MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions

Unit 1 – Major Test 1 – Functions, Relations and Transformations

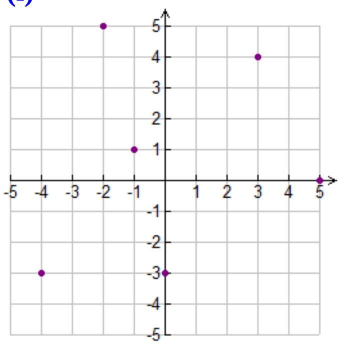
Mr. N. Nolfi

Victim: Mr. Solutions Inspiring words Mr. S.!! ☺

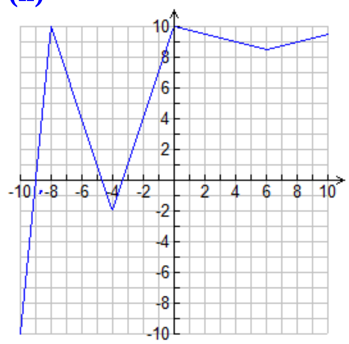
KU	APP	TIPS	COM
20 /20	25 /25	16 /16	10 /10

1. Study each graph carefully and then answer the questions found immediately below the graphs. **(20 KU)**

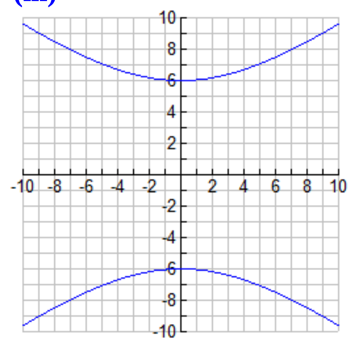
(i)



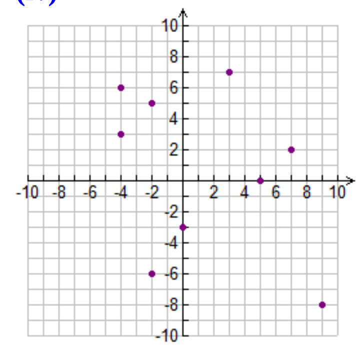
(ii)



(iii)



(iv)



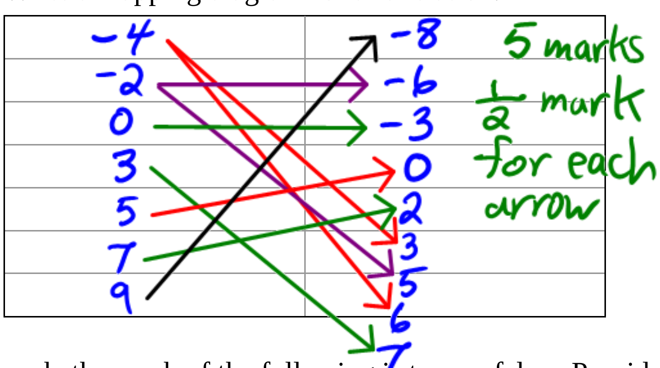
(a) Which of the above relations are continuous? ii, iii ✓ ✓

(b) Which of the above relations are functions? ✓ i, ii ✓

(c) One of the above relations is a discrete function. Write the function as a set of ordered pairs.

$\{(-4,-3), (-2,5), (-1,1), (0,-3), (3,4)\}$ ✓ ✓

(d) One of the above discrete relations ***is not*** a function. Write a mapping diagram for this relation.



5 marks
½ mark for each arrow

(e) Suppose that the continuous function given above is called f. Evaluate each of the following.

$f(0) =$ 10 ✓	$f(a)=1 \quad \therefore a =$ -9, -5, -3 ✓ ✓ ✓
$f(3) \doteq$ 9.2 ✓	$f(-b)=0 \quad \therefore b \doteq$ 9, 4.8, -3.2 ✓ ✓ ✓
$f(-4) =$ -2 ✓	$f(3-5) =$ f(-2) = 4 ✓

2. State whether each of the following is true or false. Provide an explanation to support each response. **(8 TIPS)**

Statement	***True or False?***	***Explanation***
For all functions f and all real numbers u and c, $f(u+c) = f(u) + f(c)$ ✓ = ½ mark	F ✓	e.g. $f(x)=\sqrt{x}$, $u=16$, $c=9$ $f(u+c)=f(16+9)=f(25)=\sqrt{25}=5$ $f(u)+f(c)=f(16)+f(9)=\sqrt{16}+\sqrt{9}=4+3=7$ ∵ $5 \neq 7$, the given statement must be false! ✓✓
The equation $\frac{x^2}{16}+\frac{y^2}{9}=1$ describes a function.	F ✓	e.g. Let $x=3$. Then $\frac{3^2}{16}+\frac{y^2}{9}=1$ ✓✓ $\therefore \frac{y^2}{9}=\frac{16}{16}-\frac{9}{16}=\frac{7}{16}$ $\therefore y^2=\frac{63}{16} \quad \therefore y=\pm\sqrt{\frac{63}{16}}$ For a single value of x, there are two values of y ∴ not a function

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– 0	–	–	– 0

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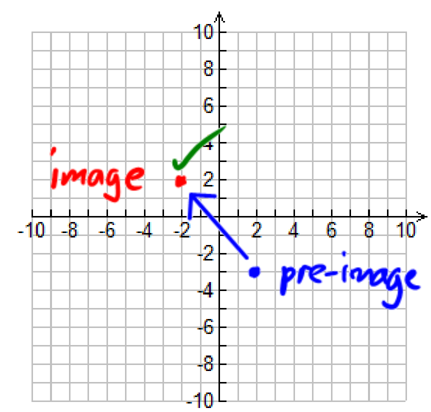
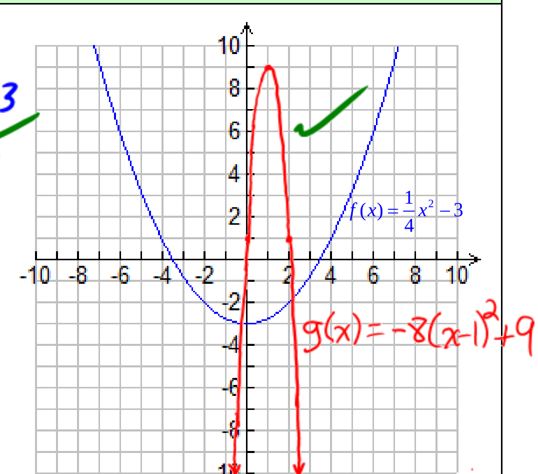
Wrong order!

Statement	*True or False?*	*Explanation*
For the function $g(x)=\sqrt{x+3}-5$, $D=\{x\in\mathbb{R}:x\geq -3\}$ and $R=\{y\in\mathbb{R}:y\geq -5\}$. (Here D and R represent domain and range respectively.)	T	For $\sqrt{x+3}$ to be defined, $x+3\geq 0$ $\therefore x\geq -3$. Since $\sqrt{x+3}\geq 0$, $\sqrt{x+3}-5\geq -5$
Suppose that $g(x)=-3f(2x-8)+6$. To obtain the graph of g, the following transformations must be performed to f: • Vertical stretch by a factor of –3 followed by a shift up by 6 units • Horizontal compression by a factor of 1/2 followed by a shift 8 units right.	F	The horizontal transformations are obtained by transforming f's input $2x-8$ to x: $\boxed{2x-8}\xrightarrow{+8}\boxed{2x}\xrightarrow{\times\frac{1}{2}}\boxed{x}$ $\therefore$ the correct horiz. trans. is $x\rightarrow\frac{1}{2}(x+8)$, which is a shift 8 units right FOLLOWED by a compression by a factor of $\frac{1}{2}$.

3. Complete the following table. **(5 APP)**

Pre-image	$(2,-3)$	*Trans-formation in Mapping Notation*	$(x,y)\rightarrow(-3x+4,-y-1)$	*Graph*	
Trans-formation	*Horizontal* 1. Stretch by a factor of –3. 2. Translate 4 units right *Vertical* 1. Reflect in the x-axis. 2. Translate down 1 unit.	*Image*	$-3(2)+4=-2$ $-(-3)-1=2$ $\therefore$ image is $(-2,2)$		

4. Complete the following table. **(8 APP)**

Equation of Pre-image Function	*Transformation*	*Equation of Image Function*	*Graph of y = g(x)*
$f(x)=\frac{1}{4}x^2-3$	*Verbal* Horizontal 1. Compress by factor of $\frac{1}{4}$ 2. Translate 1 right Vertical 1. Stretch by factor of –2 2. Translate 3 up *Function Notation* $g(x)=-2f(4(x-1))+3$ *Mapping Notation* $(x,y)\rightarrow(\frac{1}{4}x+1,-2y+3)$	$g(x)=-2f(4(x-1))+3$ $=-2\left[\frac{1}{4}(4(x-1))^2-3\right]+3$ $=-\frac{1}{2}(16)(x-1)^2+6+3$ $=-8(x-1)^2+9$ $\therefore g(x)=-8(x-1)^2+9$	

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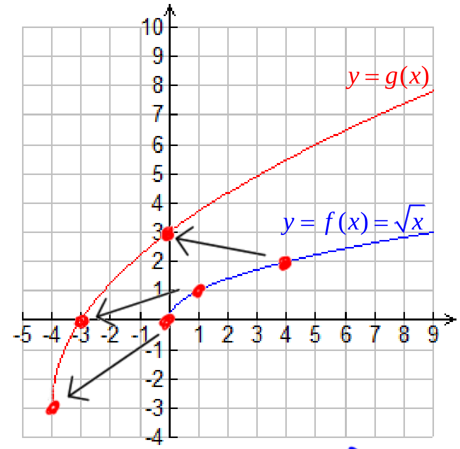
5. The graph of $y = g(x)$ is a transformation of the graph of $y = f(x) = \sqrt{x}$



$+1$ $(0,0) \rightarrow (-4,-3)$ $+1$
$+1$ $+3$
$(1,1) \rightarrow (-3,0)$
$+3$ $+1$ $+3$
$(4,2) \rightarrow (0,3)$ $+3$

vert. stretch by factor of 3

(a) Using both ***function notation*** and ***mapping notation***, state how $y = f(x) = \sqrt{x}$ can be transformed into $y = g(x)$. **(4 APP)**

$g(x) =$ 3 $f($ $x+4$ $)$ -3

(or $g(x) = f(9(x+4)) - 3$)

$(x, y) \rightarrow ($ $x-4$, $3y-3$ $)$

(or $(x,y) \rightarrow (\frac{1}{9}x - 4, y-3)$)

(b) State an equation of g. **(2 APP)**

$g(x) = 3\sqrt{x+4} - 3$

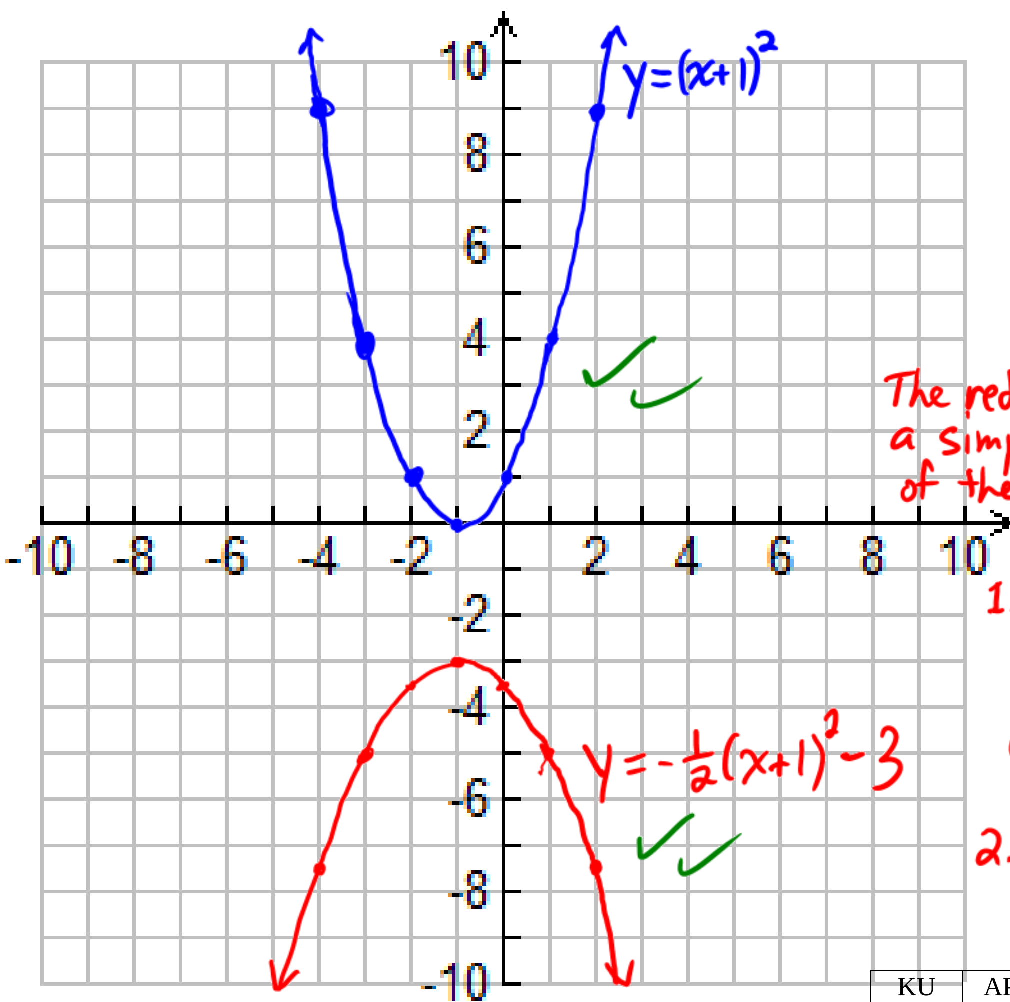
(or $g(x) = \sqrt{9(x+4)} - 3$)

(c) State the domain and range of g. **(2 APP)**

$D = \{x \in \mathbb{R} \mid x \geq -4\}$

$R = \{y \in \mathbb{R} \mid y \geq -3\}$

6. Sketch the graphs of $y = (x+1)^2$ and $y = -\frac{1}{2}(x+1)^2 - 3$ on the same set of axes. **(4 APP)**



The red graph is a simple transformation of the blue graph:

1. Compress vertically by a factor of $-\frac{1}{2}$ (compression & reflection)
2. Translate down 3 units

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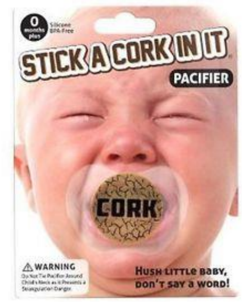
7. In an attempt to reduce the amount of noise in room 224, Mr. Nolfi decided to give his most talkative student a surprise "gift," a beautifully decorated box containing a cork pacifier. Unbeknown to the student, the box was fitted with a spring-loaded device that was designed to launch the cork vertically upwards upon raising of the box's lid.

Mr. Nolfi tested two different springs to see which would perform better. The results of his experiment are listed below. In each case, the function gives the height of the cork above the ground in metres, *t* seconds after the box's lid is raised.



Spring 1	***Spring 2***
$h_1(t) = -4.9t^2 + 5t$	$h_2(t) = -4.9t^2 + 7t$

(a) The most talkative student, ______________________, is about 1.7 m tall. Assuming that the box rests on the ground when the lid is raised by ______________________ and that he/she is standing upright, which spring would have a better chance of launching the cork directly into his/her mouth? **(4 TIPS)**

$h_1(t) = -4.9\left(t^2 - \frac{5}{4.9}t\right)$

$= -4.9\left[t^2 - \frac{5}{4.9}t + \left(\frac{5}{9.8}\right)^2 - \left(\frac{5}{9.8}\right)^2\right]$

$= -4.9\left(t - \frac{5}{9.8}\right)^2 + 4.9\left(\frac{5}{9.8}\right)^2$

$\therefore$ max height for spring 1 is $4.9\left(\frac{5}{9.8}\right)^2 \doteq 1.3$ ✓

$h_2(t) = -4.9\left(t^2 - \frac{7}{4.9}t\right)$

$= -4.9\left[t^2 - \frac{7}{4.9}t + \left(\frac{7}{9.8}\right)^2 - \left(\frac{7}{9.8}\right)^2\right]$

$= -4.9\left(t - \frac{7}{9.8}\right)^2 + 4.9\left(\frac{7}{9.8}\right)^2$

$\therefore$ max height for spring 2 is $4.9\left(\frac{7}{9.8}\right)^2 \doteq 2.5$ ✓

A case can be made for either spring! ✓✓

Spring 1: Can't reach the top of ____'s head BUT it might just be able to reach ____'s mouth. To know for certain, we would need to know the distance from the top of ____'s head to his/her mouth.

Spring 2: Certainly has a great enough upward velocity to reach ____'s mouth BUT it could overshoot the target because the max height is far above ______'s head

(b) How can the function h_1 be transformed into the function h_2? **(4 TIPS)**

From the vertex forms of the equations of h_1 and h_2, it's clear that both functions have the same ✓ vertical stretch factor (-4.9) and the same horizontal stretch factor (1). Thus, it is only necessary to ✓ translate the vertex of h_1 to the vertex of h_2:

Mapping Notation $(x, y) \rightarrow \left(x + \frac{2}{9.8},\ y + \underbrace{4.9\left(\frac{7}{9.8}\right)^2 - 4.9\left(\frac{5}{9.8}\right)^2}\right)$ ✓ ✓

It's only necessary to provide one of these 2.

approx 0.2 approx 1.2

Function Notation $h_2(t) = h_1\left(t - \frac{2}{9.8}\right) + 4.9\left(\frac{7}{9.8}\right)^2 - 4.9\left(\frac{5}{9.8}\right)^2$

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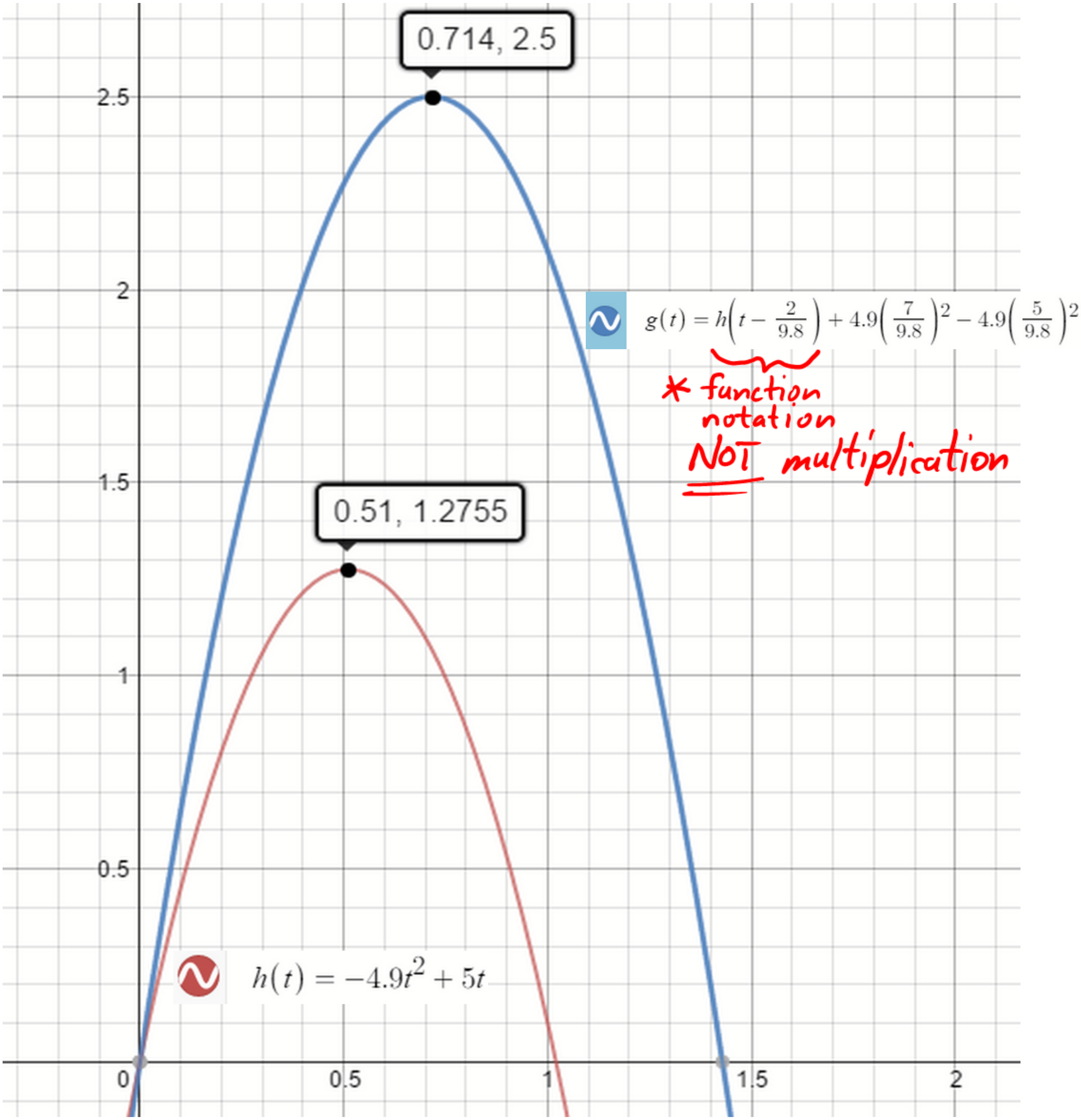
(see graph on next page →)



0.714, 2.5
0.51, 1.2755
$g(t) = h\left(t - \frac{2}{9.8}\right) + 4.9\left(\frac{7}{9.8}\right)^2 - 4.9\left(\frac{5}{9.8}\right)^2$
* function notation
NOT multiplication
$h(t) = -4.9t^2 + 5t$
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2