

Grade 11 Functions (University Preparation)
Unit 2 – Test 2 – Trigonometric Functions

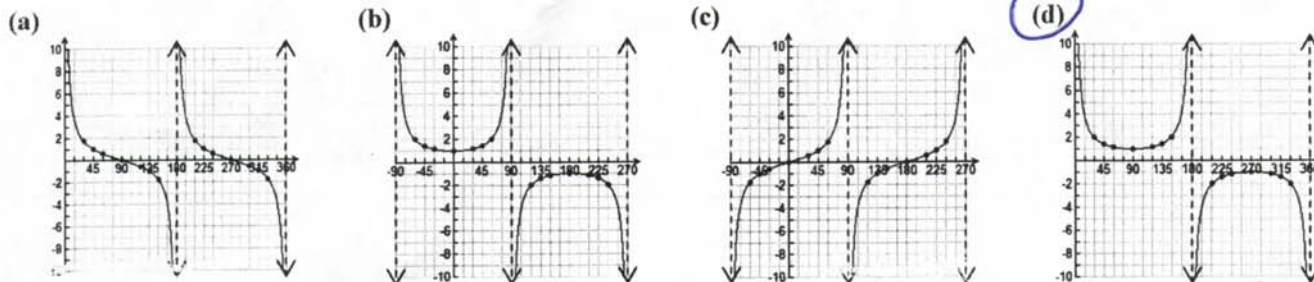
Mr. N. Nolfi

Victim:

M. solutions

KU	APP	TIPS	COM
13 /13	16 /16	10 /10	10 /10

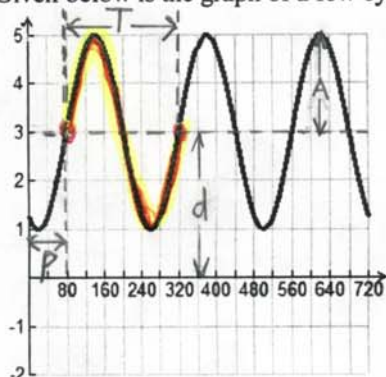
1. Circle the graph of $f(x) = \csc x$. Then explain your choice. (1 KU, 2 COM)



Explain your choice.

Since $\sin 0^\circ = 0$, $\csc 0^\circ = \frac{1}{0}$, which is undefined. Since $\sin 90^\circ = 1$, $\csc 90^\circ = \frac{1}{1} = 1$. The graph in (d) is the only one that satisfies the above.

2. Given below is the graph of a few cycles of a sinusoidal function.



- (a) Given that the base function is $f(x) = \sin x$, state the values of A , d , T , p and k . (5 KU)

$A = 2$ $d = 3$ $T = 240^\circ$ $p = 80^\circ$ $k = \frac{360^\circ}{240^\circ} = \frac{3}{2}$

- (b) State an equation of the given sinusoidal function. (2 KU)

$f(x) = 2 \sin\left[\frac{3}{2}(x - 80^\circ) + 3\right]$

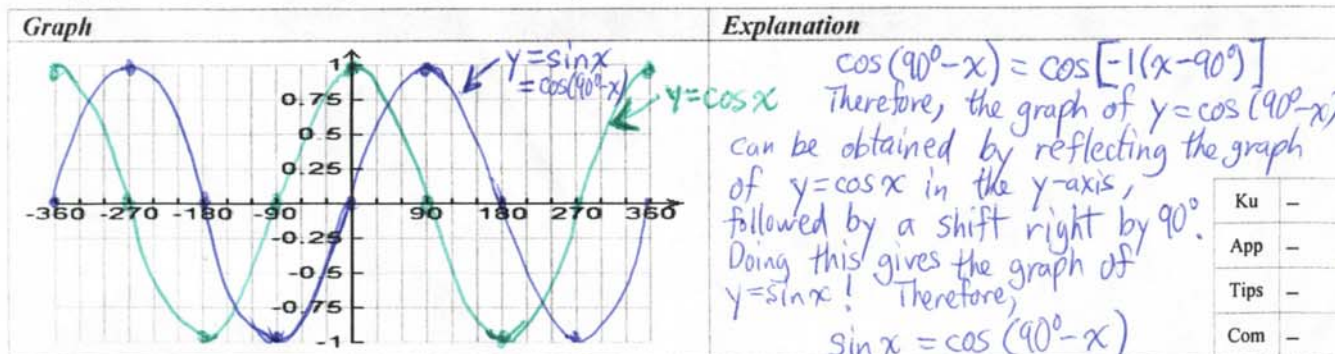
- (c) State the domain and the range of the given sinusoidal function. (2 KU)

$D = \mathbb{R}$, $R = \{y \in \mathbb{R} \mid 1 \leq y \leq 5\}$

- (d) What transformations would need to be applied to $f(x) = \sin x$ to obtain the given graph? (2 KU)

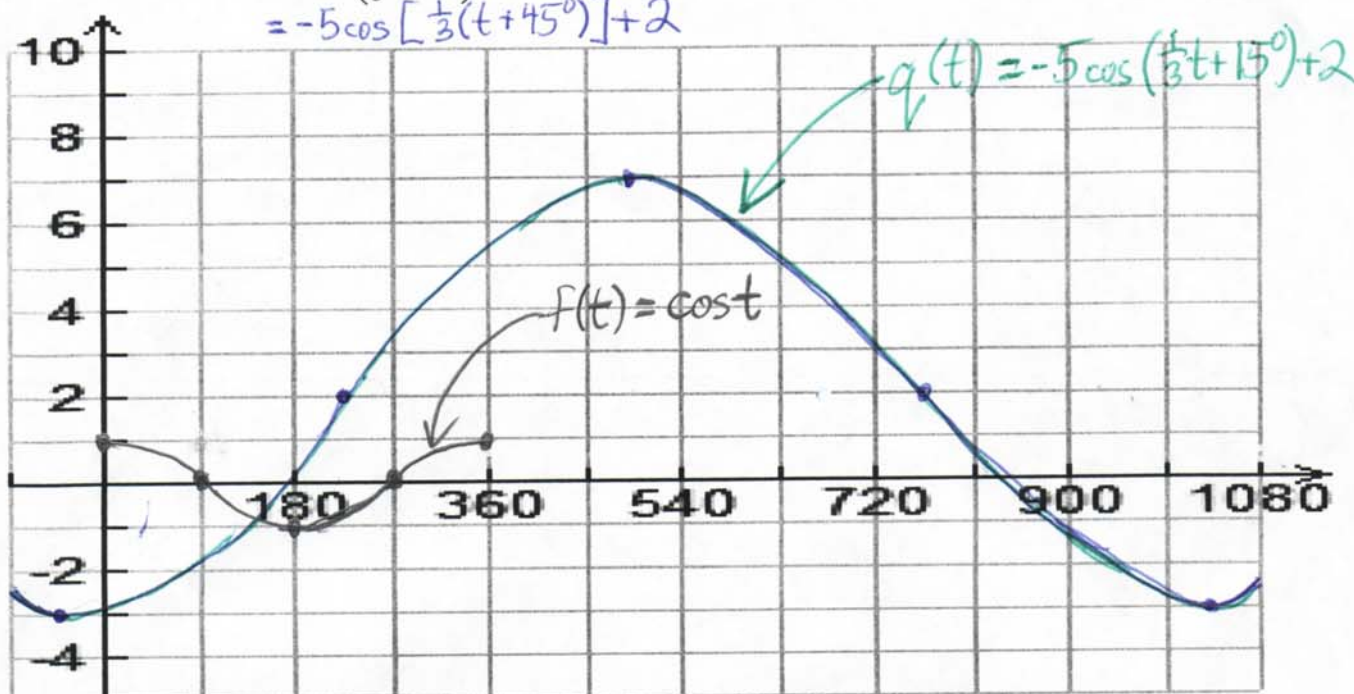
Vertical	Horizontal
1. Stretch vertically by a factor of 2	1. Compress horizontally by a factor of $\frac{2}{3}$
2. Shift (translate) up 3 units	2. Shift right by 80°

3. Use a graphical argument to explain why $\sin x = \cos(90^\circ - x)$ for all $x \in \mathbb{R}$. (2 TIPS, 1 COM)



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Tips	-
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4. Sketch the graph of $q(t) = -5\cos\left(\frac{1}{3}t + 15^\circ\right) + 2$ for $-90^\circ \leq t \leq 1080^\circ$. (4 APP, 2 COM)



Rough Work

$$(x, y) \rightarrow (3x - 45^\circ, -5y + 2)$$

$$\therefore (0, 1) \rightarrow (-45^\circ, -3)$$

$$(90^\circ, 0) \rightarrow (225^\circ, 2)$$

$$(180^\circ, -1) \rightarrow (495^\circ, 7)$$

$$(270^\circ, 0) \rightarrow (765^\circ, 2)$$

$$(360^\circ, 1) \rightarrow (1035^\circ, -3)$$

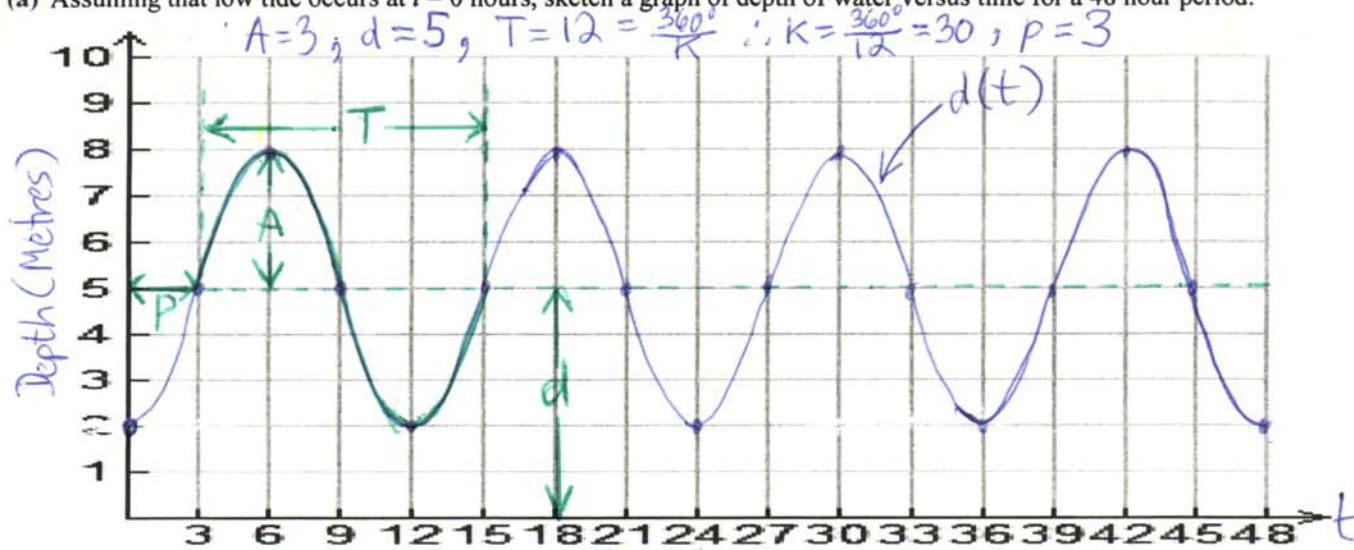
5. State whether each of the following is true or false. Provide an explanation to support each response. (4 TIPS, 2 COM)

Statement(s)	True or False?	Explanation
$\sin(90^\circ - x) = \sin 90^\circ - \sin x$ $\therefore \sin(90^\circ - x) = 1 - \sin x$	F	Remember that for most functions f, $f(x+y) \neq f(x) + f(y)$ Consider $x = 45^\circ$ L.S. $= \sin(90^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$ R.S. $= 1 - \sin 45^\circ = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} \neq \frac{1}{\sqrt{2}}$ $\therefore$ L.S. $\neq$ R.S.
Let f represent any periodic function and let T represent the period of f. Then, $f(x) = f(x) + f(T)$	F	If a function f is periodic, $f(x+T) = f(x)$ To show that the given statement is false, consider $f(x) = \sin 3x$ for which $T = \frac{360^\circ}{3} = 120^\circ$ $f(45^\circ) = \sin 135^\circ = \frac{1}{\sqrt{2}}$ $f(45^\circ) + f(T) = f(45^\circ) + f(120^\circ) = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \neq \frac{1}{\sqrt{2}}$

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6. The average depth of the water in a port on a tidal river is 5 m. At low tide, the depth of the water is 2 m and one cycle is completed every 12 hours. (6 APP, 3 COM) Low tide = 2m, Average = 5m $\therefore$ high tide = 8m $\frac{8+2}{2} = 5$

(a) Assuming that low tide occurs at $t = 0$ hours, sketch a graph of depth of water versus time for a 48 hour period.



(b) Write an equation of the function $d(t)$, the depth of the water in metres, t hours after low tide.

$$d(t) = 3 \sin[30(t-3^\circ)] + 5$$

7. The rodent population in a particular region varies with the number of predators that inhabit the region. At any time t years since 1976, the number of rodents $r(t)$ is given by the function $r(t) = 1500 \sin(45^\circ t) + 2500$.

(a) What is the period of this function? (1 KU)

$$T = \frac{360^\circ}{K} = \frac{360^\circ}{45^\circ} = 8 \text{ years}$$



(b) Rewrite the function $r(t)$ in such a way that the rodent population cycle lasts for 12 years. (2 APP)

$$T = \frac{360^\circ}{K} \therefore K = \frac{360^\circ}{T} = \frac{360^\circ}{12 \text{ years}} = 30^\circ \text{ per year}$$

$$\therefore r(t) = 1500 \sin(30^\circ t) + 2500$$

(c) What is the maximum number of rodents in any given cycle? (2 APP)

Since each cycle lasts 8 years, a max must occur 2 years after the beginning of each cycle.

$\therefore$ a max occurs at $t = 2$ and

$$d(2) = 1500 \sin(45^\circ \times 2) + 2500 = 4000$$

(d) Between 1976 and 2004, in which years did the rodent population reach a minimum? (4 TIPS)

Since each cycle lasts 8 years, a min occurs 6 years after the beginning of each cycle. Therefore, rodent population should reach a minimum at $t = 6, t = 6 + 8 = 14, t = 6 + 2(8) = 22$, etc. This corresponds to the years 1982, 1990 and 1998.

(e) How many rodents would you predict for the year 2018? (2 APP)

$$t = \# \text{ years} = 2018 - 1976 = 42$$

$$r(42) = 1500 \sin(45^\circ(42)) + 2500 = 4000$$

For 2018, the model predicts a rodent population of 4000. //

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