MCR3U9: Unit 1 Minor Test – The Good, the Bad and the UGLY

Insanity: doing the same thing over and over again and expecting different results.

(This quotation is often attributed to Einstein but there is no solid evidence to support this claim.)

Good Responses	***Bad and UGLY Responses***
$B(n)=\begin{cases}0, & 0\le n<10\\ 50, & 10\le n<30\\ 100, & 30\le n<50\\ 150, & n\ge 50\end{cases}$ **(5 APP)** $B(n)=\begin{cases}0, & n<10\\ 50, & 10\le n<30\\ 100, & 30\le n<50\\ 150, & n\ge 50\end{cases}$	**(5 APP)** $B(n)=\begin{cases}n=0, & \to n<10 \quad 0\le n<10\\ n=50, & \to 10\le n<30\\ n=100, & \to 30\le n<50\\ n=150 & \to 50\le n\end{cases}$ −1C −1C $B(n)=\begin{cases}y=0, & 0\le n<10\\ y=50, & 10\le n<30\\ y=100, & 30\le n<50\\ y=150, & n\ge 50\end{cases}$ $B(n)=\begin{cases}0n, & 0\le n<10\\ 50n, & 10\ge n<30\\ 100n, & 30\ge n<50\\ 150n, & n>50.\end{cases}$
(a) Show that the function $g(x)=af(x)$, where a represents any non-zero real number, has exactly the same x-intercepts as f. **(5 TIPS)** This is true for any function f (f doesn't need to be a quadratic function). The co-ordinates of the x-intercepts are $(r_1,0)$ and $(r_2,0)$. The transformation given above can be expressed in mapping notation as follows: $(x,y)\to(x,ay)$ $\therefore (r_1,0)\to(r_1,a(0))=(r_1,0)$ and $(r_2,0)\to(r_2,a(0))=(r_2,0)$ $\therefore$ the points $(r_1,0)$ and $(r_2,0)$ are invariant under the transformation $\therefore$ g has the same x-intercepts as f	(a) Show that the function $g(x)=af(x)$, where a represents any non-zero real number, has exactly the same x-intercepts as f. (5 TIPS) $f(x)=a(x-r_1)(x-r_2)$ $=a(x-(1-\sqrt5))(x-(1+\sqrt5))$ $5=a(4-(1-\sqrt5))(4-(1+\sqrt5))$ $5=a(5.23)(0.77)$ $\frac{5}{4}=\frac{a(4)}{4}$ $5/4=a$ $f(x)=\frac{5}{4}(x-(1-\sqrt5))(x-(1+\sqrt5))$? −6T How did $1-\sqrt5$ and $1+\sqrt5$ suddenly enter the picture?

Good Responses

(a) Determine the equation of $T_E^{-1}(x)$. (4 APP)

Apply the transformation $(x,y)\to(y,x)$:

$\therefore x = 0.03y^2 + 245.50$ ✓

$\therefore x - 245.50 = 0.03y^2$

$\therefore \dfrac{x-245.50}{0.03} = y^2$ ✓

$\therefore \pm\sqrt{\dfrac{x-245.50}{0.03}} = y$ ✓

Since $y>0$, $T_E^{-1}(x) = \sqrt{\dfrac{x-245.50}{0.03}}$

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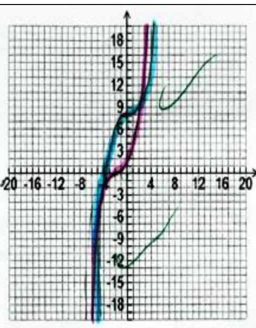
4. A pig-headed grade-9 student insists that $(x+2)^3 = x^3+8$. You, being a far more mature, experienced and wiser grade-11 AP student obviously know better. Use your knowledge of transformations to prove that the grade-9 student is wrong! Note that a grid is provided so that you can illustrate your answer with graphs. (**Hint:** Use $f(x)=x^3$ as the base function.) (5 COM)



Transformations applied to $f(x)=x^3$ to get...

$(x+2)^3$

following the order opposite to BEDMAS to figure out horizontal transformations

$x+2$

$\downarrow$

$\boxed{-2}$ means shift 2 units left

$\downarrow$

x

✓

x^3+8

$x^3\ \boxed{+8}$

means shift 8 units up.

$\therefore (x+2)^3 \neq x^3+8$ because $(x+2)^3$ translates the graph of $f(x)=x^3$ 2 units to the left, whereas x^3+8 translates $f(x)=x^3$ 8 units up. ✓

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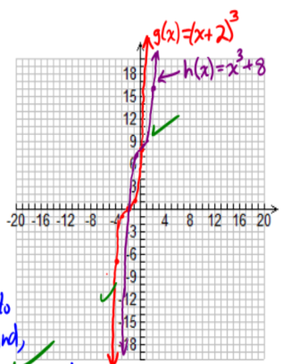


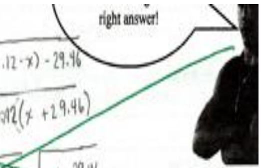
Let $g(x)=(x+2)^3 = f(x+2)$ and $h(x)=x^3+8 = f(x)+8$.

The graph of g is obtained by translating the graph of f 2 units to the left. ✓ The graph of h, on the other hand, is obtained by translating f 8 units upward. ✓

As shown in the diagram, the two graphs intersect at only two points, meaning that $(x+2)^3 \neq x^3+8$ except for $x=-2$ and $x=0$. ✓ Thus, the expressions $(x+2)^3$ and x^3+8 are NOT equivalent. The grade-9 student, as often is the case, was WRONG!!

Bad and UGLY Responses

(a) Determine the equation of $T_E^{-1}(x)$. (4 APP)



$T_E(x) = 0.03x^2 + 245.50$

$y = 0.03x^2 + 245.50$

$x = 0.03y^2 + 245.50$

$0 = 0.03y^2 + 245.50 - x$

$a=0.03$ $b=0$ $c=245.50-x$

$y = \dfrac{-(0)\pm\sqrt{0^2-4(0.03)(245.50-x)}}{2(0.03)}$

$= \dfrac{\pm\sqrt{(-0.12-x)-29.46}}{0.06}$

$= \frac{1}{0.06}\pm\sqrt{(-0.12-x)-29.46}$

$= \frac{1}{0.06}\pm\sqrt{0.12(x+29.46)}$

$= \frac{1}{0.06}\pm\sqrt{0.12}\sqrt{-x-29.46}$

$= \frac{1}{0.06}\pm 0.346\sqrt{-x-29.46}$ ← x is positive, $\therefore \{0<x<100\}$

$= \frac{1}{0.06} - 0.346\sqrt{-x-29.46}$

$= 16.321 - \sqrt{-x-29.46}$

$T_E^{-1}(x) = -\sqrt{x+29.46} + 16.321$

−4A

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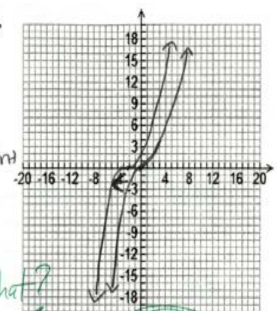

$(x+2)^3$

$=(x+2)(x+2)(x+2)$

? the 3rd exponent affects everything that is in the bracket.

$(x+2)^3$

Translate 2 units to the left Translate what?

$f(x) = af(b(x-h))+k$

−4C

$(x+2)$ affects the x co-ordinates on the horizontal axis and we know that whatever operation shown on the horizontal axis must be reversed

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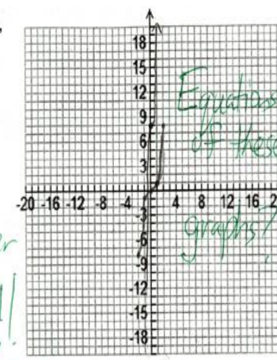

$(x+2)^3 \neq x^3+8$

$(x+2)(x+2)(x+2)$

$(x^2+2x+2x+4)(x+2)$

$(x^2+4x+4)(x+2)$

$(x^3+2x^2+4x^2+8x+4x+8)$

$(x^3+6x^2+12x+8) \neq x^3+8$

$\therefore (x+2)^3 \neq x^3+8$

You must answer the question being asked!!

−4C