MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions

Unit 2 – Mid-unit Test (Radian Measure, Trig Ratios, Transformations, Modelling)

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Victim: Mr. Solutions Your work never fail to impress Mr. S.!

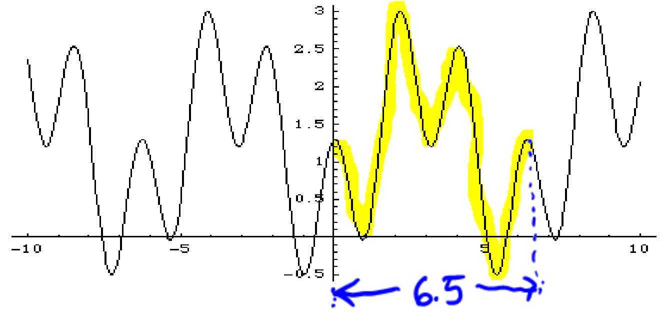
KU	APP	TIPS	COM
11/10	12/12	16/16	14/13

v.1

- Unless otherwise noted, ***radian measure must be used***. COM marks will be deducted for using degree measure.
- Up to 10 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

Part 1: Multiple Choice (11 KU)

Identify the choice that ***best*** answers the question.



1. __d__ The graph of a periodic function is shown at the right. What is the approximate ***period*** of the function?

(a) 1.75 **(b)** 13 **(c)** 3.5 **(d)** 6.5

2. __c__ Which of the following ***is most unlikely*** to produce a periodic graph?

(a) Little Anshul's height above the floor as he jumps up and down in a playpen.

(b) The height above the floor of Ashutosh's (naughty former student) mother's hand as she "disciplines" him.

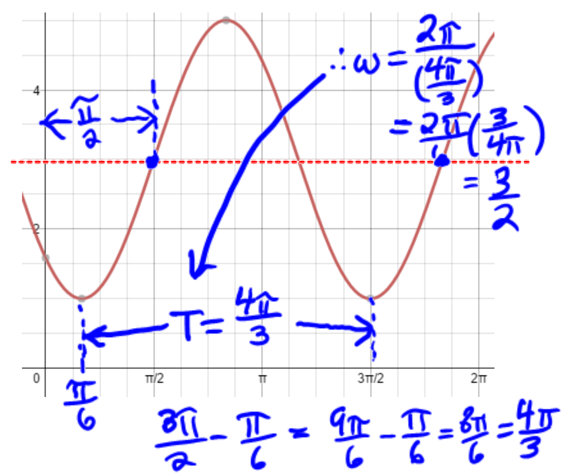
(c) The height above the ground of Victoria's airplane as it descends toward a runway for a landing. obviously not periodic

(d) Sumeet's height above the ground as she rides an extremely fast Ferris wheel.

3. __d__ Which of the following is a ***correct*** equation for the graph at the right?

(a) $f(x)=2\sin\left(\frac{2}{3}\left(x+\frac{\pi}{2}\right)\right)+3$ **(b)** $f(x)=2\sin\left(\frac{2}{3}\left(x-\frac{\pi}{2}\right)\right)+3$

(c) $f(x)=2\sin\left(\frac{3}{2}\left(x+\frac{\pi}{2}\right)\right)+3$ **(d)** $f(x)=2\sin\left(\frac{3}{2}\left(x-\frac{\pi}{2}\right)\right)+3$



4. __c__ The function shown at the right has domain and range

(a) $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\geq y\geq 5\}$ **(b)** $D=\{x\in\mathbb{R}:1\geq x\leq 5\},\ R=\mathbb{R}$

(c) $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\leq y\leq 5\}$ **(d)** $D=\{x\in\mathbb{R}:1\geq x\geq 5\},\ R=\mathbb{R}$

5. __c__ Which of the following ***does not*** make sense?

(a) A sinusoidal function has a period of π. **(b)** A sinusoidal function has an amplitude of $1/1000$.

(c) A sinusoidal function has an amplitude of –3. **(d)** A sinusoidal function is compressed by a factor of 0.001.

cannot be negative

6. __d__ Mahrufat is jumping up and down on a trampoline. Her height in metres above the ground after t seconds is given by the function $h(t)=1.5\sin(2\pi t)+1$. What does the "1.5" in the equation represent?

(a) Mahrufat's maximum height above the ground. **(b)** Mahrufat's average height above the ground.

(c) Mahrufat's minimum height above the ground. **(d)** Mahrufat's maximum displacement from the average.

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

7. __c__ ✓ A sinusoidal function has an amplitude of 0.75 units, a period of 8π and a maximum at $(0,-3)$. Which of the following ***is not*** a possible equation of the function?

$A = \frac{3}{4}$

$T = 8\pi \rightarrow \omega = \frac{2\pi}{8\pi} = \frac{1}{4}$

(a) $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}x\right)-\frac{15}{4}$ **(b)** $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}(x-8\pi)\right)-\frac{15}{4}$

(c) $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}x\right)-\frac{15}{4}$ **(d)** $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}(x+2\pi)\right)-\frac{15}{4}$

$f(0) = \frac{3}{4}\sin 0 - \frac{15}{4}$
$= -\frac{15}{4} \neq -3$

8. __d__ ✓ Let $f(x)=\sin x$ and $g(x)=A\sin(\omega(x-p))+d$. Knowing that the period of *f* is 2π, we can deduce that the period of *g* must be $\frac{2\pi}{\omega}$. Why is this true?

(a) This information is found in Mr. Nolfi's notes as well as the textbook. Everyone knows that neither source can ever be wrong. Textbooks and teachers are right about everything!

(b) It is true because period is calculated by dividing 2π by ω. Restating the question ✗

(c) To obtain the graph of *g*, the graph of *f* must be stretched or compressed horizontally by the factor ω, which means that the period of *f* is also stretched or compressed horizontally by the same factor.

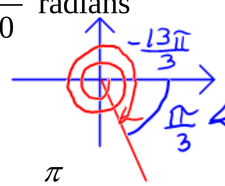
(d) To obtain the graph of *g*, the graph of *f* must be stretched or compressed horizontally by the factor $\frac{1}{\omega}$, ✓ which means that the period of *f* is also stretched or compressed horizontally by the same factor.

9. __a__ ✓ 542° is equal to

$542\left(\frac{\pi}{180}\right) = \frac{542\pi}{180} = \frac{271\pi}{90}$

(a) $\frac{271\pi}{90}$ radians **(b)** $\frac{271}{90}$ radians **(c)** $\frac{90\pi}{271}$ radians **(d)** $\frac{90}{271}$ radians

10. __c__ ✓ ~~sec~~ cos $\frac{-13\pi}{3}$ is equal to



related first quad angle +1 **Bonus** (KU)

(a) $\cos\frac{\pi}{6}$ **(b)** $-\cos\frac{\pi}{6}$ **(c)** $\cos\frac{\pi}{3}$ **(d)** $-\cos\frac{\pi}{3}$

11. __b__ ✓ Consider two ***coterminal*** angles *x* and *y*, and their ***principal*** angle θ. If $2\pi < x < 4\pi$, $-4\pi < y < -2\pi$, $\cos\theta = -\frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then

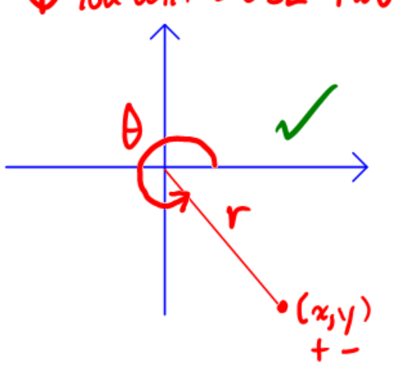
$\therefore \frac{\pi}{2} < \theta < \pi$ (θ is in quadrant II)

(a) $x=\frac{7\pi}{3}$, $y=\frac{-11\pi}{3}$ quad. I **(b)** $x=\frac{8\pi}{3}$, $y=\frac{-10\pi}{3}$ quad II **(c)** $x=\frac{10\pi}{3}$, $y=\frac{-8\pi}{3}$ quad III **(d)** $x=\frac{11\pi}{3}$, $y=\frac{-7\pi}{3}$ quad IV

Part 2: Written Responses

12. For ***negative two marks***, evaluate $\sin\frac{-13\pi}{3}$. For ***three communication marks***, explain why both the tangent and cotangent of any angle in quadrant IV must be negative. (Use a diagram to illustrate your answer.) For ***one bonus mark***, write ***one sentence*** that describes the purpose of mathematical modelling. **(3 COM)**

You will LOSE two marks! Don't do it!!



In quad. IV,
$x > 0$ and $y < 0$ ✓
$\therefore \tan\theta = \frac{y}{x} < 0$ ✓
and $\cot\theta = \frac{x}{y} < 0$

Bonus +1 Bonus (COM)
The purpose of mathematical modelling is to create some system of equations whose behaviour closely approximates (i.e. simulates, mimics) that of some real-world system.

KU	APP	TIPS	COM
+1	− 0	− 0	+ 1

13. Suppose that $g(x) = -2\tan\left(\frac{1}{4}(x+\pi/4)\right)$. **(12 APP)**

(a) State the transformations required to obtain g from the base/parent/mother function $f(x) = \tan x$.

Horizontal	***Vertical***
1. Stretch by a factor of 4.	**1.** Stretch by a factor of −2. (Stretch by a factor of 2 followed by a reflection in the x-axis)
2. Translate $\frac{\pi}{4}$ radians to the left.	**2.** No vertical translation

(b) Express the transformation in ***mapping notation***.

$(x, y) \to \left(4x - \frac{\pi}{4}, -2y\right)$

(c) Apply the transformation to a few key points on the graph of the base function $f(x) = \tan x$

Pre-image Point on $y = f(x)$	***Image Point on $y = g(x)$***
$(0, 0)$	$\left(-\frac{\pi}{4}, 0\right)$
$\left(\frac{\pi}{6}, \frac{1}{\sqrt{3}}\right)$	$\left(\frac{5\pi}{12}, -\frac{2}{\sqrt{3}}\right)$
$\left(\frac{\pi}{4}, 1\right)$	$\left(\frac{3\pi}{4}, -2\right)$
$\left(\frac{\pi}{3}, \sqrt{3}\right)$	$\left(\frac{13\pi}{12}, -2\sqrt{3}\right)$
$\left(-\frac{\pi}{6}, -\frac{1}{\sqrt{3}}\right)$	$\left(-\frac{11\pi}{12}, \frac{2}{\sqrt{3}}\right)$
$\left(-\frac{\pi}{4}, -1\right)$	$\left(-\frac{5\pi}{4}, 2\right)$
$\left(-\frac{\pi}{3}, -\sqrt{3}\right)$	$\left(-\frac{19\pi}{12}, 2\sqrt{3}\right)$

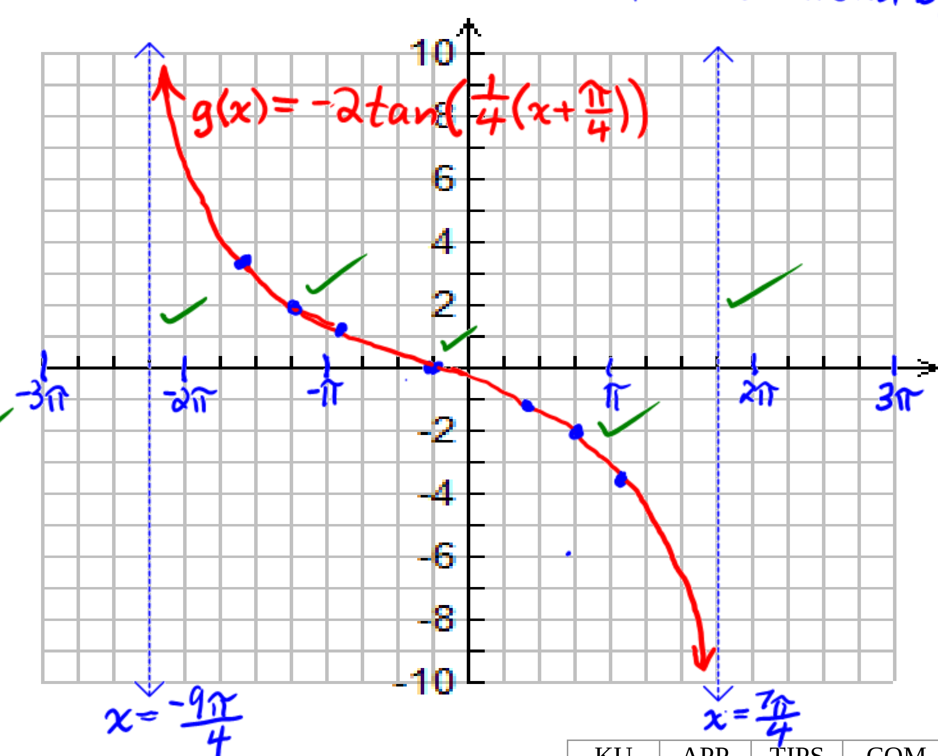
(d) Apply the transformation to the asymptotes of $f(x) = \tan x$

$x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

Pre-image Asymptote of $y = f(x)$	***Image Asymptote of $y = g(x)$***
$x = -\frac{\pi}{2}$	$x = 4\left(-\frac{\pi}{2}\right) - \frac{\pi}{4} = -\frac{4\pi}{2} - \frac{\pi}{4} = -\frac{9\pi}{4}$
$x = \frac{\pi}{2}$	$x = 4\left(\frac{\pi}{2}\right) - \frac{\pi}{4} = \frac{4\pi}{2} - \frac{\pi}{4} = \frac{7\pi}{4}$

(e) Finally, sketch the graph of ***one cycle*** of $y = g(x)$.

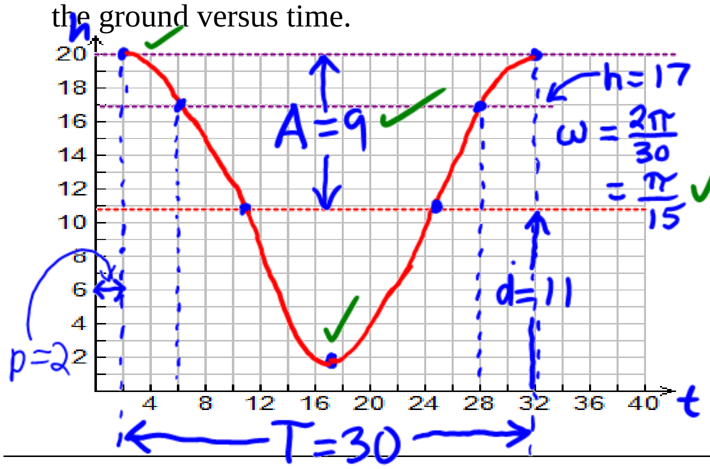
$T = 4\pi$ because of horiz. stretch by 4



KU	APP	TIPS	COM
+1	−0	−0	+1

14. A certain town has a windmill with the tip of one of its blades painted red. The owner of the windmill notices that at $t = 2$ s the red tip is 20 m above the ground. Then, over a period of 30 seconds, the red tip moves from 20 m above the ground down to 2 m above the ground and back up to 20 m. **(16 TIPS)**

(a) Sketch the graph of height of the red mark above the ground versus time.



(b) Write ***two different equations***, one using "cos" and the other using "sin," of a sinusoidal function that models the height of the red mark above the ground versus time.

$h(t) = 9\cos\left(\frac{\pi}{15}(t-2)\right)+11$

$h(t) = 9\sin\left(\frac{\pi}{15}\left(t+\frac{11}{2}\right)\right)+11$

($\frac{1}{4}$ cycle $= \frac{30}{4} = \frac{15}{2}$. Shift graph of $h = 9\sin\left(\frac{\pi}{15}(t-2)\right)+11$ $\frac{15}{2}$ units to the left.
$t-2+\frac{15}{2} = t+\frac{11}{2}$)

(c) What is the equation of the horizontal axis of this sinusoidal function? What does the horizontal axis represent in this context?

$h = 11$

This represents the red mark's average height above the ground.

(d) How high above the ground is the red mark after 13 seconds?

$h(13) = 9\cos\left(\frac{\pi}{15}(13-2)\right)+11$
$\doteq 4.98$

The red mark is about 4.98 m above the ground at 13 s.

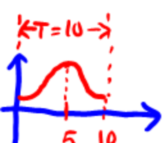
(e) At approximately what time(s) during the 30-second period is the red mark 17 m above the ground?

The red mark is 17 m above the ground at about 6 s and 28 s. (Estimated from graph)

(f) What is the diameter of the windmill? Explain.

The diameter of the windmill is twice the amplitude of h.
$\therefore d = 2(9) = 18$ m

(g) The owner of the town's mini-golf course would like to install a small 2m-diameter version of the windmill. The small version of the windmill is designed to ***scale*** with the original, but rotates more rapidly than the original. It completes a full rotation in 10 s. Write an equation to model the height above the ground of the red mark on the small windmill given that at $t = 5$ s, the red mark is at its maximum height above the ground.



Scaled down by a factor of $\frac{18}{2} = 9$

$\therefore$ average height above ground $= \frac{11}{9}$ ← average height of large windmill; ← scale-down factor

$T = 10$, $A = -1$, $\omega = \frac{2\pi}{10} = \frac{\pi}{5}$, $d = \frac{11}{9}$

Also $p = 0$ since min height is at $t = 0$, $t = 10$ ← see diagram

$h(t) = -\cos\left(\frac{\pi}{5}t\right)+\frac{11}{9}$

(h) A golf ball has a diameter of 4.3 cm. If the red mark on the ***small windmill*** is as close to the ground as possible, would the golf ball be able to pass through the space between the bottom of the blade and the ground? Explain.

Min height
$= h(0)$
$= -\cos\left(\frac{\pi}{5}(0)\right) + \frac{11}{9}$
$= -1 + \frac{11}{9}$
$= \frac{2}{9} \doteq 0.22$ m $= 22$ cm

OR

min height
$= \frac{2}{9}$ ← min height of large windmill; ← scale-down factor
$\doteq 0.22$

The golf ball can easily pass through the space between the bottom of the blade and the ground.

KU	APP	TIPS	COM
+1	−0	−0	+1