MCR 3U0 — **Semester 1, 2010 - 2011**

Grade 11 Functions (University Preparation)
Unit 6 – Major Test – Trigonometric Functions

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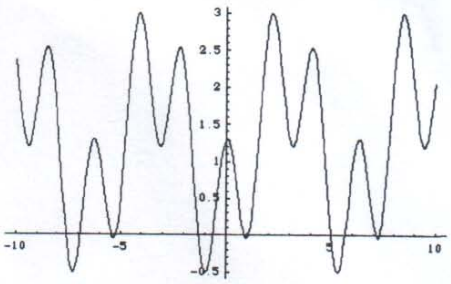
Victim: Mr. Solutions

KU	APP	TIPS	COM
/14	/19	/20	/10

Questions 1 – 8: Multiple Choice **(8 KU)**
Identify the choice that best completes the statement or answers the question.

1. d The graph of a periodic function is shown at the right. What is the approximate ***period*** of the function?



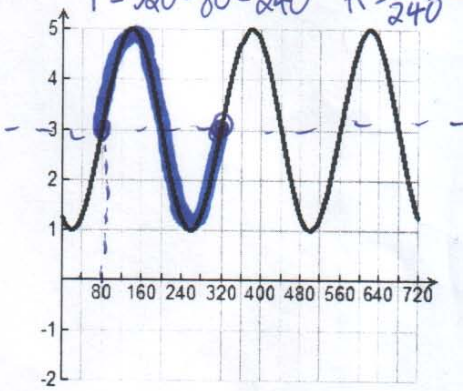
   a. 1.75 b. 13
   c. 3.5 (d.) 6.5

2. c Which of the following ***is most unlikely*** to produce a periodic graph?

   a. Eslam's height above the floor as he jumps up and down in a playpen.
   b. The height above the floor of Ashutosh's mother's hand as she "disciplines" him with a roti-maker.
   (c.) The height above the ground of Katy's airplane as it descends toward a runway for a landing.
   d. Aesha's height above the ground as she rides an extremely fast Ferris wheel. (One rotation per second!)

3. d Which of the following is a ***correct*** equation for the graph at the right?

T = 320 − 80 = 240 $k = \frac{360^\circ}{240} =$



   ~~a.~~ $f(x) = 2\sin\left(\frac{2}{3}(x+80)\right)+3$ ~~b.~~ $f(x) = 2\sin\left(\frac{2}{3}(x-80)\right)+3$
   ~~c.~~ $f(x) = 2\sin\left(\frac{3}{2}(x+80)\right)+3$ (d.) $f(x) = 2\sin\left(\frac{3}{2}(x-80)\right)+3$

4. c The function, whose graph is shown at the right, has domain and range

   ~~a.~~ $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\geq y\geq 5\}$ ~~b.~~ $D=\{x\in\mathbb{R}:1\leq x\leq 5\},\ R=\mathbb{R}$
   (c.) $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\leq y\leq 5\}$ ~~d.~~ $D=\{x\in\mathbb{R}:1\geq x\geq 5\},\ R=\mathbb{R}$

5. b Which of the following ***does not*** make sense?

   a. A sinusoidal function has a period of π°.
   (b.) A sinusoidal function has an amplitude of −3. → Amplitude cannot be negative!!
   c. A sinusoidal function has an amplitude of 1/1000000.
   d. A sinusoidal function is compressed horizontally by a factor of 0.001.

6. d Zohra is jumping up and down on a trampoline. Her height in metres above the ground after t seconds is given by the function $h(t) = 1.5\sin(360t)+2$. What does the "1.5" in the equation represent?

   ~~a.~~ Zohra's maximum height above the ground. ~~b.~~ Zohra's average height above the ground.
   ~~c.~~ Zohra's minimum height above the ground. (d.) Zohra's maximum displacement from the average height.

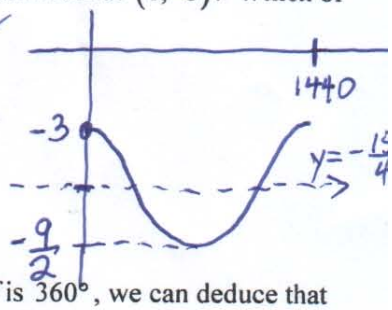
7. a A sinusoidal function has an amplitude of 0.75 units, a period of 1440° and a maximum at $(0,-3)$. Which of the following ***is not*** an equation of the function.

a. $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}x\right)-\frac{15}{4}$ ✓ $f(0)=-\frac{15}{4}\neq -3$

b. $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}(x-1440)\right)-\frac{15}{4}$ ✓

c. $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}x\right)-\frac{15}{4}$ ✗

d. $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}(x+360)\right)-\frac{15}{4}$



8. d Let $f(x)=\sin x$ and $g(x)=A\sin\left(k(x-p)\right)+d$. Knowing that the period of f is 360°, we can deduce that the period of g must be $\frac{360°}{k}$. Why is this true?

a. ✗ This information is found in Mr. Nolfi's notes as well as the textbook. Everyone knows that neither source can ever be wrong. Textbooks and teachers are right about everything!

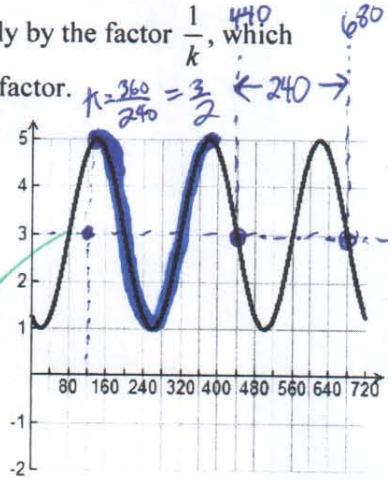
b. ✗ It is true because period is calculated by dividing 360° by k.

c. ✗ To obtain the graph of g, the graph of f must be stretched or compressed horizontally by the factor k, which means that the period of f is also stretched or compressed horizontally by the same factor.

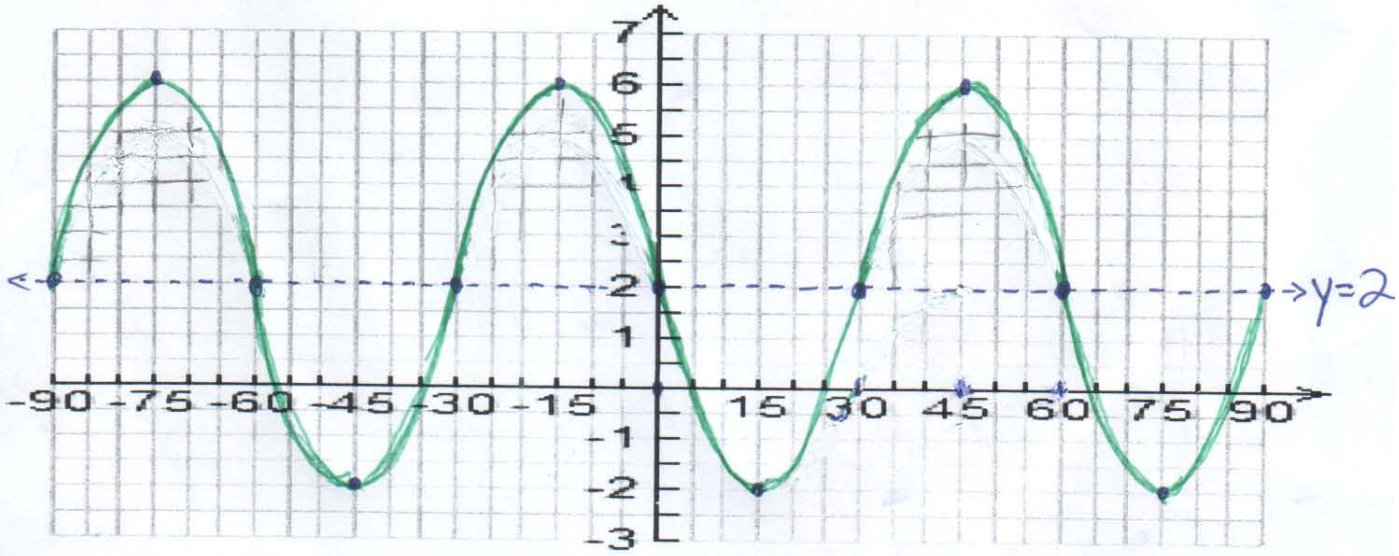
d. ✓ To obtain the graph of g, the graph of f must be stretched or compressed horizontally by the factor $\frac{1}{k}$, which means that the period of f is also stretched or compressed horizontally by the same factor.

9. What transformations would need to be applied to $f(x)=\cos x$ to obtain the given graph? **(2 KU)**

Horizontal	*Vertical*
1. Compress by a factor of $\frac{2}{3}$	1. Stretch by a factor of 2
2. Translate 120 units right.	2. Translate 3 units up

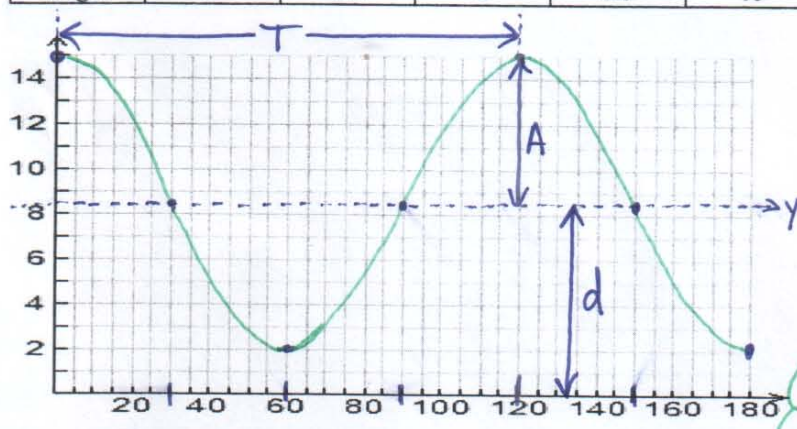


10. Sketch ***two cycles*** of a sinusoidal function having amplitude 4, period 60°, horizontal axis $y=2$ and passing through the point with co-ordinates $(15,-2)$. **(4 KU)**



11. The following table lists the height of an object $h(t)$ (in metres), after t seconds. Sketch a graph of height of the object versus time. Then determine an equation of the function that models the height of this object. **(4 APP)**

t	0	30	60	90	120	150	180
height	15	8.5	2	8.5	15	8.5	2



$A = 15 - 8.5 = 6.5$

$d = 8.5$

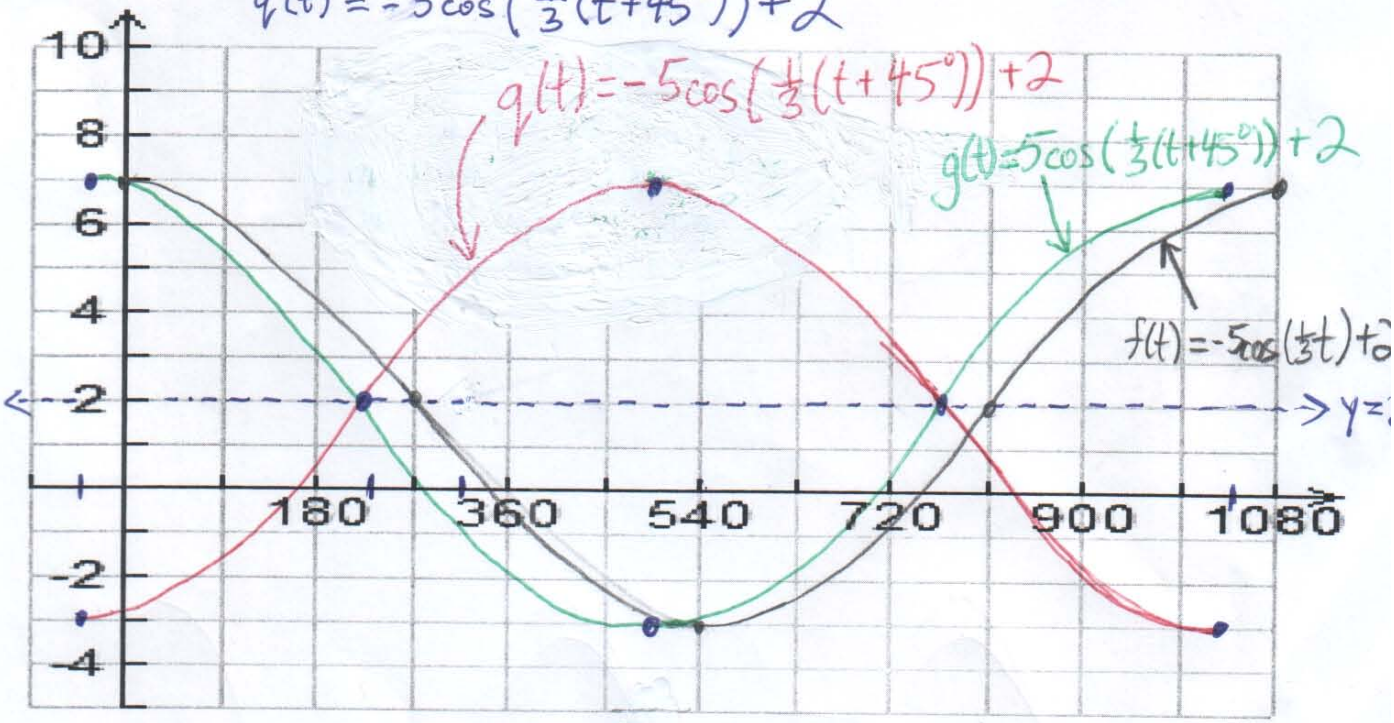
$p = 0$ (base function cos)

$T = 120° \Rightarrow K = \frac{360°}{120°} = 3$

$h(t) = 6.5\cos(3t) + 8.5$ is an equation of the curve.

12. Sketch the graph of $q(t) = -5\cos\left(\frac{1}{3}t + 15°\right) + 2$ for $-90° \le t \le 1080°$. **(4 APP)**

$q(t) = -5\cos\left(\frac{1}{3}(t + 45°)\right) + 2$



$A = 5$

$d = 2$ → horizontal axis is $y = 2$

$p = -45°$ (45° left)

$K = \frac{1}{3}$

$\therefore T = \frac{360°}{(\frac{1}{3})} = 1080°$

Transformations of $y = \cos t$

Horizontal

1. Stretch by a factor of 3
2. Translate 45° left

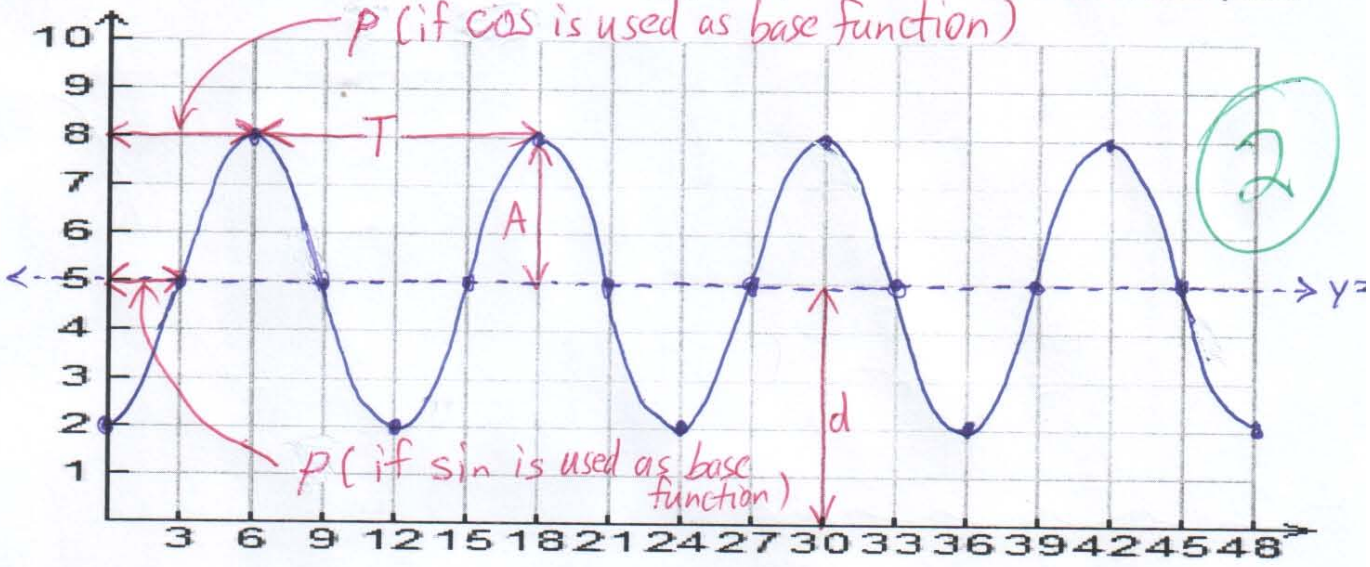
Vertical

1. Stretch by a factor of 5 and reflect in x-axis
2. Translate 2 units up

$(x, y) \rightarrow (3x - 45, -5y + 2)$

13. The average depth of the water in a port on a tidal river is 5 m. At low tide, the depth of the water is 2 m and one cycle is completed every 12 hours. **(6 APP)**

(a) Assuming that low tide occurs at $t = 0$ hours, sketch a graph of depth of water versus time for a 48 hour period.



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(b) Write an equation of the function $d(t)$, the depth of the water in metres, t hours after low tide.

$T = 12 \quad A = 3$

$\therefore K = \frac{360}{12} \quad d = 3$

$= 30 \quad P = \begin{cases} 3 \rightarrow \sin \\ 6 \rightarrow \cos \end{cases}$

$d(t) = 3\sin(30(t-3)) + 5$

OR

$d(t) = 3\cos(30(t-6)) + 5$

4

14. The rodent population in a particular region varies with the number of predators that inhabit the region. At any time t years since 1976, the number of rodents $r(t)$ is given by the function $r(t) = 1500\sin(45t) + 2500$.

(a) What is the period of this function? **(1 APP)**

$T = \frac{360}{45}$

$= 8$ years

(b) Rewrite the function $r(t)$ in such a way that the rodent population cycle lasts for 12 years. **(2 APP)**

$r(t) = 1500\sin(30t) + 2500$

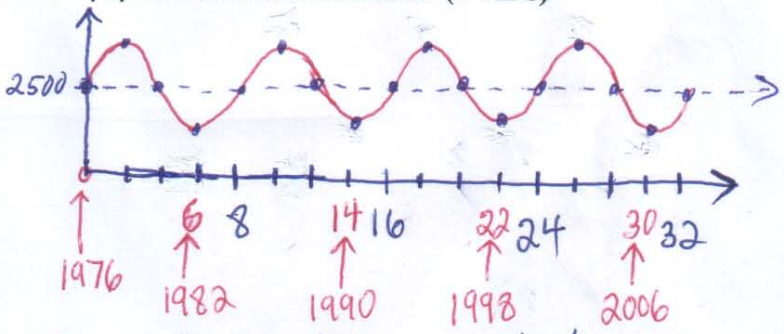
(c) What is the maximum number of rodents in any given cycle? **(2 APP)**

$\max = d + A$

$= 2500 + 1500$

$= 4000$

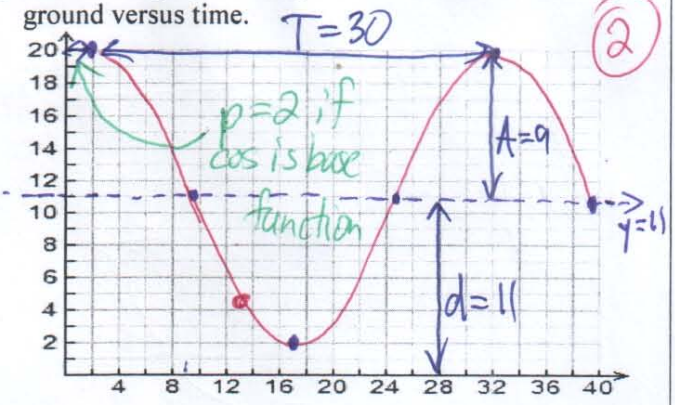
(d) Between 1976 and 2004, in which years did the rodent population reach a minimum? **(4 TIPS)**



The rodent population reached a minimum in 1982, 1990 and 1998.

15. A certain town has a windmill with the tip of one of its blades painted red. The owner of the windmill notices that at $t = 2$s the red tip is 20 m above the ground. Then, over a period of 30 seconds, the red tip moves from 20 m above the ground down to 2 m above the ground and back up to 20 m. **(16 TIPS)**

(a) Sketch the graph of height of the red mark above the ground versus time.



(2)

(b) Write an equation of the sinusoidal function that models the height of the red mark above the ground versus time.

$h(t) = 9\cos(12(t-2)) + 11$

(2) OR

$K = \frac{360}{30} = 12$

(c) What is the equation of the horizontal axis of this sinusoidal function? What does the horizontal axis represent in this context?

Horizontal Axis: $y = 11$ ✓

It represents the average height above the ground of the red mark. ✓

(2)

(d) How high above the ground is the red mark after 13 seconds?

$h(13) = 9\cos(12(13-2)) + 11$

$= 9\cos 132° + 11$

$\doteq 5$ m (agrees with graph)

(2)

(e) At what time during the 30-second period is the red mark 17 m above the ground?

(3)

$9\cos(12(t-2)) + 11 = 17$

$\therefore \cos(12(t-2)) = \frac{6}{9} = \frac{2}{3}$

$\therefore 12(t-2) = \cos^{-1}\left(\frac{2}{3}\right)$

$\therefore 12(t-2) \doteq 48$

$\therefore t \doteq 6$, $t \doteq 28$ (agrees with graph)

(f) What is the diameter of the windmill?

$\text{diameter} = 2A$

$= 2(9) = 18$ m

(1)

(g) The owner of the town's mini-golf course would like to install a small 2m-diameter version of the windmill. The small version of the windmill is designed to ***scale*** with the original, but rotates more rapidly than the original. It completes a full rotation in 10 s. Write an equation to model the height above the ground of the red mark on the small windmill given that at $t = 5$s, the red mark is at its maximum height above the ground.

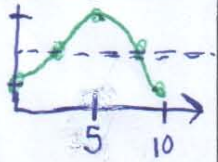
(3)

$A = d \div 2 = 2 \div 2 = 1$

$T = 10 \Rightarrow K = \frac{360}{10} = 36$

$p = 0$ Scale factor $= \frac{2}{18} = \frac{1}{9}$

min. height $= \frac{1}{9}(2) = \frac{2}{9}$ (scales with original)



$h(t) = \sin(36(t-2.5)) + \frac{11}{9}$

(h) A golf ball has a diameter of 4.3 cm. If the red mark on the small windmill is as close to the ground as possible, would the golf ball be able to pass through the space between the bottom of the blade and the ground? Explain.

(1)

The ball will be able to pass through the space because its diameter is 0.043 m, which is significantly smaller than the minimum height of the red mark (about 0.22 m)