MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Mathematics
Trigonometry Unit Final Test

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Victim: Mr. Solutions

Marvellously and masterfully done!!

KU	APP	TIPS	COM
21/21	23/23	21/21	10/10

v.2

1. Use the three methods indicated below to demonstrate that the equation $\cos(2\pi-\theta)=\cos\theta$ ***is*** an identity.

(a) Compound angle identity **(3 KU)**

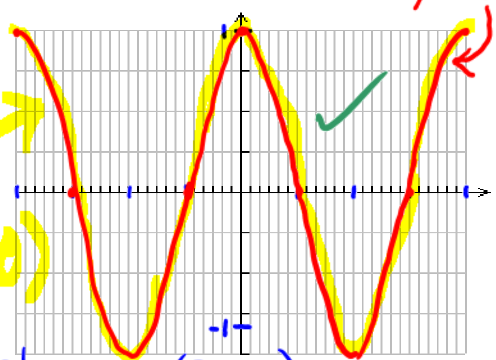
$\cos(2\pi-\theta)$

$=\cos 2\pi\cos\theta+\sin 2\pi\sin\theta$ ✓

$=1(\cos\theta)+0(\sin\theta)$ ✓

$=\cos\theta$ ✓

(b) Graphical (Transformations) **(3 KU)**

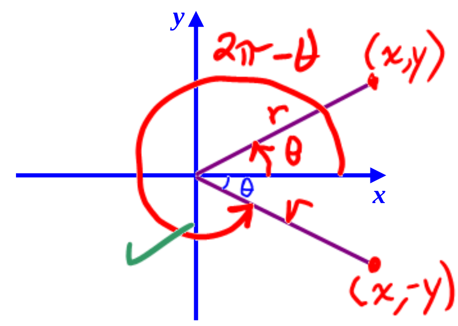


Let $y=\cos(2\pi-\theta)$

$=\cos(-1(\theta-2\pi))$ ✓

The graph of this function can be obtained by reflecting $y=\cos\theta$ in the y-axis, then shifting 2π to the right. Doing this simply produces the graph of $y=\cos\theta$!

(c) Angles of Rotation **(3 KU)**



$\cos\theta=\frac{x}{r}$ ✓

$\cos(2\pi-\theta)=\frac{x}{r}$ ✓

$\therefore \cos(2\pi-\theta)=\cos\theta$

2. Using the methods listed below, demonstrate that the equation $\tan\left(x+\frac{\pi}{4}\right)=\tan x+\tan\frac{\pi}{4}$ ***is not*** an identity.

(a) ***Counterexample*** **(3 KU)**

Let $x=\frac{\pi}{6}$

Then,

$L.S.=\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$=\tan\left(\frac{5\pi}{12}\right)$ ✓

$\doteq 3.732$

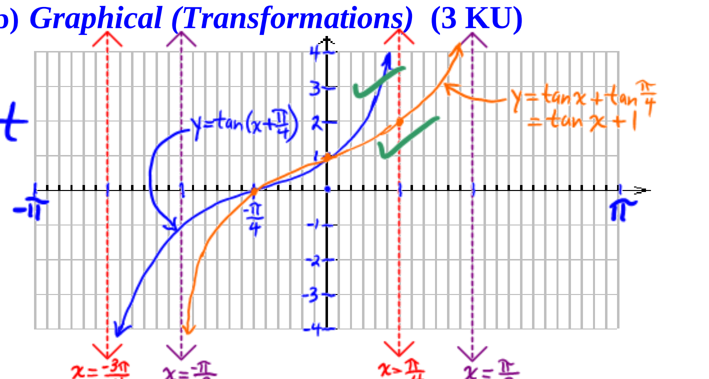
$R.S.=\tan\frac{\pi}{6}+\tan\frac{\pi}{4}$

$=\frac{1}{\sqrt{3}}+1$

$=\frac{\sqrt{3}+3}{3}\doteq 1.911$ ✓

$\therefore L.S.\neq R.S.$, the given equation cannot be an identity. ✓

(b) ***Graphical (Transformations)*** **(3 KU)**



$y=\tan(x+\frac{\pi}{4})\rightarrow$ graph of $y=\tan x$ shifted $\frac{\pi}{4}$ to the left

$y=\tan x+\tan\frac{\pi}{4}=\tan x+1\rightarrow$ graph of $y=\tan x$ shifted 1 up

Graphs are NOT coincident $\rightarrow$ the given equation cannot be an identity ✓

3. Evaluate the following trig ratios without using a calculator. Exact values are required!

(a) $\sin\frac{7\pi}{12}$ **(4 APP)**

$=\sin\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right)$ ✓

$=\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$=\sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$ ✓

$=\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{\sqrt{3}+1}{2\sqrt{2}}$

$=\frac{\sqrt{6}+\sqrt{2}}{4}$

✓✓ Either form is accepted.

(b) $\cos 157.5°$ **(4 APP)**

$157.5°=\frac{315°}{2}$

$\cos 2\theta=2\cos^2\theta-1$

$\therefore \cos 315°=2\cos^2 157.5°-1$ ✓

$\therefore \frac{\cos 315°+1}{2}=\cos^2 157.5°$ ✓

$\therefore -\sqrt{\frac{\cos 315°+1}{2}}=\cos 157.5°$

since 157.5° is in quad. II

$\therefore \cos 157.5°=-\sqrt{\frac{\frac{1}{\sqrt{2}}+1}{2}}$

$=-\sqrt{\frac{1}{2}\left(\frac{1+\sqrt{2}}{\sqrt{2}}\right)}$ ✓

$=-\sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}}=-\sqrt{\frac{\sqrt{2}+2}{4}}=-\frac{\sqrt{\sqrt{2}+2}}{2}$ ✓

Any form is accepted

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

4. For each of the following, write an identity entirely in terms of the given trigonometric ratio.

(a) $\cos 4\theta$ entirely in terms of $\cos\theta$ **(5 APP)**

$\cos 4\theta = \cos(2(2\theta))$ ✓

$= 2\cos^2 2\theta - 1$ ✓

$= 2(2\cos^2\theta - 1)^2 - 1$

$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 2 - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

✓✓✓

(b) $\cos 3\theta$ entirely in terms of $\cos\theta$ **(5 APP)**

$\cos 3\theta$

$= \cos(2\theta + \theta)$ ✓

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ ✓

$= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$

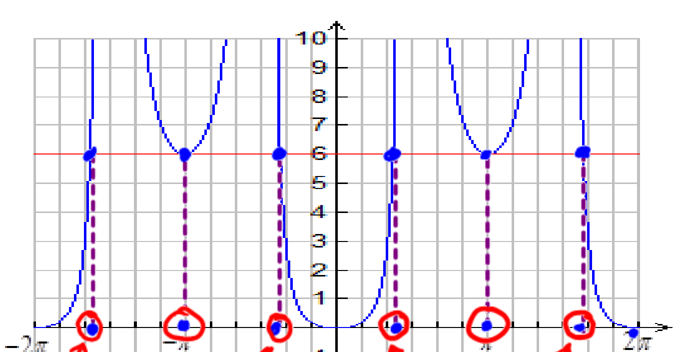
$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

✓✓

5. The following question deals with solving trigonometric equations both graphically and algebraically.

(a) Shown below are the graphs of $y = (\sec x - 1)(2\sec x - 1)$ and $y = 6$. State ***approximate*** solutions to the equation $(\sec x - 1)(2\sec x - 1) = 6$ for $x \in [-2\pi, 2\pi]$. In addition, ***mark*** the solutions on the graph. **(6 KU)**



Close to $\frac{-5\pi}{3}$ Close to $\frac{-\pi}{3}$ Close to $\frac{\pi}{3}$ Close to $\frac{5\pi}{3}$

$x \doteq -5, -\pi, -1.1, 1.1, \pi, 5$

KU { 3 marks → marking solutions on graph; 3 marks → stating approximate solutions }

(b) Use an algebraic method to solve the equation $(\sec x - 1)(2\sec x - 1) = 6$, where $x \in [-\pi, \pi]$. **(5 APP)**

$\therefore 2\sec^2 x - 3\sec x + 1 = 6$

$\therefore 2\sec^2 x - 3\sec x - 5 = 0$

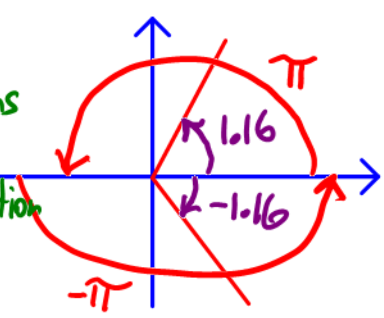
$\therefore (2\sec x - 5)(\sec x + 1) = 0$

$\therefore 2\sec x - 5 = 0$ or $\sec x = -1$

$\therefore \sec x = \frac{5}{2}$ or $\sec x = -1$

$\therefore x \doteq 1.16$ or $x \doteq -1.16$ or $x = \pi$ or $x = -\pi$ } agree with estimates from 5(a)

APP { 2 marks → solutions; 3 marks → justification }



KU	APP	TIPS	COM
–0	–0	–0	–0

6. Solve the equation $4\cos 2x + 10\sin x = 7$ for the interval $0 \le x \le 2\pi$. If necessary, and only if necessary, round your answers to 1 decimal place. **(7 TIPS)**

$4\cos 2x + 10\sin x = 7$

$\therefore 4(1-2\sin^2 x) + 10\sin x - 7 = 0$

$\therefore 4 - 8\sin^2 x + 10\sin x - 7 = 0$

$\therefore -8\sin^2 x + 10\sin x - 3 = 0$

$\therefore 8\sin^2 x - 10\sin x + 3 = 0$

$\therefore 8\sin^2 x - 4\sin x - 6\sin x + 3 = 0$

$\therefore 4\sin x(2\sin x - 1) - 3(2\sin x - 1) = 0$

$\therefore (2\sin x - 1)(4\sin x - 3) = 0$

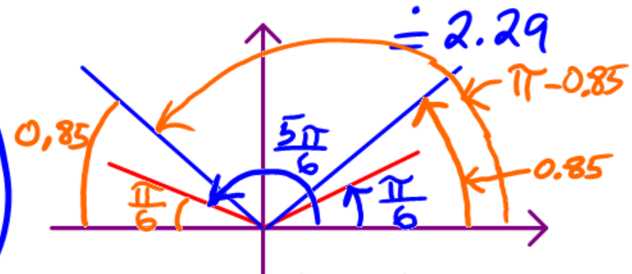
$\therefore 2\sin x - 1 = 0$ or $4\sin x - 3 = 0$

$\therefore \sin x = \frac{1}{2}$ or $\sin x = \frac{3}{4}$

$\therefore x = \sin^{-1}\left(\frac{1}{2}\right)$ or $x = \sin^{-1}\left(\frac{3}{4}\right)$

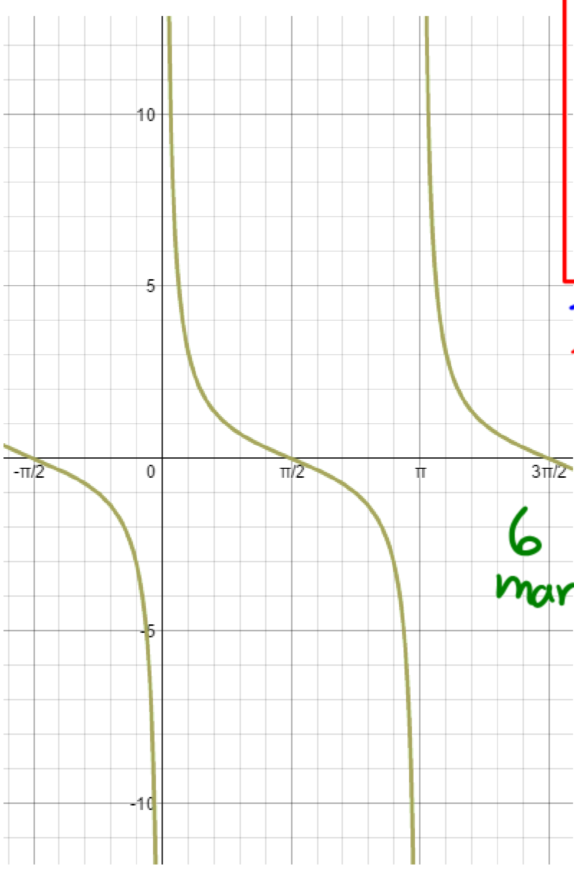
$\therefore x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

or $x \doteq 0.85$ or $x \doteq \pi - 0.85$
$\doteq 2.29$



2 marks → solutions
5 marks → justification

7. The graph of $f(x) = \dfrac{2\cot 2x(\csc^2 x + 1) - 4\cot 2x}{\cot^2 x - 1}$ is shown below. Write an equation for the identity suggested by this graph then prove that the equation is indeed an identity. **(10 TIPS)**



The graph suggests that the equation $\dfrac{2\cot 2x(\csc^2 x + 1) - 4\cot 2x}{\cot^2 x - 1} = \cot x$ is an identity 4 marks

Proof: 6 marks

$L.S = \dfrac{2}{\cot^2 x - 1}\left[\cot 2x(\csc^2 x + 1 - 2)\right]$

$= \left(\dfrac{2}{\cot^2 x - 1}\right)\left(\dfrac{\cot^2 x - 1}{2\cot x}\right)(\csc^2 x - 1)$

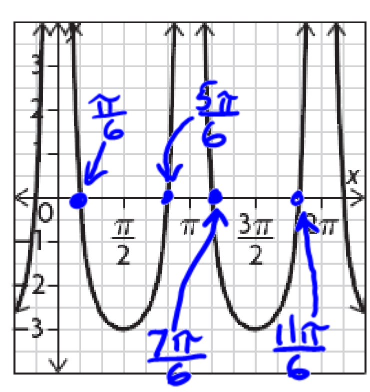
$= \left(\dfrac{1}{\cot x}\right)(\cot^2 x)$

$= \cot x$

$= R.S.$

$\therefore$ the given equation is an identity

8. Write a quadratic trigonometric equation involving $\csc x$ whose solutions in the interval $[0, 2\pi]$ are the same as the x-intercepts of the graph shown at the right. Show that your equation yields the correct solutions. **(4 TIPS)**



1 mark → equation
3 marks → justification

x	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$
$\csc x$	2	2	−2	−2

This shows that the solutions must be $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

For the given solutions in $[0, 2\pi]$,

$\csc x = 2$ or $\csc x = -2$

$\therefore \csc x - 2 = 0$ or $\csc x + 2 = 0$

$\therefore (\csc x - 2)(\csc x + 2) = 0$

$\therefore \csc^2 x - 4 = 0$

$\therefore \csc^2 x = 4$

Any of these forms is accepted provided that justification is given

KU	APP	TIPS	COM
− 0	− 0	− 0	− 0