MCR3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions

Minor Test – Unit 1 – Inverses of Functions, Using Transformations to Deepen Insight

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Victim: Mr. Solutions Another example of superior mathematical reasoning Mr. S!

KU	APP	TIPS	COM
14/14	21/21	8/8	15/7+8

v.2

1. Andeep, the heating system repair expert, is paid as described below:

- \$24/h for working up to 45 h per week
- time-and-a-half (\$24/h + \$12/h = \$36/h) for working overtime (hours worked beyond 45 in a single week)

For instance, if Andeep works 50 hours in a single week, he is paid a total of $45(24)+5(36)=1260$ dollars.

(a) Let $P(t)$ represent how much Andeep is paid per week for working t hours. Complete the definition of $P(t)$ found below. **(4 APP)**

Hint: The calculation shown above should be used as a guide for writing the expression for $t > 45$.

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$$P(t)=\begin{cases} 24t, & 0\le t\le 45 \\ 36(t-45)+1080, & t>45 \\ (\text{OR } 36t-540, & t>45) \end{cases}$$

(b) Sketch a graph of P. **(3 APP)**

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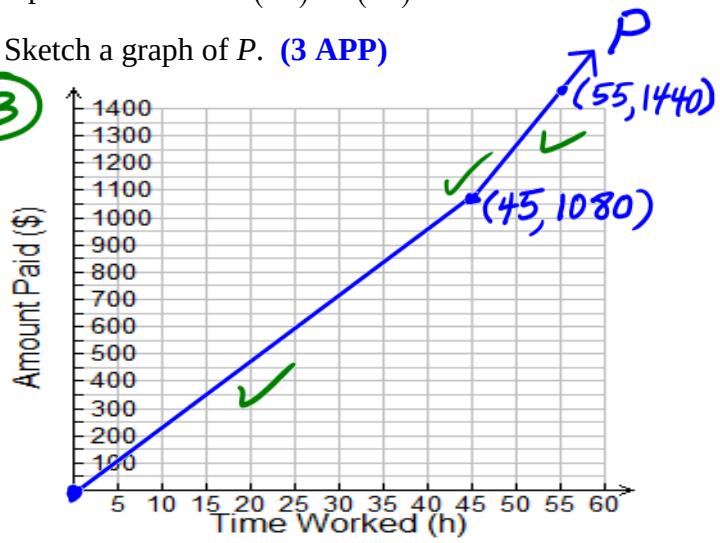

(c) Because of Andeep's exceptional customer service, his boss, Ms. I. M. Cotu, decided to give him a monthly bonus. As shown in the table below, the bonus paid is a function of the number of customers who give Andeep a five-star rating.

Number of Customers who give Andeep a 5-Star Rating	*Bonus*
Fewer than 10	\$0
At least 10 but fewer than 30	\$75
At least 30 but fewer than 50	\$125
Fifty or more	\$175

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Let $B(n)$ represent the monthly bonus Andeep receives if n customers give him a 5-star rating. Complete the definition of $B(n)$ found below. **(5 APP)**

$$B(n)=\begin{cases} 0, & 0\le n<10 \\ 75, & 10\le n<30 \\ 125, & 30\le n<50 \\ 175, & n\ge 50 \end{cases}$$

(d) Sketch a graph of B. **(5 APP)**

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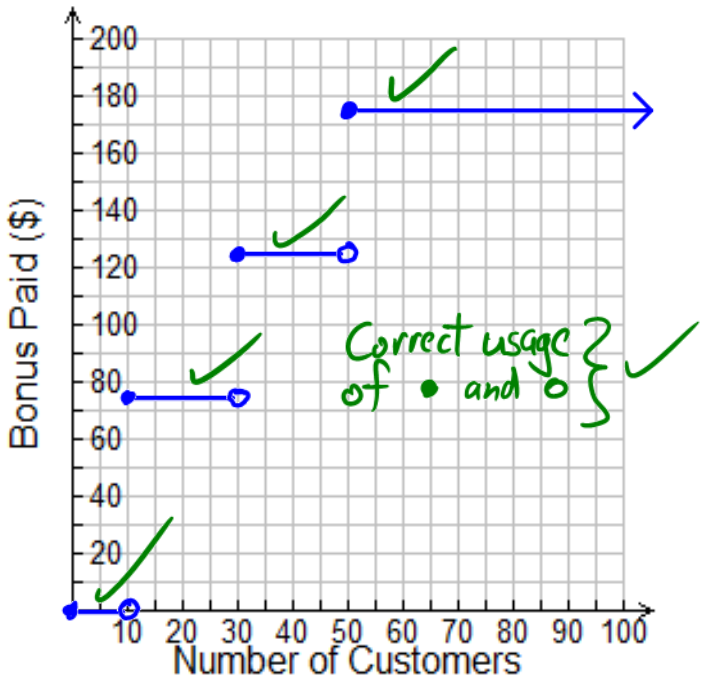

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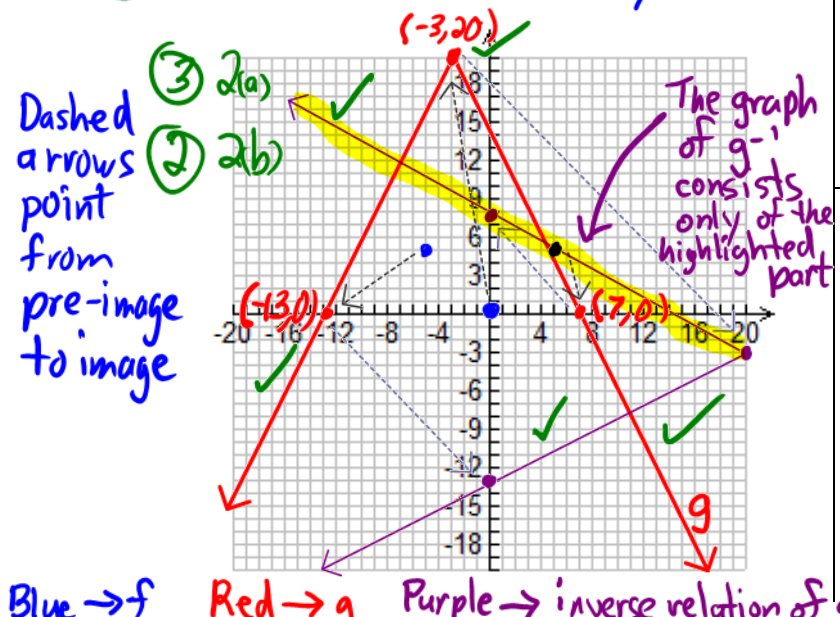
2. Consider the function g defined by the equation $g(x) = -4\left|\frac{1}{2}(x+3)\right| + 20$.

(a) Use mapping notation to describe the transformations of the mother function $f(x) = |x|$ that would produce the function g. Then use the transformations to sketch the graph of g. **(5 KU)**

② $(x, y) \rightarrow (2x - 3, -4y + 20)$



(b) Using the grid given in (a), sketch the graph of the ***inverse relation*** of g. **(2 KU)**

(c) State a restriction on the domain of g that ensures that g is one-to-one on the restricted domain. ***Briefly*** explain how you arrived at your answer. **(2 KU)**

$x \geq -3$

g is one-to-one for all $x \geq -3$ because it is strictly decreasing

(d) Determine the equation of $g^{-1}(x)$ for the restricted domain stated in (c). **(5 KU)**

For $x \geq -3$, $g(x) = -4\left(\frac{1}{2}\right)(x+3) + 20$

$= -2x + 14$

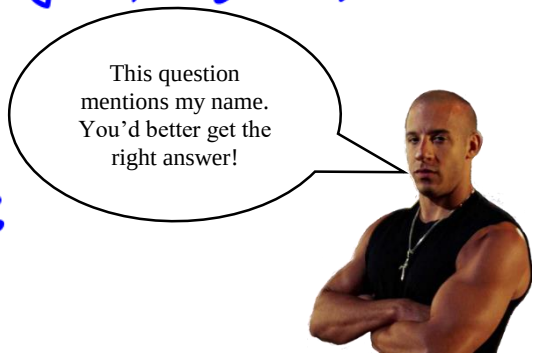
To find g^{-1}, apply the transf. $(x,y) \rightarrow (y,x)$

$\therefore x = -2y + 14$

$\therefore y = -\frac{1}{2}x + 7, x \leq 20$

$\therefore g^{-1}(x) = -\frac{1}{2}x + 7, x \leq 20$

3. The function $T_E(x) = 0.07x^2 + 215.50$ approximates the ***exhaust temperature***, in Fahrenheit degrees, of a diesel engine operating at x % of the maximum load on the engine ($0 < x < 100$).



(a) Determine the equation of $T_E^{-1}(x)$. **(4 APP)**

Apply the transformation $(x,y) \rightarrow (y,x)$:

$x = 0.07y^2 + 215.50$

$\therefore x - 215.50 = 0.07y^2$

$\therefore \frac{x - 215.50}{0.07} = y^2$

$\therefore \pm\sqrt{\frac{x - 215.50}{0.07}} = y$

y represents the % load on the engine

Since $y > 0$, $T_E^{-1}(x) = \sqrt{\frac{x - 215.50}{0.07}}$, $x \geq 215.50$

(b) In the equation of $T_E^{-1}(x)$, what does x represent? Explain. **(2 COM)**

In $T_E^{-1}(x)$, x represents the exhaust temperature because for T_E^{-1}, the input is the same as the output for T_E. Therefore, $x \geq 215.50$ because this is the minimum exhaust temperature according to the equation of T_E.

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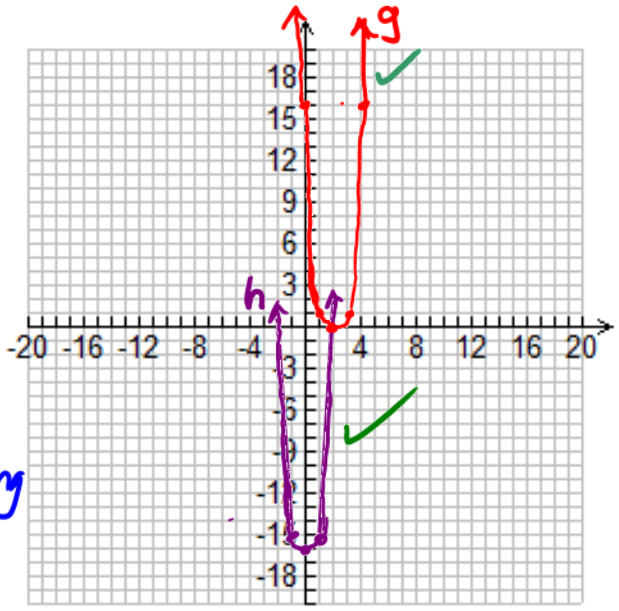
4. A pig-headed grade-9 student insists that $(x-2)^4 = x^4 - 2^4 = x^4 - 16$. You, being a far more mature, experienced and wiser grade-11 AP student obviously know better. Use your knowledge of transformations to prove that the grade-9 student is wrong! Note that a grid is provided so that you can illustrate your answer with graphs. (**Hint:** Use $f(x) = x^4$ as the base function.) **(5 COM)**



Let $g(x) = (x-2)^4 = f(x-2)$ and $h(x) = x^4 - 16 = f(x) - 16$.

The graph of g is obtained by translating the graph of f two units to the right. The graph of h, on the other hand, is obtained by translating the graph of f 16 units downward. As shown in the diagram, the two graphs intersect at only one point, meaning that $(x-2)^4 \neq x^4 - 2^4$ except for $x=2$. Thus the expressions $(x-2)^4$ and $x^4 - 2^4$ are NOT equivalent. The grade-9 student, as often is the case, was WRONG!!

5. Let r_1 and r_2 represent the x-intercepts of the quadratic function $f(x) = x^2 + bx + c$.

(a) What are the x-intercepts of the function $g(x) = f(ax)$, where a represents any non-zero real number? **(5 TIPS)**

The following holds for any function f (f doesn't need to be a quadratic function.)

The co-ordinates of the x-intercepts are $(r_1, 0)$ and $(r_2, 0)$.

The transformation given above can be expressed in mapping notation as follows: $(x, y) \longrightarrow \left(\frac{1}{a}x, 0\right)$

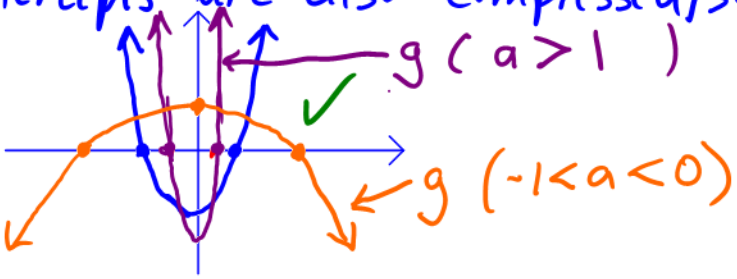
$\therefore (r_1, 0) \longrightarrow \left(\frac{1}{a}r_1, 0\right)$ and $(r_2, 0) \longrightarrow \left(\frac{1}{a}r_2, 0\right)$

$\therefore$ the x-intercepts of g are $\frac{1}{a}r_1$ and $\frac{1}{a}r_2$.

(b) Interpret this geometrically (i.e. graphically). Include a diagram to illustrate your answer. **(3 TIPS)**

$|a|>1$ $0<|a|<1$

Under a horizontal compression/stretch by a factor of $\frac{1}{a}$ $(a \neq 0)$, the x-intercepts are also compressed/stretched by the same factor.



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