}{\sin^2 x - \cos^2 x}$$ Pythagorean Identity

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x - \cos^2 x}$$

$$= 1$$

$$= R.S.$$

$$\therefore L.S. = R.S.$$

(b) $\sin x \tan x + \dfrac{\sin x}{\tan x} = \dfrac{1}{\cos x}$

$$L.S. = \left(\frac{\sin x}{1}\right)\left(\frac{\sin x}{\cos x}\right) + \left(\frac{\sin x}{1}\right) \div \left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\sin^2 x}{\cos x} + \left(\frac{\sin x}{1}\right)\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\sin x \cos x}{\sin x}$$

$$= \frac{(\sin^2 x)(\sin x) + (\sin x \cos x)(\cos x)}{\cos x \sin x}$$

$$= \frac{\sin x(\sin^2 x) + \sin x(\cos^2 x)}{\cos x \sin x}$$

$$= \frac{\cancel{\sin x}(\sin^2 x + \cos^2 x)}{\cos x \cancel{\sin x}}$$

$$= \frac{1}{\cos x} \quad \text{(Pythagorean Identity)}$$

$$= R.S.$$

$$\therefore L.S. = R.S.$$

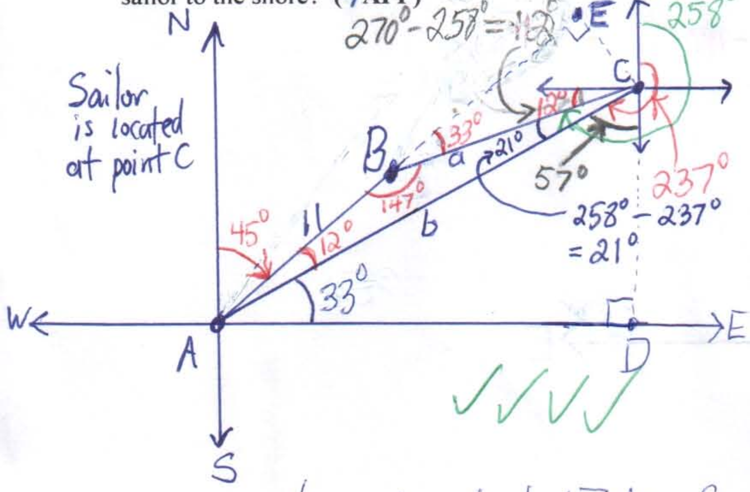
14. Is the equation $\sin x = \cos x$ an identity? If it is an identity, prove it! Otherwise, provide a counterexample to show that it is not. **(4 APP)**

This equation is NOT an identity. For instance, if $x = 30^\circ$, then $\sin x = \frac{1}{2}$ and $\cos x = \frac{\sqrt{3}}{2}$

In fact, for the interval $0 \le x \le 360^\circ$, $\sin x$ and $\cos x$ agree only for two values of x ($x = 45^\circ$ and $x = 180^\circ + 45^\circ = 225^\circ$).

This question is now only worth 4 APP marks due to errors in the question.

15. Two lighthouses are 11 km apart along the shore of a lake. From lighthouse A, lighthouse B has a bearing of 45°. A sailor gets a bearing of 237° from ~~his~~ present position for lighthouse A and 258° for lighthouse B. How far, to the nearest kilometre, is the sailor from both lighthouses? What is the shortest distance, to the nearest kilometre, from the sailor to the shore? **(4APP)**



(a) How far is the sailor from both lighthouses?

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\therefore a = \frac{c \sin A}{\sin C} = \frac{11 \sin 12^\circ}{\sin 21^\circ} \doteq 6.38$$

Similarly,

$$b = \frac{c \sin B}{\sin C} = \frac{11 \sin 147^\circ}{\sin 21^\circ} = 16.72$$

The sailor is about 17 km from lighthouse A and 6 km from lighthouse B.

(b) What is the shortest distance from the sailor to the shore?

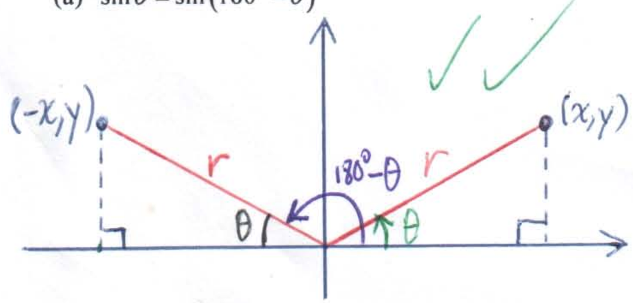
The shortest distance is CE (see diagram)

$$\frac{CE}{a} = \sin 33^\circ \longrightarrow CE = a \sin 33^\circ = \left(\frac{11 \sin 12^\circ}{\sin 21^\circ}\right)(\sin 33^\circ) \doteq 3.5$$

The shortest distance to the shore is about 3.5 km

16. Are the following equations identities? If so, prove that they are by using angle of rotation diagrams. If not, provide counterexamples. Assume that θ is a principal angle that lies in the first quadrant. **(6 TIPS)**

(a) $\sin\theta = \sin(180° - \theta)$

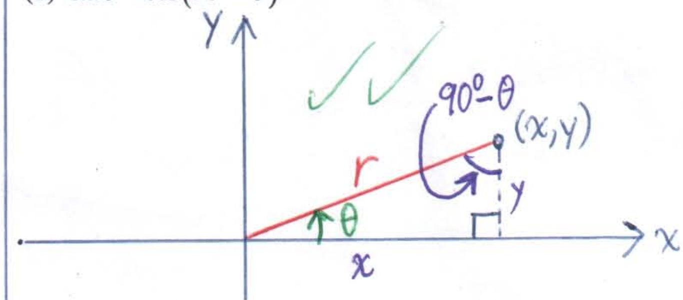


$$\sin\theta = \frac{y}{r}$$

$$\sin(180^\circ - \theta) = \frac{y}{r}$$

$$\therefore \sin\theta = \sin(180^\circ - \theta)$$

(b) $\sin\theta = \cos(90° - \theta)$

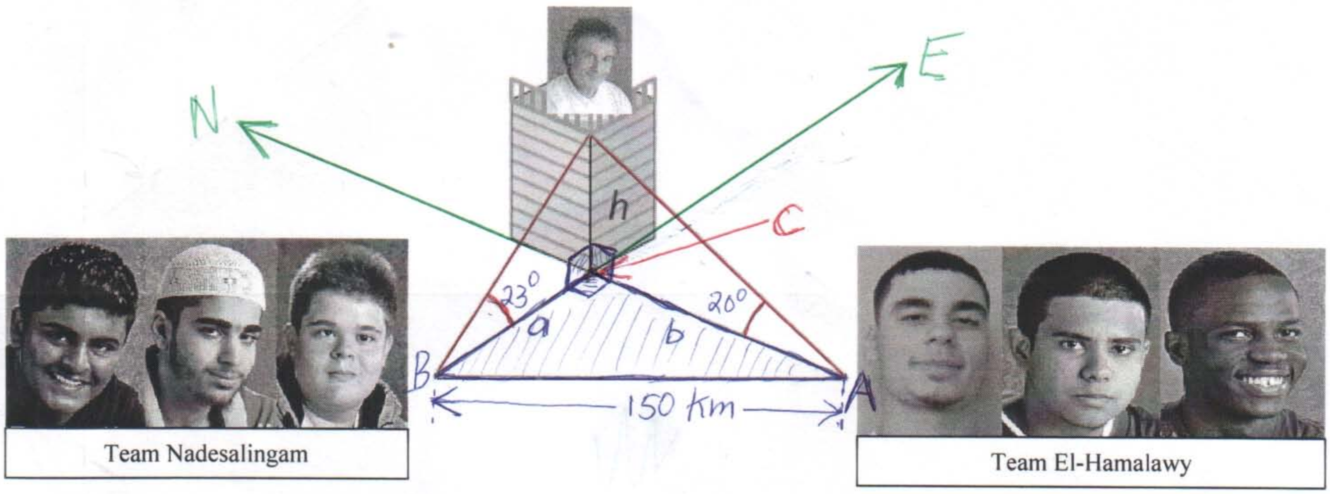


$$\sin\theta = \frac{opp.}{hyp.} = \frac{y}{r}$$

$$\cos(90^\circ - \theta) = \frac{adj.}{hyp.} = \frac{y}{r}$$

$$\therefore \sin\theta = \cos(90^\circ - \theta)$$

17. Team Nadesalingam and Team El-Hamalawy have been asked by Mr. Nolfi to make measurements to be used for a math test. Mr. Nolfi stood on a tower while both teams stood on the ground. While facing directly East, Team Nadesalingam measures the angle of elevation to Mr. Nolfi to be $23°$. While facing directly North, Team El-Hamalawy measures the angle of elevation to Mr. Nolfi to be $20°$. If the teams are 150 m apart, calculate the height of the tower. **(9 TIPS)**



(a) $\dfrac{h}{a} = \tan 23°$

$\therefore a = \dfrac{h}{\tan 23°}$

$\dfrac{h}{b} = \tan 20°$

$\therefore b = \dfrac{h}{\tan 20°}$

(b) Since Team N. faces East and Team E. faces North, $\triangle ABC$ must be a right triangle

$\therefore a^2 + b^2 = 150^2$

$\therefore \left(\dfrac{h}{\tan 23°}\right)^2 + \left(\dfrac{h}{\tan 20°}\right)^2 = 150^2$

$\therefore \dfrac{h^2}{\tan^2 23°} + \dfrac{h^2}{\tan^2 20°} = 150^2$

$\therefore h^2 \tan^2 20° + h^2 \tan^2 23° = 150^2 \tan^2 23° \tan^2 20°$

$\therefore h^2(\tan^2 20° + \tan^2 23°) = 150^2 \tan^2 23° \tan^2 20°$

$\therefore h^2 = \dfrac{150^2 \tan^2 23° \tan^2 20°}{\tan^2 20° + \tan^2 23°}$

$\therefore h = \sqrt{\dfrac{150^2 \tan^2 23° \tan^2 20°}{\tan^2 20° + \tan^2 23°}}$

$\doteq 41.4$

The height of the tower is about 41 m.