MCR 3U0 | **Semester 1, 2007 - 2008**

Grade 11 Functions (University Preparation)
Unit 1 – Applications of Quadratics and Transformations

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Victim: Mr. Solutions

KU	APP	TIPS	COM
5 /5	19/19	14/14	18/18

1. Explain the meaning of the following sentence. "Your success in school is a function of your priorities." **(2 COM)**

This sentence means that a student's level of success in school is dependent upon the priorities that he/she sets. From the point of view of a function, the priorities are the input and level of success is the output.

2. In the following questions, f refers to the quadratic function defined by the equation $f(u) = 10u^2 - 7u + 1$.
(There are 5 COM marks allotted for all of question 2.)

$a = 10,\ b = -7,\ c = 1$

(a) Use the discriminant to determine the number of x-intercepts of f. **(3 APP)**

$D = b^2 - 4ac$
$= (-7)^2 - 4(10)(1)$
$= 9$

Since $D > 0$, f has two real roots. Therefore, f has two x-intercepts. //

(b) Use the discriminant to determine whether f factors over the integers. If so, factor f and use the factored form to determine how many roots f has. Does you answer agree with the answer that you obtained in (a)? **(3 APP)**

From 2(b), $D = 9$. Since D is a perfect square, f factors over the integers.

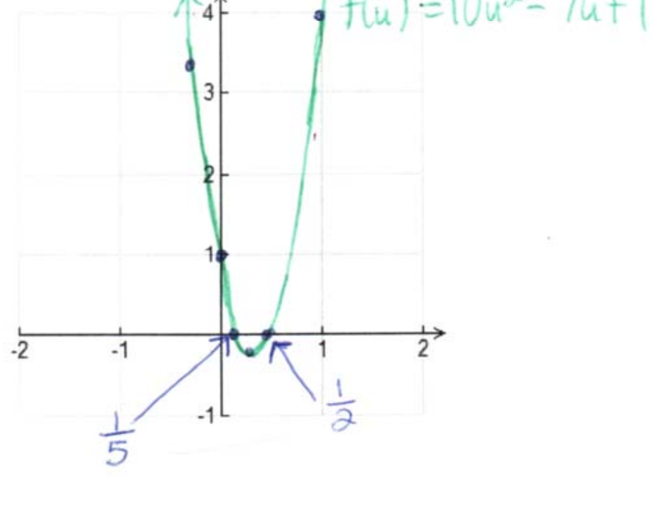
$f(u) = 10u^2 - 7u + 1$
$= 10u^2 - 5u - 2u + 1$
$= 5u(2u-1) - 1(2u-1)$
$= (2u-1)(5u-1)$

$10(1) = 10 \quad (-2)(-5) = 10$
$-2 + (-5) = -7$

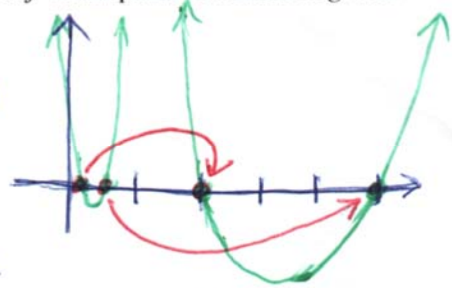
Rough work

From the factored form it's clear that f has two roots, which agrees with the answer to 2b).

(c) Sketch the graph of f. In addition, clearly mark the x-intercepts of f. **(3 APP)**



(d) The roots of $f(u)=10u^2-7u+1$ are $u=\frac{1}{5}$ and $u=\frac{1}{2}$. What transformation of f would produce a function g with roots $u=2$ and $u=5$? Find the equation of g. **(4 TIPS)**



Consider the horizontal stretch $(x,y)\to(10x,y)$

Then $(\frac{1}{5},0)\to(10(\frac{1}{5}),10(0))=(2,0)$

and $(\frac{1}{2},0)\to(10(\frac{1}{2}),10(0))=(5,0)$

Clearly, this transformation satisfies the requirements.

Therefore, an equation of g is

$\frac{1}{5}\xrightarrow{\times 10}2$

$\frac{1}{2}\xrightarrow{\times 10}5$

$$g(u)=f\left(\tfrac{1}{10}u\right)=10\left(\tfrac{1}{10}u\right)^2-7\left(\tfrac{1}{10}u\right)+1$$

$$\therefore g(u)=\tfrac{1}{10}u^2-\tfrac{7}{10}u+1$$

Note: There are an infinite # of correct answers since g can be stretched vertically by any factor without changing the roots

3. Determine the equation of the quadratic function that passes through $(3\sqrt{2},5)$ if the roots of the corresponding quadratic equation are $\sqrt{2}+3$ and $\sqrt{2}-3$. **(4 APP, 2 COM)**

Let f represent the required quadratic function.

Then $f(x)=a(x-(\sqrt{2}+3))(x-(\sqrt{2}-3))$ for some $a\in\mathbb{R}$.

To allow for easy expansion, we can rewrite f as

$$f(x)=a((x-\sqrt{2})-3)((x-\sqrt{2})+3)$$

Since $(3\sqrt{2},5)$ lies on the required parabola, $f(3\sqrt{2})=5$.

$$\therefore a((3\sqrt{2}-\sqrt{2})-3)((3\sqrt{2}-\sqrt{2})+3)=5$$

$$\therefore a(2\sqrt{2}-3)(2\sqrt{2}+3)=5$$

$$\therefore a(8-9)=5$$

$$\therefore -1a=5$$

$$\therefore a=-5$$

$$\therefore f(x)=-5((x-\sqrt{2})+3)((x-\sqrt{2})-3)$$

$$\therefore f(x)=-5[(x-\sqrt{2})^2-9]$$

$$\therefore f(x)=-5(x-\sqrt{2})^2+45$$

4. The following table lists the approximate accelerations due to gravity near the surface of the Earth, moon and sun.

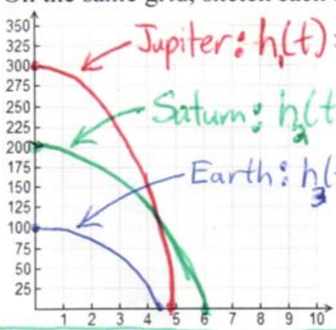
Earth	*Jupiter*	*Saturn*
9.87 m/s^2	25.95 m/s^2	11.08 m/s^2

The data in the above table lead to the following equations for the height of an object dropped near the surface of each of the celestial bodies given above. In each case, $h(t)$ represents the height, in metres, of an object above the surface of the body t seconds after it is dropped from an initial height h_0.

Earth	*Jupiter*	*Saturn*
$h(t) = -4.94t^2 + h_0$	$h(t) = -12.97t^2 + h_0$	$h(t) = -5.54t^2 + h_0$

In questions (a) to (d), use an initial height of 100 m for the Earth, 200 m for Saturn and 300 m for Jupiter.

(a) On the same grid, sketch each function. **(3 KU)**



(b) Explain how the "Jupiter function" can be transformed into the "Saturn function." **(3 TIPS)**

$$\frac{5.54}{12.97} h_1(t) \doteq \frac{5.54}{12.97}(-12.97t^2 + 300)$$
$$\doteq -5.54t^2 + 128.14$$
$$\therefore \frac{5.54}{12.97} h_1(t) + 71.86 \doteq -5.54t^2 + 128.14 + 71.86$$
$$= -5.54t^2 + 200 = h_2(t)$$
$$\therefore h_2(t) \doteq \frac{5.54}{12.97} h_1(t) + 71.86$$

$\therefore$ the Saturn function can be obtained by stretching h_1 vertically by a factor of $\frac{5.54}{12.97}$ followed by a shift up 71.86 units.

(c) Consider the graphs for Jupiter and Saturn. Explain the *physical meaning* of the point(s) of intersection of the two graphs. **(2 KU, 2 COM)**

The Jupiter and Saturn graphs intersect where $t \doteq 3.7$ s. At this time, the objects have the same height above the surface of each respective planet

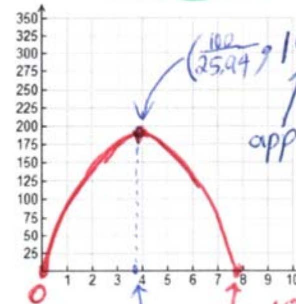
Roughwork

$$-12.97t^2 + 300 = 0$$
$$t^2 = \frac{300}{12.97}$$
$$t = \sqrt{\frac{300}{12.97}}$$

(d) State the domain and range of the *Jupiter function*. **(2 APP, 1 COM)**

$$D = \left\{ t \in \mathbb{R} \mid 0 \le t \le \sqrt{\frac{300}{12.97}} \right\}$$
$$R = \{ h \in \mathbb{R} \mid 0 \le h \le 300 \}$$

(e) Imagine that you could stand on the surface of Jupiter. You perform an experiment that involves firing cannonballs vertically into the air with different initial velocities. Suppose that you fired a cannonball with an initial velocity of 100 m/s. Then the height (in metres) of the cannonball at a given time t (in seconds) is modelled by the function $h(t) = -12.97t^2 + 100t$. What is the maximum height reached by the cannonball? How long does it take to return to the ground? **(4 APP, 2 COM)**



The cannonball is at ground level when $h(t) = 0$

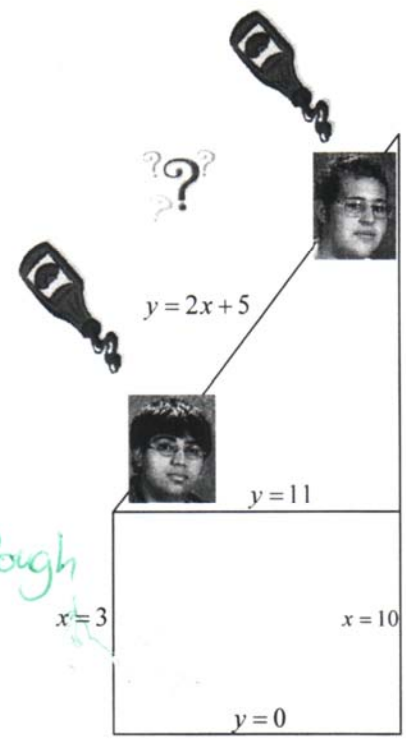
$$\therefore -12.97t^2 + 100t = 0$$
$$\therefore -12.97t\left(t - \frac{100}{12.97}\right) = 0$$
$$\therefore t = 0 \quad \text{or} \quad t = \frac{100}{12.97} \doteq 7.7\text{s}$$

The t-co-ordinate of the vertex is halfway between the roots:

$$\frac{1}{2}\left(0 + \frac{100}{12.97}\right) = \frac{100}{25.94} \doteq 3.86$$
$$h\left(\frac{100}{25.94}\right) = -12.97\left(\frac{100}{25.94}\right)^2 + 100\left(\frac{100}{25.94}\right) \doteq 193.$$

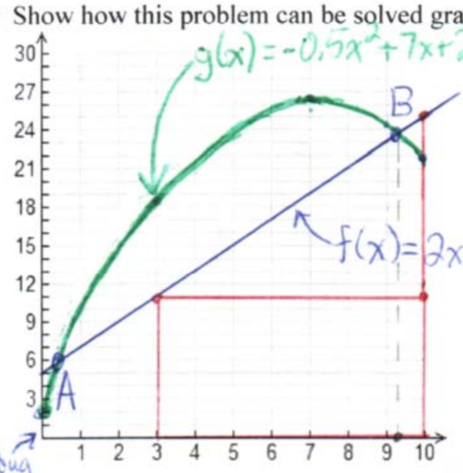
Therefore, the cannonball achieves a maximum height of about 193 m. It takes about 7.7 s for the cannonball to return to the ground.

5. Instead of paying attention during class discussions, Divye and Andrew constantly daydream about their favourite pastimes ("shoot 'em up" video games and eating respectively). Mr. Nolfi became so fed up with them that he decided that something needed to be done. Realizing that a detention would probably accomplish very little, Mr. Nolfi conceived a devious scheme that was sure to get their attention. As shown in the diagram, he forced Divye and Andrew to climb onto the roof of a house. Then Mr. Nolfi gave Doug a supply of balloons filled with ketchup and allowed him to fling them at the two daydreamers.



Suppose that the roofline is modelled by the linear function $y = f(x) = 2x + 5$ and that Doug flings a balloon whose *path* is modelled by the quadratic function $y = g(x) = -0.5x^2 + 7x + 2$. Assuming that Doug is standing at the origin of the co-ordinate system when he flings the balloons, who gets splattered with Ketchup, Andrew or Divye?

(a) Show how this problem can be solved graphically. **(2 TIPS, 1 COM)**



$g(x) = -0.5x^2 + 7x + 2$ Rough

x	$g(x)$
0	2
3	18.5
9	24.5
10	22

The graph of $y = g(x)$ intersects the graph of $y = f(x)$ at two points A and B as shown. Obviously, the balloon hits the roof at point B, so Andrew will get splattered!

(b) Now give an algebraic to this problem. **(5 TIPS, 3 COM)**

The balloon hits the roof when $g(x) = f(x)$.

$\therefore -0.5x^2 + 7x + 2 = 2x + 5$

$\therefore -0.5x^2 + 5x - 3 = 0$

$\therefore x^2 - 10x + 6 = 0$ (multiply B.S. of previous eq'n by -2)

$\therefore x = \dfrac{10 \pm \sqrt{(-10)^2 - 4(1)6}}{2} = \dfrac{10 \pm \sqrt{76}}{2}$

$\therefore x = \cancel{\dfrac{10 - \sqrt{76}}{2} \doteq 0.64}$ or $x = \dfrac{10 + \sqrt{76}}{2} \doteq 9.36$

Since the roofline begins at $x = 3$, $x \doteq 0.64$ is an irrelevant solution. Therefore, the ketchup balloon hits the roof approximately at the point with x-co-ordinate 9.36. Since Andrew is positioned very close to that point, he is splattered with ketchup. //