2015-11-17-Solutions-Unit 2 - Identities Test v1.pdf

2015-11-17-Solutions-Unit 2 - Identities Test v2.pdf

2015-11-17-Solutions-Unit 2 - Identities Test v2a.pdf

MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions
Unit 2 – Trigonometric Identities Mini-Test

Mr. N. Nolfi

Victim: Mr. Solutions

Your deductive powers are impressive Mr. S.! Mr. N.

KU	APP	TIPS	COM
5 /5	6 /6	10/10	13/13

v.1

- Up to 5 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

1. Prove that the equation $\dfrac{\tan^2\theta-\sin^2\theta}{\tan^2\theta}=\sin^2\theta$ is an identity. **(6 APP)**

$$\begin{aligned}
L.S. &= \frac{\tan^2\theta-\sin^2\theta}{\tan^2\theta}\\
&= \frac{\tan^2\theta}{\tan^2\theta}-\frac{\sin^2\theta}{\tan^2\theta}\\
&= 1-\left(\frac{\sin^2\theta}{1}\right)\left(\frac{1}{\tan^2\theta}\right)\\
&= 1-\left(\frac{\sin^2\theta}{1}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\\
&= 1-\cos^2\theta\\
&= \sin^2\theta = R.S.
\end{aligned}$$

$\therefore$ L.S. = R.S.

$\therefore$ the given equation is an identity.

2. Prove that the equation $\dfrac{1+\cos x}{1-\cos x}-\dfrac{1-\cos x}{1+\cos x}=4\cot x\csc x$ is an identity. **(10 TIPS)**

$$\begin{aligned}
L.S. &= \frac{1+\cos x}{1-\cos x}-\frac{1-\cos x}{1+\cos x}\\
&= \frac{(1+\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}-\frac{(1-\cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)}\\
&= \frac{1+2\cos x+\cos^2 x-(1-2\cos x+\cos^2 x)}{1-\cos^2 x}\\
&= \frac{4\cos x}{\sin^2 x}\\
&= \frac{4}{1}\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\sin x}\right)\\
&= 4\cot x\csc x\\
&= R.S.
\end{aligned}$$

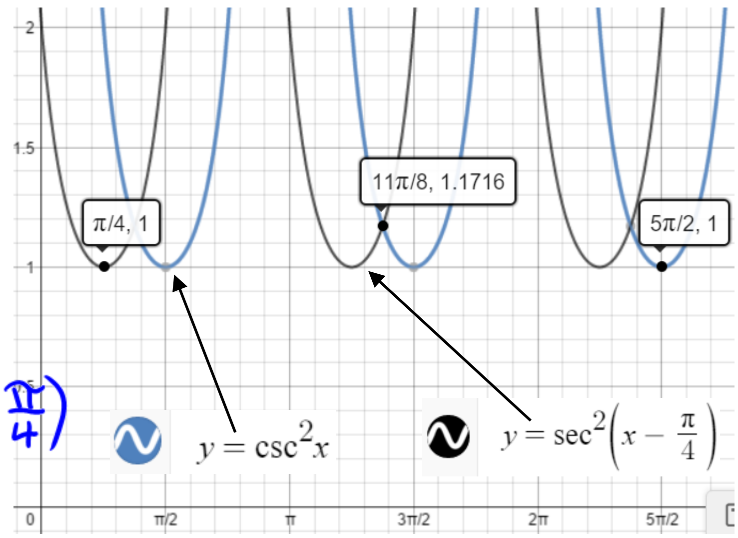
$\therefore$ L.S. = R.S.

$\therefore$ the given equation is an identity

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

Additional Space for Question 2 if Needed

3. Use a ***counterexample*** to prove that the equation $\csc^2 x = \sec^2\left(x - \frac{\pi}{4}\right)$ is ***not*** an identity. In addition, explain how you used the provided graph to find a counterexample. **(5 KU, 3 COM)**



Let $x = \frac{\pi}{4}$

Then,

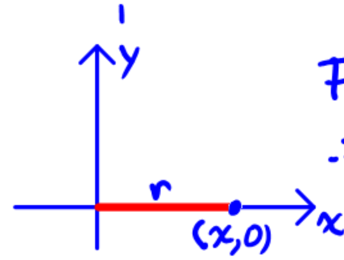
$$L.S. = \csc^2 \frac{\pi}{4} \qquad R.S. = \sec^2\left(\frac{\pi}{4} - \frac{\pi}{4}\right)$$
$$= (\sqrt{2})^2 \qquad\qquad = \sec^2 0$$
$$= 2 \qquad\qquad = 1^2$$
$$= 1$$

$\therefore$ L.S. $\neq$ R.S., the given equation is NOT an identity.

A counterexample can be found at any value of x for which the graphs do not intersect. For such values of x, the expressions cannot agree for otherwise, the y-co-ordinates would be equal.

Rough Work

$$\csc \frac{\pi}{4} = \frac{\text{hyp.}}{\text{opp.}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$



For $\theta = 0$, $r = x$

$\therefore \sec 0 = \frac{r}{x} = \frac{r}{r} = 1$

KU	APP	TIPS	COM
–0	–0	–0	–0

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v.2

- Up to 5 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

1. Prove that the equation $\dfrac{\tan^2\theta}{\tan^2\theta-\sin^2\theta}=\csc^2\theta$ is an identity. **(6 APP)**

$$\text{L.S.}=\frac{\tan^2\theta}{\tan^2\theta-\sin^2\theta}$$
$$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta}$$
$$=\frac{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{\left(\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}\right)}$$
$$=\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{\sin^2\theta-\sin^2\theta\cos^2\theta}\right)$$
$$=\frac{\sin^2\theta}{\sin^2\theta(1-\cos^2\theta)}$$
$$=\frac{1}{\sin^2\theta}$$
$$=\csc^2\theta$$
$$=\text{R.S.}$$

$\therefore$ L.S. = R.S.

$\therefore$ the given equation is an identity

2. Prove that the equation $\dfrac{\tan^2 x}{1-\cos^2 x}+\dfrac{\sin x}{\sec^2 x-1}=\cos x\left(\sec^3 x+\cot x\right)$ is an identity. **(10 TIPS)**

$$\text{L.S.}=\frac{\tan^2 x}{1-\cos^2 x}+\frac{\sin x}{\sec^2 x-1}$$
$$=\frac{\tan^2 x}{\sin^2 x}+\frac{\sin x}{\tan^2 x}$$
$$=\left(\frac{\tan^2 x}{1}\right)\left(\frac{1}{\sin^2 x}\right)+\left(\frac{\sin x}{1}\right)\left(\frac{1}{\tan^2 x}\right)$$
$$=\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{1}{\sin^2 x}\right)+\left(\frac{\sin x}{1}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right)$$
$$=\frac{1}{\cos^2 x}+\frac{\cos^2 x}{\sin x}$$
$$=\sec^2 x+\left(\frac{\cos x}{1}\right)\left(\frac{\cos x}{\sin x}\right)$$
$$=\sec^2 x+\cos x\cot x$$

$$\text{R.S.}=\cos x\left(\sec^3 x+\cot x\right)$$
$$=\left(\frac{1}{\sec x}\right)\left(\frac{\sec^3 x}{1}\right)+\cos x\cot x$$
$$=\sec^2 x+\cos x\cot x$$

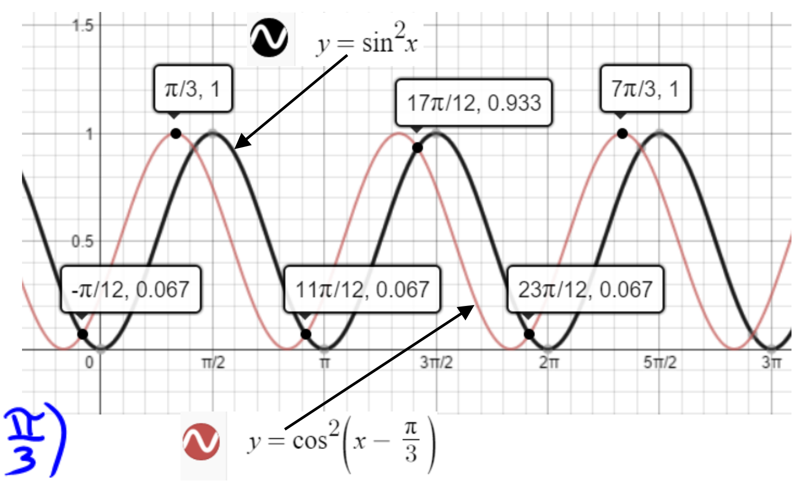
$\therefore$ L.S. = R.S.

$\therefore$ the given equation is an identity.

KU	APP	TIPS	COM
− 0	− 0	− 0	− 0

Additional Space for Question 2 if Needed

3. Use a ***counterexample*** to prove that the equation $\sin^2 x = \cos^2\left(x - \frac{\pi}{3}\right)$ is ***not*** an identity. In addition, explain how you used the provided graph to find a counterexample.
(5 KU, 3 COM)



Let $x = \frac{\pi}{3}$

Then,

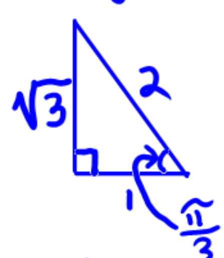
$$L.S. = \sin^2\frac{\pi}{3} = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$R.S. = \cos^2\left(\frac{\pi}{3} - \frac{\pi}{3}\right) = \cos^2 0 = 1^2 = 1$$

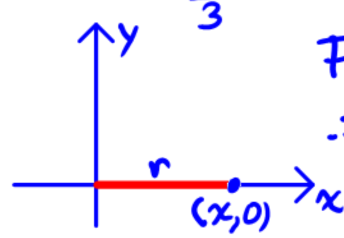
$\therefore$ L.S. $\neq$ R.S., the given equation is NOT an identity.

A counterexample can be found at any value of x for which the graphs do not intersect. For such values of x, the expressions cannot agree for otherwise, the y-co-ordinates would be equal.

Rough Work



$\sin\frac{\pi}{3} = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2}$



For $\theta = 0$, $r = x$

$\therefore \cos 0 = \frac{x}{r} = \frac{r}{r} = 1$

KU	APP	TIPS	COM
−0	−0	−0	−0

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v.2a

- Up to 5 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

1. Prove that the equation $\dfrac{\cot^2\theta}{\cot^2\theta-\csc^2\theta}=-\cot^2\theta$ is an identity. **(6 APP)**

$$L.S.=\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-\frac{1}{\sin^2\theta}}$$

$$=\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta-1}\right)$$

$$=\frac{\cos^2\theta}{-(1-\cos^2\theta)}$$

$$=-\frac{\cos^2\theta}{\sin^2\theta}$$

$$=-\cot^2\theta$$

$\therefore L.S.=R.S.$

$\therefore$ the given equation is an identity.

2. Prove that the equation $\dfrac{1}{1-\cos^2 x}+\dfrac{\sec x}{\cot^2 x+1}=\sin x\left(\csc^3 x+\tan x\right)$ is an identity. **(10 TIPS)**

$$L.S.=\frac{1}{\sin^2 x}+\frac{\left(\frac{1}{\cos x}\right)}{\csc^2 x}$$

$$=\frac{1}{\sin^2 x}+\left(\frac{1}{\cos x}\right)\left(\frac{1}{\csc^2 x}\right)$$

$$=\frac{1}{\sin^2 x}+\left(\frac{1}{\cos x}\right)\left(\frac{\sin^2 x}{1}\right)$$

$$=\frac{1}{\sin^2 x}+\frac{\sin^2 x}{\cos x}$$

$$R.S.=\sin x\left(\frac{1}{\sin^3 x}+\frac{\sin x}{\cos x}\right)$$

$$=\frac{\sin x}{\sin^3 x}+\frac{\sin^2 x}{\cos x}$$

$$=\frac{1}{\sin^2 x}+\frac{\sin^2 x}{\cos x}$$

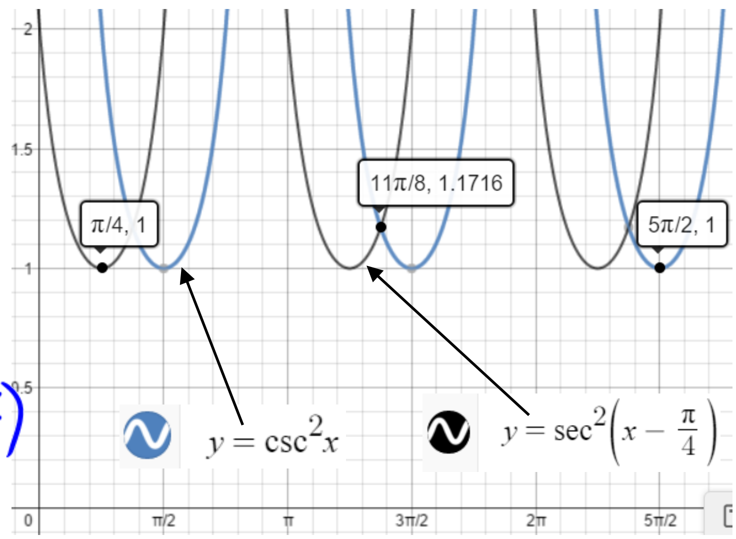
$\therefore L.S.=R.S.$

$\therefore$ the given equation is an identity.

KU	APP	TIPS	COM
−0	−0	−0	−0

Additional Space for Question 2 if Needed

3. Use a ***counterexample*** to prove that the equation $\csc^2 x = \sec^2\left(x - \frac{\pi}{4}\right)$ is ***not*** an identity. In addition, explain how you used the provided graph to find a counterexample. **(5 KU, 3 COM)**



Let $x = \frac{\pi}{4}$

Then,

$$L.S. = \csc^2 \frac{\pi}{4} = (\sqrt{2})^2 = 2$$

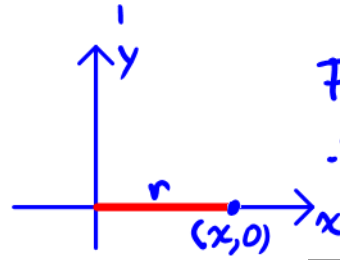
$$R.S. = \sec^2\left(\frac{\pi}{4} - \frac{\pi}{4}\right) = \sec^2 0 = 1^2 = 1$$

$\because$ L.S. $\neq$ R.S., the given equation is NOT an identity.

A counterexample can be found at any value of x for which the graphs do not intersect. For such values of x, the expressions cannot agree for otherwise, the y-co-ordinates would be equal.

Rough Work

$$\csc \frac{\pi}{4} = \frac{\text{hyp.}}{\text{opp.}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$



For $\theta = 0$, $r = x$

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−0	−0	−0	−0