MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions

Unit 2 – Mid-unit Test (Radian Measure, Trig Ratios, Transformations, Modelling)

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Victim: Mr. Solutions (Your work never fails to impress Mr. S.!)

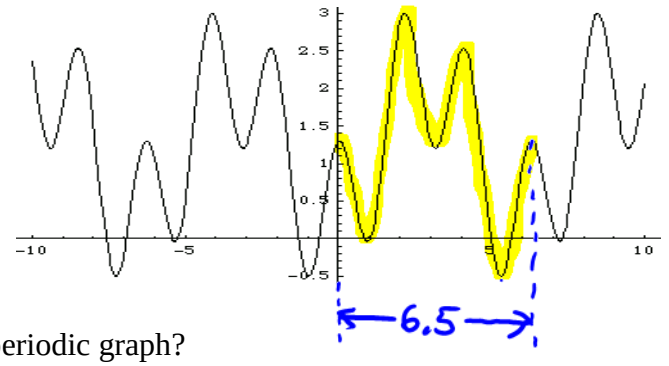
KU	APP	TIPS	COM
11/10	12/12	16/16	14/13

v.2

- Unless otherwise noted, ***radian measure must be used***. COM marks will be deducted for using degree measure.
- Up to 10 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

Part 1: Multiple Choice (11 KU)

Identify the choice that ***best*** answers the question.



1. ___c___ ✓ The graph of a periodic function is shown at the right. What is the approximate ***period*** of the function?

(a) 13 **(b)** 1.75 **(c)** 6.5 **(d)** 3.5

2. ___d___ ✓ Which of the following ***is most unlikely*** to produce a periodic graph?

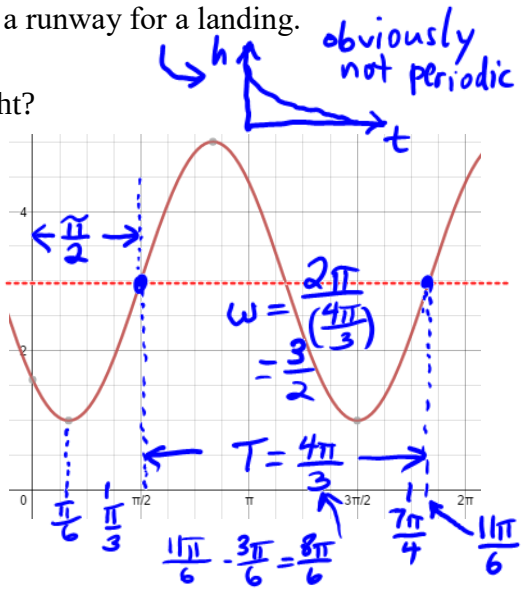
(a) Little Anshul's height above the floor as he jumps up and down in a playpen.

(b) The height above the floor of Ashutosh's (naughty former student) mother's hand as she "disciplines" him.

(c) Naxi's height above the ground as she rides an extremely fast Ferris wheel.

(d) The height above the ground of Angel's airplane as it descends toward a runway for a landing.

3. ___a___ ✓ Which of the following is a ***correct*** equation for the graph at the right?



(a) $f(x)=2\sin\left(\frac{3}{2}\left(x-\frac{\pi}{2}\right)\right)+3$ ✓ ✓ ✓

(b) $f(x)=2\sin\left(\frac{3}{2}\left(x+\frac{\pi}{2}\right)\right)+3$ ✗

(c) $f(x)=2\sin\left(\frac{2}{3}\left(x-\frac{\pi}{2}\right)\right)+3$ ✗

(d) $f(x)=2\sin\left(\frac{2}{3}\left(x+\frac{\pi}{2}\right)\right)+3$ ✗

4. ___a___ ✓ The function shown at the right has domain and range

(a) $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\le y\le 5\}$

(b) $D=\{x\in\mathbb{R}:1\ge x\ge 5\},\ R=\mathbb{R}$

(c) $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\ge y\ge 5\}$

(d) $D=\{x\in\mathbb{R}:1\le x\le 5\},\ R=\mathbb{R}$

5. ___a___ ✓ Which of the following ***does not*** make sense?

(a) A sinusoidal function has a period of $-\pi$. (can't be negative)

(b) A sinusoidal function has an amplitude of $1/1000$.

(c) A sinusoidal function has an amplitude of 3.

(d) A sinusoidal function is compressed by a factor of 0.001.

6. ___b___ ✓ Smrithi is jumping up and down on a trampoline. Her height in metres above the ground after t seconds is given by the function $h(t)=1.5\sin(2\pi t)+1$. What does the "1" in the equation represent?

(a) Smrithi's maximum displacement from the average.

(b) Smrithi's average height above the ground.

(c) Smrithi's minimum height above the ground.

(d) Smrithi's maximum height above the ground.

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

7. ___b___ A sinusoidal function has an amplitude of 0.75 units, a period of 8π and a maximum at $(0,-3)$. Which of the following ***is not*** a possible equation of the function?

(a) $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}(x+2\pi)\right)-\frac{15}{4}$ **(b)** $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}x\right)-\frac{15}{4}$

(c) $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}(x-8\pi)\right)-\frac{15}{4}$ **(d)** $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}x\right)-\frac{15}{4}$

$\rightarrow f(0)=\frac{3}{4}\sin 0-\frac{15}{4}$
$=\frac{3}{4}(0)-\frac{15}{4}$
$=-\frac{15}{4}\neq -3$

8. ___c___ Let $f(x)=\sin x$ and $g(x)=A\sin(\omega(x-p))+d$. Knowing that the period of f is 2π, we can deduce that the period of g must be $\frac{2\pi}{\omega}$. Why is this true?

(a) This information is found in Mr. Nolfi's notes as well as the textbook. Everyone knows that neither source can ever be wrong. Textbooks and teachers are right about everything!

(b) It is true because period is calculated by dividing 2π by ω. Restates the question!

(c) To obtain the graph of g, the graph of f must be stretched or compressed horizontally by the factor $\frac{1}{\omega}$, which means that the period of f is also stretched or compressed horizontally by the same factor.

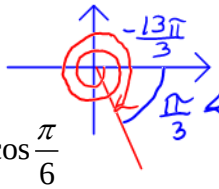
(d) To obtain the graph of g, the graph of f must be stretched or compressed horizontally by the factor ω, which means that the period of f is also stretched or compressed horizontally by the same factor.

9. ___d___ 542° is equal to

$542\left(\frac{\pi}{180}\right)=\frac{542\pi}{180}=\frac{271\pi}{90}$

(a) $\frac{90}{271}$ radians **(b)** $\frac{90\pi}{271}$ radians **(c)** $\frac{271}{90}$ radians **(d)** $\frac{271\pi}{90}$ radians

10. ___c___ $\cos\left(-\frac{13\pi}{3}\right)$ is equal to



related first quad angle

+1 Bonus

(a) $\cos\frac{\pi}{6}$ **(b)** $-\cos\frac{\pi}{6}$ **(c)** $\cos\frac{\pi}{3}$ **(d)** $-\cos\frac{\pi}{3}$

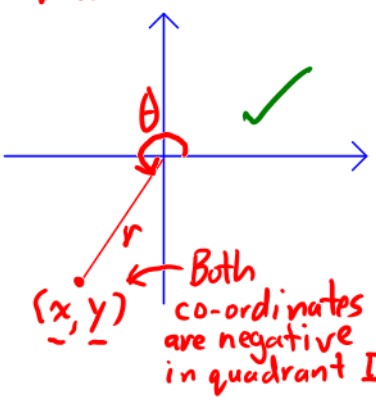
11. ___d___ Consider two ***coterminal*** angles x and y, and their ***principal*** angle θ. If $2\pi < x < 4\pi$, $-4\pi < y < -2\pi$, $\cos\theta=-\frac{1}{2}$ and $\sin\theta=\frac{\sqrt{3}}{2}$, then $\therefore \frac{\pi}{2}<\theta<\pi$ (θ is in quadrant II)

(a) $x=\frac{10\pi}{3}$, $y=\frac{-8\pi}{3}$ quad. III **(b)** $x=\frac{11\pi}{3}$, $y=\frac{-7\pi}{3}$ quad IV **(c)** $x=\frac{7\pi}{3}$, $y=\frac{-11\pi}{3}$ quad I **(d)** $x=\frac{8\pi}{3}$, $y=\frac{-10\pi}{3}$ quad II

Part 2: Written Responses

12. For ***negative two marks***, evaluate $\sin\frac{-13\pi}{3}$. For ***three communication marks***, explain why both the cosine and secant of any angle in quadrant III must be negative. (Use a diagram to illustrate your answer.) For ***one bonus mark***, write ***one sentence*** that describes the purpose of mathematical modelling. **(3 COM)**

You will LOSE two marks! Don't do it!!



In quad. III, $x<0$. In addition, $r>0$ because it's a distance.
$\therefore \cos\theta=\frac{x}{r}<0$
and $\sec\theta=\frac{r}{x}<0$

Bonus +1
The purpose of mathematical modelling is to create some system of equations whose behaviour closely approximates (i.e. simulates, mimics) that of some real-world system.

KU	APP	TIPS	COM
+1	-0	-0	+1

13. Suppose that $g(x) = -2\cot\left(\frac{1}{4}(x+\pi/4)\right)$. **(12 APP)**

(a) State the transformations required to obtain g from the base/parent/mother function $f(x) = \cot x$.

Horizontal	*Vertical*
1. Stretch by a factor of 4. ✓ 2. Translate $\frac{\pi}{4}$ radians to the left.	1. Stretch by a factor of -2. (Stretch by a factor of 2 followed by a reflection in the x-axis) ✓ 2. No vertical translation

(b) Express the transformation in ***mapping notation***.

$(x, y) \rightarrow \left(4x - \frac{\pi}{4}, -2y\right)$ ✓ ✓

(c) Apply the transformation to a few key points on the graph of the base function $f(x) = \cot x$

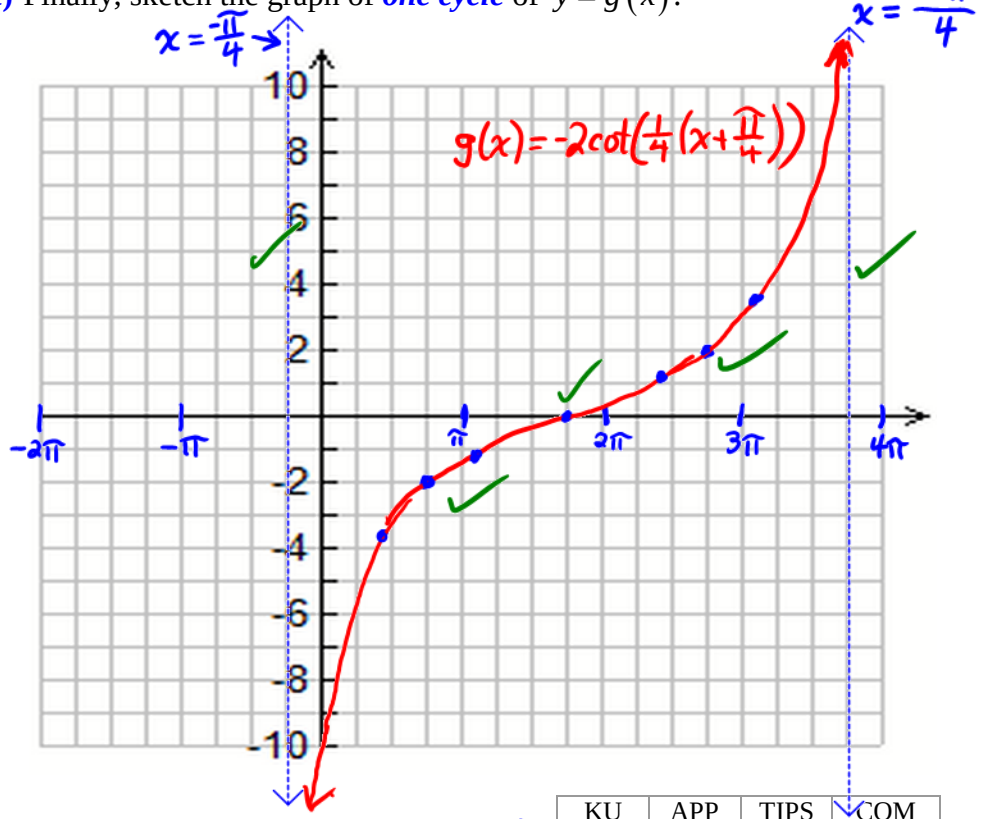
Pre-image Point on $y = f(x)$	*Image Point on* $y = g(x)$
$\left(\frac{\pi}{6}, \sqrt{3}\right)$	$\left(\frac{5\pi}{12}, -2\sqrt{3}\right)$
$\left(\frac{\pi}{4}, 1\right)$	$\left(\frac{3\pi}{4}, -2\right)$
$\left(\frac{\pi}{3}, \frac{1}{\sqrt{3}}\right)$ ✓	$\left(\frac{13\pi}{12}, \frac{-2}{\sqrt{3}}\right)$ ✓
$\left(\frac{\pi}{2}, 0\right)$	$\left(\frac{7\pi}{4}, 0\right)$
$\left(\frac{2\pi}{3}, -\frac{1}{\sqrt{3}}\right)$	$\left(\frac{29\pi}{12}, \frac{2}{\sqrt{3}}\right)$
$\left(\frac{3\pi}{4}, -1\right)$	$\left(\frac{11\pi}{4}, 2\right)$
$\left(\frac{5\pi}{6}, -\sqrt{3}\right)$	$\left(\frac{37\pi}{12}, 2\sqrt{3}\right)$

(d) Apply the transformation to the asymptotes of $f(x) = \cot x$

$x = 0$ and $x = \pi$

Pre-image Asymptote of $y = f(x)$	*Image Asymptote of* $y = g(x)$
$x = 0$	$x = 4(0) - \frac{\pi}{4}$ $= -\frac{\pi}{4}$ ✓
$x = \pi$	$x = 4\pi - \frac{\pi}{4}$ $= \frac{15\pi}{4}$

(e) Finally, sketch the graph of ***one cycle*** of $y = g(x)$.

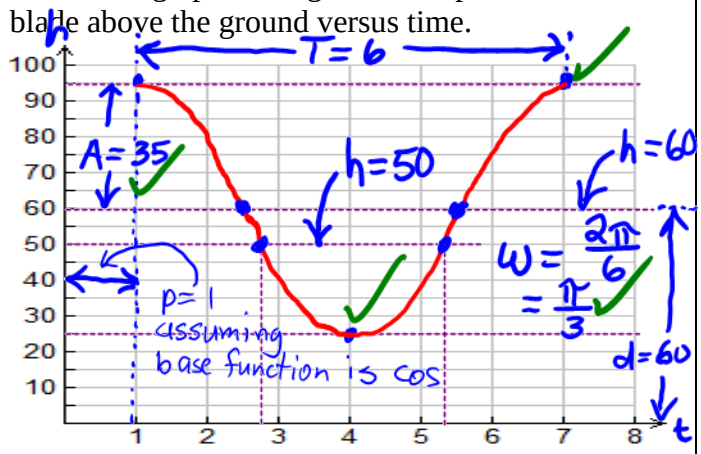


$T = 4\pi$ because of the horizontal stretch by a factor of 4.

KU	APP	TIPS	COM
+1	-0	-0	+1

14. A certain type of wind turbine has three blades, one that is red, another that is green and yet another that is blue. An observer notices that at $t = 1$ s, the tip of the blue blade is 95 m above the ground. Then, over a period of 6 seconds, the tip of the blue blade moves from 95 m above the ground down to 25 m above the ground and back up to 95 m. **(16 TIPS)**

(a) Sketch the graph of height of the tip of the blue blade above the ground versus time.



(b) Write ***two different equations***, one using "cos" and the other using "sin," of a sinusoidal function that models the height of the tip of the blue blade above the ground versus time.

$h(t) = 35\cos\left(\frac{\pi}{3}(t-1)\right)+60$ ✓

$h(t) = 35\sin\left(\frac{\pi}{3}\left(t+\frac{1}{2}\right)\right)+60$ ✓

($\frac{1}{4}$ cycle $= \frac{6}{4} = \frac{3}{2}$. Shift graph of $h = 35\sin\left(\frac{\pi}{3}(t-1)\right)+60$ $\frac{3}{2}$ units to the right, i.e. $t-1+\frac{3}{2} = t+\frac{1}{2}$)

(c) What is the equation of the horizontal axis of this sinusoidal function? What does the horizontal axis represent in this context?

$h = 60$ ✓

This represents the average height above the ground of the blue blade's tip. ✓

(d) How high above the ground is the tip of the blue blade after 4 seconds?

$h(4) = 35\cos\left(\frac{\pi}{3}(4-1)\right)+60$

$= 25$ ✓

(Can also read directly from graph)

(e) At approximately what time(s) during the 6-second period is the tip of the blue blade 50 m above the ground?

The tip of the blue blade is 50 m above the ground at about 2.8 s and 5.2 s. ✓ (estimated from graph)

(f) What is the length of one of the blades? Explain.

The length of one of the blades is equal to the amplitude. ✓

$\therefore$ length $= 35$ m ✓

(g) A small 5m-diameter version of the wind turbine is built for demonstration purposes. The small version of the wind turbine is designed to ***scale*** with the original, but rotates more rapidly, completing a full rotation in 2 s. Write an equation to model the height above the ground of the tip of the blue blade given that at $t = 1$ s, the tip is at its maximum height above the ground.

Scaled down by a factor of $\frac{70}{5} = 14$

$\therefore$ average height above ground $= \frac{60}{14} = \frac{30}{7}$ ← average height of large turbine ← scale-down factor

$T = 2$, $A = \frac{-5}{2}$, $w = \frac{2\pi}{2} = \pi$, $d = \frac{30}{7}$

Also $p = 0$ since min. height occurs at $t = 0$, $t = 2$ ← see diagram

$\therefore h(t) = \frac{-5}{2}\cos(\pi t)+\frac{30}{7}$ ✓ ✓

(h) Would it be safe for a 2-m tall individual to stand underneath the spinning blades of the ***small*** wind turbine? Explain.

Min height

$= h(0)$

$= -\frac{5}{2}\cos(0)+\frac{30}{7}$

$= -\frac{5}{2}+\frac{30}{7}$

$\doteq 1.79$ ✓

The 2-m tall person would be decapitated!

OR

min height

$= \frac{25}{14}$ ← min height of large turbine ← scale-down factor

$\doteq 1.79$

KU	APP	TIPS	COM
+1	−0	−0	+1