MCR3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions

Minor Test – Unit 1 – Inverses of Functions, Using Transformations to Deepen Insight

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Victim: Mr. Solutions   Another example of superior mathematical reasoning Mr. S.!

KU	APP	TIPS	COM
14/14	21/21	8/8	15/7+8

v.1

1. Khadeeja, the smiley auto mechanic, is paid as described below:

- \$20/h for working up to 40 h per week
- time-and-a-half (\$20/h + \$10/h = \$30/h) for working overtime (hours worked beyond 40 in a single week)

For instance, if Khadeeja works 50 hours in a single week, she is paid a total of $40(20)+10(30)=1100$ dollars.

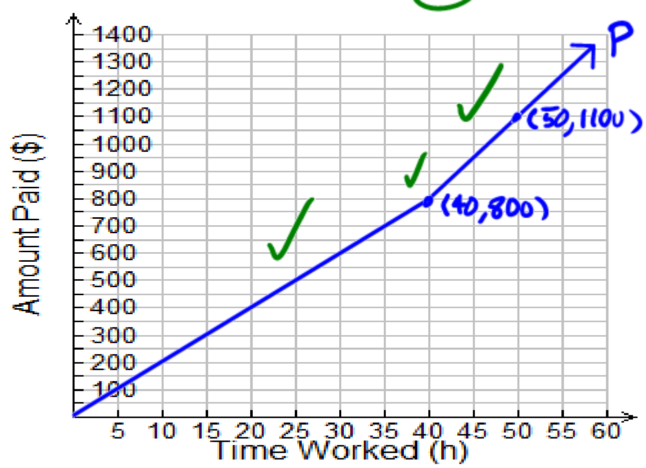
(a) Let $P(t)$ represent how much Khadeeja is paid per week for working t hours. Complete the definition of $P(t)$ found below. **(4 APP)** 4

Hint: The calculation shown above should be used as a guide for writing the expression for $t > 40$.

$$P(t)=\begin{cases} 20t, & 0\le t\le 40 \\ 30(t-40)+800, & t>40 \end{cases}$$

(or $30t-400$)

(b) Sketch a graph of P. **(3 APP)** 3



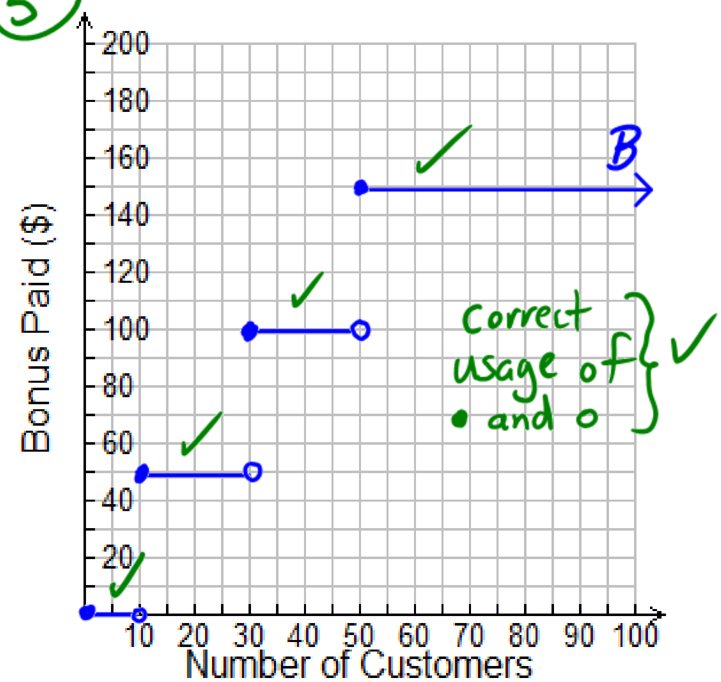
(c) Because of Khadeeja's exceptional customer service, her boss, Ms. I. Mama, decided to give her a monthly bonus. As shown in the table below, the bonus paid is a function of the number of customers who give Khadeeja a five-star rating.

Number of Customers who give Khadeeja a 5-Star Rating	***Bonus***
Fewer than 10	\$0
At least 10 but fewer than 30	\$50
At least 30 but fewer than 50	\$100
Fifty or more	\$150

Let $B(n)$ represent the monthly bonus Khadeeja receives if n customers give her a 5-star rating. Complete the definition of $B(n)$ found below. **(5 APP)** 5

$$B(n)=\begin{cases} 0, & 0\le n<10 \\ 50, & 10\le n<30 \\ 100, & 30\le n<50 \\ 150, & n\ge 50 \end{cases}$$

(d) Sketch a graph of B. **(5 APP)** 5



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– 0	– 0	– 0	– 0

2. Consider the function g defined by the equation $g(x) = -3\left|\frac{1}{2}(x-5)\right| + 15$.

(a) Use mapping notation to describe the transformations of the mother function $f(x) = |x|$ that would produce the function g. Then use the transformations to sketch the graph of g. **(5 KU)**

(2) $(x, y) \to (2x+5 ✓, -3y+15)$ ✓

(3) 2(a)

(2) 2(b)

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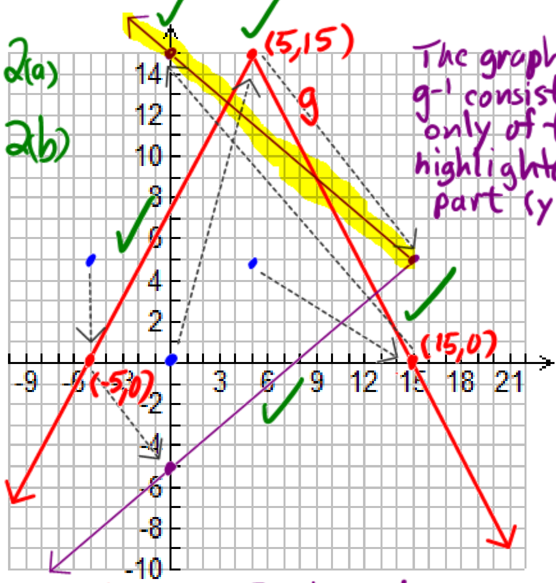

The graph of g^{-1} consists only of the highlighted part ($y \geq 5$)

Blue → f Red → g Purple → inverse relation of g

(b) Using the grid given in (a), sketch the graph of the ***inverse relation*** of g. **(2 KU)**

(c) State a restriction on the domain of g that ensures that g is one-to-one on the restricted domain. ***Briefly*** explain how you arrived at your answer. **(2 KU)**

$x \geq 5$ ✓

g is one-to-one for all $x \geq 5$ because it is strictly decreasing ✓

(d) Determine the equation of $g^{-1}(x)$ for the restricted domain stated in (c). **(5 KU)**

For $x \geq 5$, $g(x) = -\frac{3}{2}(x-5) + 15$

$= -\frac{3}{2}x + \frac{45}{2}$ ✓

To find g^{-1}, apply the transf. $(x,y) \to (y,x)$

$\therefore x = -\frac{3}{2}y + \frac{45}{2}$ ✓

$\therefore y = -\frac{2}{3}x + 15$, ✓ $x \leq 15$ ✓

$\therefore g^{-1}(x) = -\frac{2}{3}x + 15,\ x \leq 15$ ✓

3. The function $T_E(x) = 0.03x^2 + 245.50$ approximates the ***exhaust temperature***, in Fahrenheit degrees, of a diesel engine operating at x % of the maximum load on the engine ($0 < x < 100$).

This question mentions my name. You'd better get the right answer!

(a) Determine the equation of $T_E^{-1}(x)$. **(4 APP)**

Apply the transformation $(x,y) \to (y,x)$:

$\therefore x = 0.03y^2 + 245.50$ ✓

$\therefore x - 245.50 = 0.03y^2$

$\therefore \frac{x - 245.50}{0.03} = y^2$ ✓

$\therefore \pm\sqrt{\frac{x-245.50}{0.03}} = y$ ✓

y represents the % load on the engine

Since $y > 0$, $T_E^{-1}(x) = \sqrt{\frac{x-245.50}{0.03}}$, $x \geq 245.50$ ✓

(b) In the equation of $T_E^{-1}(x)$, what does x represent? Explain. **(2 COM)**

In $T_E^{-1}(x)$, x represents the exhaust ✓ temperature because for T_E^{-1}, the input is the same as the output for T_E.

Therefore, $x \geq 245.50$ because this is the ✓ minimum exhaust temperature according to the equation of T_E.

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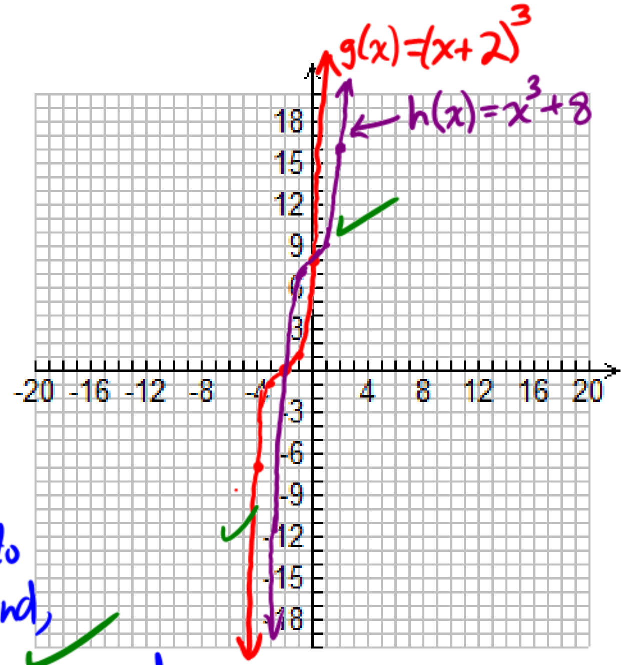
4. A pig-headed grade-9 student insists that $(x+2)^3 = x^3 + 8$. You, being a far more mature, experienced and wiser grade-11 AP student obviously know better. Use your knowledge of transformations to prove that the grade-9 student is wrong! Note that a grid is provided so that you can illustrate your answer with graphs. (**Hint:** Use $f(x) = x^3$ as the base function.) **(5 COM)**



Let $g(x) = (x+2)^3 = f(x+2)$ and
$h(x) = x^3 + 8 = f(x) + 8$.

The graph of g is obtained by translating the graph of f 2 units to the left. The graph of h, on the other hand, is obtained by translating f 8 units upward.

As shown in the diagram, the two graphs intersect at only two points, meaning that $(x+2)^3 \neq x^3+8$ except for $x=-2$ and $x=0$. Thus, the expressions $(x+2)^3$ and x^3+8 are NOT equivalent. The grade-9 student, as often is the case, was WRONG!!

5. Let r_1 and r_2 represent the x-intercepts of the quadratic function $f(x) = x^2 + bx + c$.

(a) Show that the function $g(x) = af(x)$, where a represents any non-zero real number, has exactly the same x-intercepts as f. **(5 TIPS)**

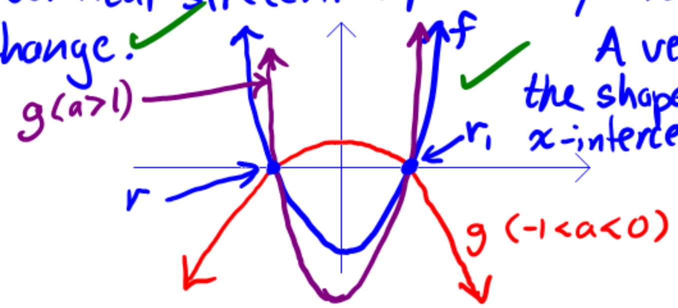
This is true for any function f (f doesn't need to be a quadratic function).
The co-ordinates of the x-intercepts are $(r_1, 0)$ and $(r_2, 0)$.
The transformation given above can be expressed in mapping notation as follows: $(x, y) \longrightarrow (x, ay)$

$\therefore (r_1, 0) \rightarrow (r_1, a(0)) = (r_1, 0)$ and $(r_2, 0) \rightarrow (r_2, a(0)) = (r_2, 0)$
$\therefore$ the points $(r_1, 0)$ and $(r_2, 0)$ are invariant under the transformation
$\therefore$ g has the same x-intercepts as f

$|a|>1$ $0<|a|<1$ $(a\neq 0)$

(b) Interpret this geometrically (i.e. graphically). Include a diagram to illustrate your answer. **(3 TIPS)**

Under a vertical stretch/compression by a factor of a, the x-intercepts do not change.



A vertical stretch/compression changes the shape of the graph but not its x-intercepts.

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