**MCR 3U0** **Semester 1, 2007 - 2008**

## Grade 11 Functions (University Preparation)
## Unit 2 – Trigonometric Ratios

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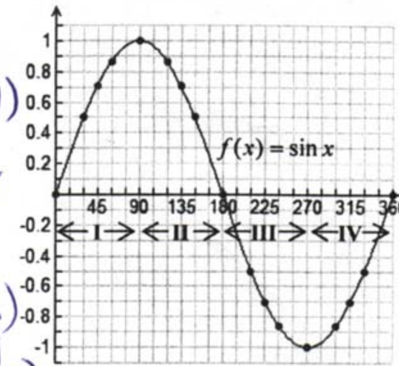
*Victim:* Mr. Solutions

| KU | APP | TIPS | COM |
|---|---|---|---|
| /20 | /17 | /10 | /22 |

1. Shown at the right is one cycle of the graph of $f(x) = \sin x$. List *four* important properties of the sine ratio that are immediately obvious from the graph. **(4 KU)**




(a) The sine of an angle must be $\geq -1$ and $\leq 1$ $(-1 \leq \sin\theta \leq 1)$

(b) Sine must be positive in quadrants I and II, negative in III & IV

(c) $\sin 0^\circ = \sin 180^\circ = 0$, $\sin 90^\circ = 1$, $\sin 270^\circ = -1$

(d) $\sin 30^\circ = \frac{1}{2}$; $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$ (and related ∠'s)

i.e. Range $= \{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$ (Sine ratio of special ∠'s)

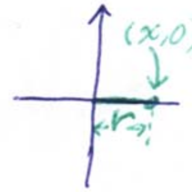
2. Use the special triangles or the unit circle to evaluate each of the following. You must give exact values. No approximations will be accepted! **(8 KU, 4 COM)**

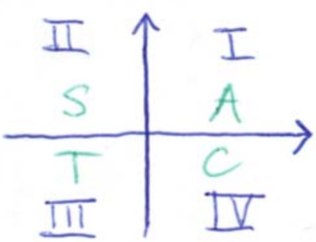
(a) $\cos 225^\circ$ (III)
$= -\cos 45^\circ$
$= -\frac{1}{\sqrt{2}}$

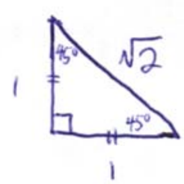
(b) $\sin 330^\circ$ (IV)
$= -\sin 30^\circ$
$= -\frac{1}{2}$

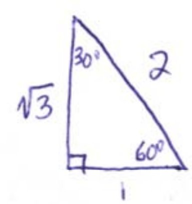
(c) $\tan 210^\circ$ (III)
$= \tan 30^\circ$
$= \frac{1}{\sqrt{3}}$

(d) $\sin 0^\circ$
$= \frac{y}{r}$
$= \frac{0}{r}$
$= 0$




(e) $\sec(315^\circ)$ (IV)
$= \sec 45^\circ$
$= \frac{1}{\cos 45^\circ}$
$= \sqrt{2}$

(f) $\cot 240^\circ$ (III)
$= \cot 60^\circ$
$= \frac{1}{\tan 60^\circ}$
$= \frac{1}{\sqrt{3}}$

(g) $\csc 120^\circ$ (II)
$= \csc 60^\circ$
$= \frac{1}{\sin 60^\circ}$
$= \frac{2}{\sqrt{3}}$

(h) $\csc 0^\circ$
$= \frac{1}{\sin 0^\circ}$
$= \frac{1}{0}$, which is undefined

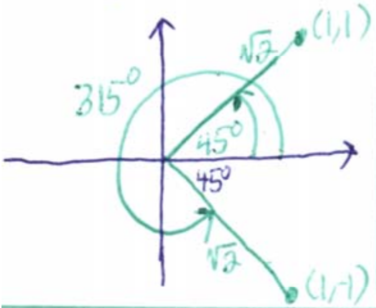
**3.** Solve for $\theta$. In each case, $0 \le \theta \le 360°$. **(8 KU, 4 COM)** Use special $\Delta$'s and ASTC (calculator needed for (c))

**(a)** $\cos\theta = \frac{1}{\sqrt{2}}$ (I, IV)

$\therefore \theta = 45°$ or

$\theta = 360° - 45°$ (IV)

$= 315°$

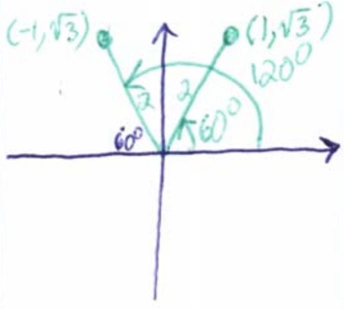



**(b)** $\csc\theta = \frac{2}{\sqrt{3}}$ (I, II)

$\therefore \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore \theta = 60°$ or

$\theta = 180° - 60° = 120°$




**(c)** $\tan\theta = -0.88345632$ (II and IV)

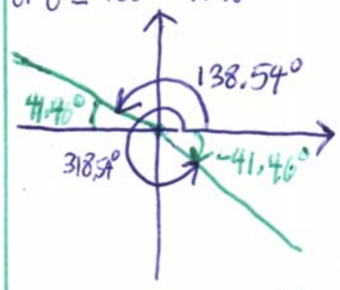
$\therefore \theta = \tan^{-1}(-0.88345632)$

$\therefore \theta \doteq -41.46°$ (calculator)

But, $0 \le \theta \le 360°$

$\therefore \theta \doteq 360° - 41.46° = 318.54°$

or $\theta \doteq 180° - 41.46° = 138.54°$




**(d)** $\sec\theta = \frac{1}{\sqrt{3}}$ (II, IV)

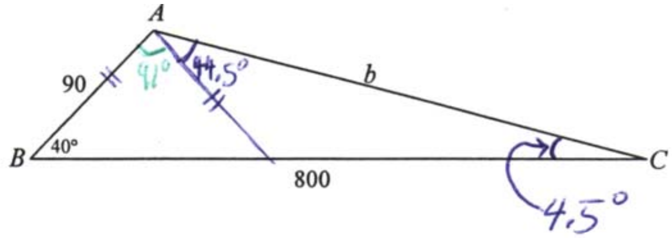
$\therefore \cos\theta = \sqrt{3}$, which is impossible because for all $\theta \in \mathbb{R}$, $-1 \le \cos\theta \le 1$.

Therefore, this equation has no solutions.

**4.** State whether each of the following is true or false. Provide an explanation to support each response. **(6 TIPS, 3 COM)**

| *Statement* | *True or False?* | *Explanation* |
|---|---|---|
| $\sin(x+y) = \sin x + \sin y$ | F | Consider the following counterexample: $\sin(30° + 150°) = \sin 180° = 0$ However, $\sin 30° + \sin 150° = \frac{1}{2} + \frac{1}{2} = 1 \ne 0$ $\therefore$ in general, $\sin(x+y) \ne \sin x + \sin y$ |
| $\frac{\cancel{\tan} x}{\cancel{\tan} 4x} = \frac{\cancel{x}}{4\cancel{x}} = \frac{1}{4}$ | F | We cannot divide both the numerator and the denominator by "tan" since it is the name of a function, not a number! Also, consider the following counterexample: $\frac{\tan 30°}{\tan 4(30°)} = \frac{\tan 30°}{\tan 120°} = \frac{\frac{1}{\sqrt{3}}}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \times \left(\frac{1}{-\sqrt{3}}\right) = -\frac{1}{3} \ne \frac{1}{4}$ |
| $\frac{\sin^{\cancel{2}} \cancel{x} \tan \cancel{x}}{\cos^{\cancel{2}} \cancel{x^2}}$ $= \frac{\sin\tan}{\cos}$ $= \frac{\sin}{\cos}\left(\frac{\tan}{1}\right)$ $= \tan(\tan)$ $= \tan^2$ | F | The given series of steps are utterly ridiculous! $\frac{\sin^2 x \tan x}{\cos^2 x^2}$ must evaluate to a number. However, $\tan^2$ by itself is NOT a number. The function "tan" must be given an "input value" (called an argument) before it can be evaluated. |

**5.** Solve the triangle shown at the right. **(6 APP, 3 COM)**




By the law of cosines,

$b^2 = 90^2 + 800^2 - 2(90)(800)\cos 40^\circ$

$\therefore\ b \doteq 733.34$

By the law of sines,

$\frac{\sin A}{800} = \frac{\sin 40^\circ}{b}$

$\therefore \sin A = \frac{800 \sin 40^\circ}{b} \doteq \frac{800 \sin 40^\circ}{733.34}$

$\therefore \angle A \doteq \sin^{-1}\left(\frac{800 \sin 40^\circ}{733.34}\right)$

$\therefore \angle A \doteq 44.5^\circ$ or $\angle A \doteq 180^\circ - 44.5^\circ = 135.5^\circ$

Clearly, $\angle A \neq 44.5^\circ$ because the longest side of a triangle must be opposite the largest angle. (see diagram)

Therefore, $\angle A \doteq 135.5^\circ$ and

$\angle C = 180^\circ - \angle A - \angle B$

$\doteq 180^\circ - 135.5^\circ - 40^\circ$

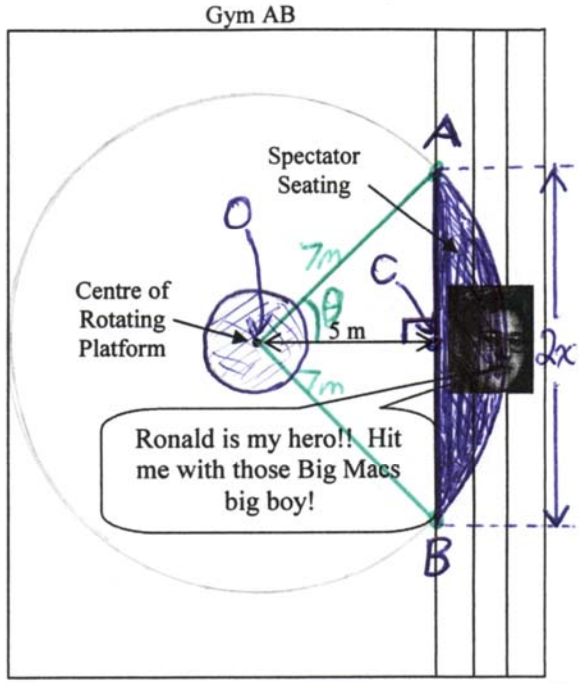
$\therefore \angle C \doteq 4.5^\circ$.

6. Prove that $\cot x \csc x(\sec x - 1) = \frac{1}{1+\cos x}$. **(6 APP, 3 COM)**

The left-hand-side appears to be much more complicated than the right-hand-side. Therefore, it's reasonable to work on the L.H.S first.

$$
\begin{aligned}
L.S. &= \cot x \csc x (\sec x - 1) \\
&= \frac{\cos x}{\sin x}\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x} - 1\right) && \text{(quotient identity and reciprocal identities)} \\
&= \frac{\cancel{\cos x}}{\sin^2 x}\left(\frac{1-\cos x}{\cancel{\cos x}}\right) && \text{(simplification)} \\
&= \frac{1-\cos x}{\sin^2 x} && \text{(divide top and bottom by } \cos x\text{)} \\
&= \frac{1-\cos x}{1-\cos^2 x} && \text{(Pythagorean identity)} \\
&= \frac{\cancel{1-\cos x}}{\cancel{(1-\cos x)}(1+\cos x)} && \text{(factor, difference of squares)} \\
&= \frac{1}{1+\cos x} && \text{(divide top and bottom by } 1-\cos x\text{)} \\
&= R.S. && \longrightarrow \therefore L.S. = R.S.
\end{aligned}
$$

7. As a promotion for McDonald's Canada, Ronald McDonald made a special guest appearance at a CPSS basketball game. At half time, he stood at the centre of a rotating platform and used a pneumatic (air-powered) cannon to fire free Big Macs toward the spectators.




(a) If the spectators are located 5 m from the centre of the rotating platform and Ronald can fire the Big Macs a maximum distance of 7m, what ***length*** (in metres) of the seating area will be hit by Big Macs? **(5 APP, 3 COM)**

Let the required length be represented by $2x$. Then, $AB = 2x$ and

$x^2 = 7^2 - 5^2$ (Pythagorean Theorem)

$\therefore x^2 = 24$

$\therefore x = \sqrt{24} = 2\sqrt{6}$

$\therefore 2x = 4\sqrt{6} \doteq 9.8$

Therefore, $4\sqrt{6}$ m of the seating area will be pelted with Big Macs. //

(b) What ***area*** (in square metres) of the spectator seating will be hit with Big Macs? (Assume that the wall behind the spectators is at least 7m away from the centre of the rotating platform.) **(4 TIPS, 2 COM)**

Let $2\theta$ represent the measure of $\angle AOB$.

Then, $\angle AOC = \theta$.

$\therefore \cos\theta = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\left(\frac{5}{7}\right) \doteq 44.4^\circ$

$\therefore 2\theta \doteq 88.8^\circ$

Now, area of sector AOB

$\doteq \frac{88.8^\circ}{360^\circ}$ (area of entire circle)

$= \frac{88.8^\circ}{360^\circ}(\pi(7^2))$

$\doteq 38.0 \text{ m}^2$

Also,

area of $\Delta AOB = \frac{bh}{2}$

$= \frac{(4\sqrt{6})(5)}{2}$

$= 10\sqrt{6}$

$\therefore$ required area

$=$ (area of sector AOB) $-$ (area $\Delta AOB$)

$\doteq 38.0 \text{ m}^2 - 10\sqrt{6} \text{ m}^2$

$\doteq 13.5 \text{ m}^2$

Therefore, about $13.5 \text{ m}^2$ of the seating area will be pelted with Big Macs. //