MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Mathematics
Trigonometry Unit Final Test

Mr. N. Nolfi

Victim: Mr. Solutions

Marvellously and masterfully done!!

KU	APP	TIPS	COM
21 /21	23/23	21 /21	10 /10

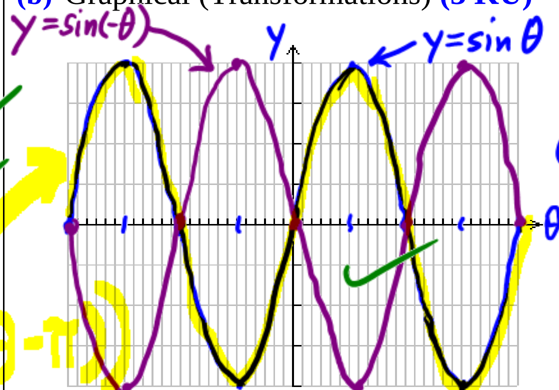
v.1

1. Use the three methods indicated below to demonstrate that the equation $\sin(\pi-\theta)=\sin\theta$ ***is*** an identity.

(a) Compound angle identity **(3 KU)**

$\sin(\pi-\theta)$
$=\sin\pi\cos\theta-\cos\pi\sin\theta$
$=0(\cos\theta)-(-1)\sin\theta$
$=\sin\theta$

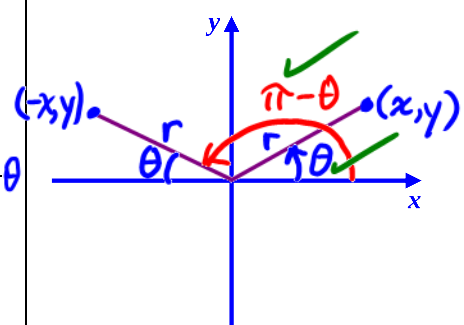
(b) Graphical (Transformations) **(3 KU)**



Let $y=\sin(\pi-\theta)$
$=\sin(-1(\theta-\pi))$

The graph of $y=\sin(\pi-\theta)$ can be obtained by reflecting $y=\sin\theta$ in the y-axis and shifting π to the right. Doing this produces the graph of $y=\sin\theta$.

(c) Angles of Rotation **(3 KU)**



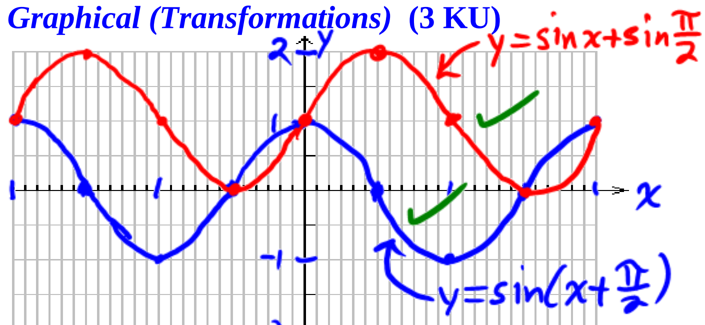
$\sin\theta=\frac{y}{r}$
$\sin(\pi-\theta)=\frac{y}{r}$
$\therefore \sin(\pi-\theta)=\sin\theta$

2. Using the methods listed below, demonstrate that the equation $\sin\left(x+\frac{\pi}{2}\right)=\sin x+\sin\frac{\pi}{2}$ ***is not*** an identity.

(a) ***Counterexample*** **(3 KU)**

Let $x=\frac{\pi}{2}$. Then
$L.S.=\sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=\sin\pi=0$
$R.S.=\sin\frac{\pi}{2}+\sin\frac{\pi}{2}=1+1=2$
$\therefore L.S.\neq R.S.$
$\therefore$ the given equation is not an identity

(b) ***Graphical (Transformations)*** **(3 KU)**



$y=\sin(x+\frac{\pi}{2})\rightarrow$ shift $y=\sin x$ $\frac{\pi}{2}$ left
$y=\sin x+\sin\frac{\pi}{2}\rightarrow$ shift $y=\sin x$ $\frac{\pi}{2}=1$ up
Graphs are not the same $\rightarrow$ equation can't be an identity.

3. Evaluate the following trig ratios without using a calculator. Exact values are required!

(a) $\cos\frac{7\pi}{12}$ **(4 APP)**

$=\cos\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right)$
$=\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$
$=\cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}$
$=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)-\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$
$=\left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$
$=\frac{\sqrt{2}-\sqrt{6}}{4}$

(b) $\cos 67.5°$ **(4 APP)**

$\cos 2\theta=2\cos^2\theta-1$
$\therefore \cos(2(67.5°))=2\cos^2 67.5°-1$
$\therefore \cos 135°+1=2\cos^2 67.5°$
$\therefore \frac{\cos 135°+1}{2}=\cos^2 67.5°$
$\therefore \sqrt{\frac{\cos 135°+1}{2}}=\cos 67.5°$ (positive $\sqrt{\ }$ since $67.5°$ is in quad. I)
$\sqrt{\frac{1}{2}\left(-\frac{1}{\sqrt{2}}+1\right)}=\cos 67.5°$
$\therefore \cos 67.5°=\sqrt{\frac{1}{2}-\frac{1}{2\sqrt{2}}}$
$=\sqrt{\frac{2}{4}-\frac{\sqrt{2}}{4}}$
$=\frac{1}{2}\sqrt{2-\sqrt{2}}$

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

4. For each of the following, write an identity entirely in terms of the given trigonometric ratio.

(a) $\cos 4\theta$ entirely in terms of $\sin\theta$ **(5 APP)**

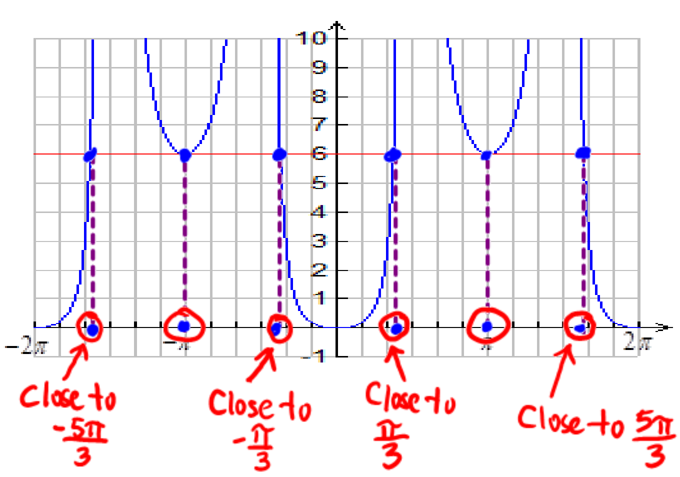
$\cos 4\theta$
$= \cos(2(2\theta))$ ✓
$= 2\cos^2 2\theta - 1$ ✓
$= 2(1-2\sin^2\theta)^2 - 1$
✓✓✓ $= 2(1-4\sin^2\theta + 4\sin^4\theta) - 1$
$= 2 - 8\sin^2\theta + 8\sin^4\theta - 1$
$= 8\sin^4\theta - 8\sin^2\theta + 1$

(b) $\cos 3\theta$ entirely in terms of $\cos\theta$ **(5 APP)**

$\cos 3\theta$
$= \cos(2\theta + \theta)$ ✓
$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ ✓
$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$
$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$ ✓✓✓
$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$
$= 4\cos^3\theta - 3\cos\theta$

5. The following question deals with solving trigonometric equations both graphically and algebraically.

(a) Shown below are the graphs of $y = (\sec x - 1)(2\sec x - 1)$ and $y = 6$. State ***approximate*** solutions to the equation $(\sec x - 1)(2\sec x - 1) = 6$ for $x \in [-2\pi, 2\pi]$. In addition, ***mark*** the solutions on the graph. **(6 KU)**



$x \doteq -5, -\pi, -1.1, 1.1, \pi, 5$

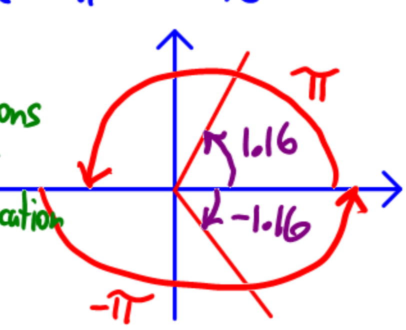
KU { 3 marks → marking solutions on graph
3 marks → stating approximate solutions }

(b) Use an algebraic method to solve the equation $(\sec x - 1)(2\sec x - 1) = 6$, where $x \in [-\pi, \pi]$. **(5 APP)**

$\therefore 2\sec^2 x - 3\sec x + 1 = 6$
$\therefore 2\sec^2 x - 3\sec x - 5 = 0$
$\therefore (2\sec x - 5)(\sec x + 1) = 0$
$\therefore 2\sec x - 5 = 0$ or $\sec x = -1$
$\therefore \sec x = \frac{5}{2}$ or $\sec x = -1$
$\therefore \cos x = \frac{2}{5}$ or $\cos x = -1$
$\therefore x = \cos^{-1}\left(\frac{2}{5}\right)$ or $x = \cos^{-1}(-1)$

$x \doteq 1.16$ or $x \doteq -1.16$
or $x = \pi$ or $x = -\pi$ } agree with estimates from 5(a)

APP { 2 marks → solutions
3 marks → justification }



KU	APP	TIPS	COM
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6. Solve the equation $4\cos 2x + 10\sin x = 7$ for the interval $0 \le x \le 2\pi$. If necessary, and only if necessary, round your answers to 1 decimal place. **(7 TIPS)**

$4\cos 2x + 10\sin x = 7$

$\therefore 4(1-2\sin^2 x) + 10\sin x - 7 = 0$

$\therefore 4 - 8\sin^2 x + 10\sin x - 7 = 0$

$\therefore -8\sin^2 x + 10\sin x - 3 = 0$

$\therefore 8\sin^2 x - 10\sin x + 3 = 0$

$\therefore 8\sin^2 x - 4\sin x - 6\sin x + 3 = 0$

$\therefore 4\sin x(2\sin x - 1) - 3(2\sin x - 1) = 0$

$\therefore (2\sin x - 1)(4\sin x - 3) = 0$

$\therefore 2\sin x - 1 = 0$ or $4\sin x - 3 = 0$

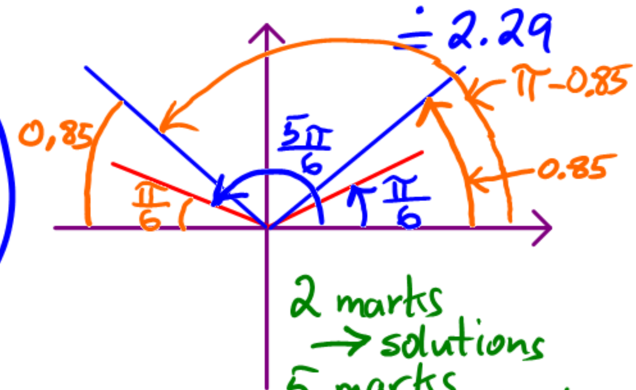
$\therefore \sin x = \frac{1}{2}$ or $\sin x = \frac{3}{4}$

$\therefore x = \sin^{-1}\left(\frac{1}{2}\right)$ or $x = \sin^{-1}\left(\frac{3}{4}\right)$

$\therefore x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

or $x \doteq 0.85$ or $x \doteq \pi - 0.85$

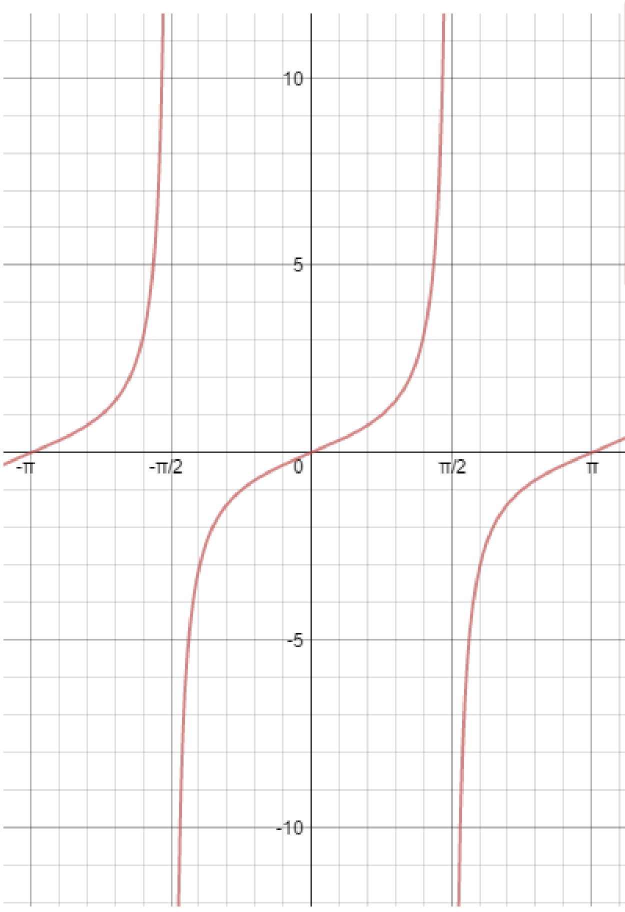
$\doteq 2.29$



2 marks → solutions

5 marks → justification

7. The graph of $f(x) = \dfrac{2\tan 2x - \sec^2 x \tan 2x}{2}$ is shown below. Write an equation for the identity suggested by this graph then prove that the equation is indeed an identity. **(10 TIPS)**



The graph suggests that

$$\frac{2\tan 2x - \sec^2 x \tan 2x}{2} = \tan x$$

is an identity   4 marks

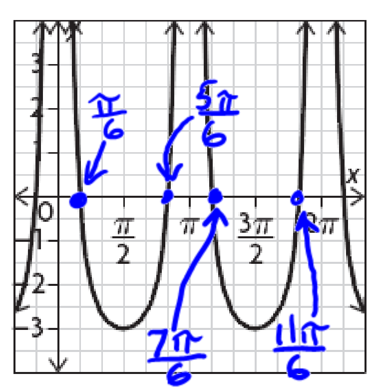
Proof:

$$\begin{aligned} L.S. &= \tfrac{1}{2}\tan 2x(2 - \sec^2 x) \\ &= \tfrac{1}{2}\left(\frac{2\tan x}{1-\tan^2 x}\right)\left[2 - (1+\tan^2 x)\right] \\ &= \left(\frac{\tan x}{1-\tan^2 x}\right)\left(\frac{1-\tan^2 x}{1}\right) \\ &= \tan x \\ &= R.S. \end{aligned}$$

$\therefore$ the given equation is an identity

6 marks

8. Write a quadratic trigonometric equation involving $\sin x$ whose solutions in the interval $[0, 2\pi]$ are the same as the x-intercepts of the graph shown at the right. Show that your equation yields the correct solutions. **(4 TIPS)**



x	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$
$\sin x$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

This shows that the solutions must be $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

1 mark → equation

3 marks → justification

For the given solutions in $[0, 2\pi]$,

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$

$\therefore \sin x - \frac{1}{2} = 0$ or $\sin x + \frac{1}{2} = 0$

$\therefore (\sin x - \frac{1}{2})(\sin x + \frac{1}{2}) = 0$

$\therefore \sin^2 x - \frac{1}{4} = 0$

$\therefore 4\sin^2 x - 1 = 0$

Any of these forms is accepted provided that justification is given

KU	APP	TIPS	COM
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