MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 AP Mathematics

Unit 3 – Solving Polynomial Equations and Inequalities

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Victim: Mr. Solutions — a dazzling display of mathematical mastery!!

KU	APP	TIPS	COM
16/16	16/16	14/14	10/10

1. Consider the quintic polynomial function $p(x) = 4x^5 + 32x^2$.

(a) ***Fully*** factor the polynomial. **(3 KU)**

$4x^5 + 32x^2$ ← sum of cubes $x^3+y^3 = (x+y)(x^2-xy+y^2)$

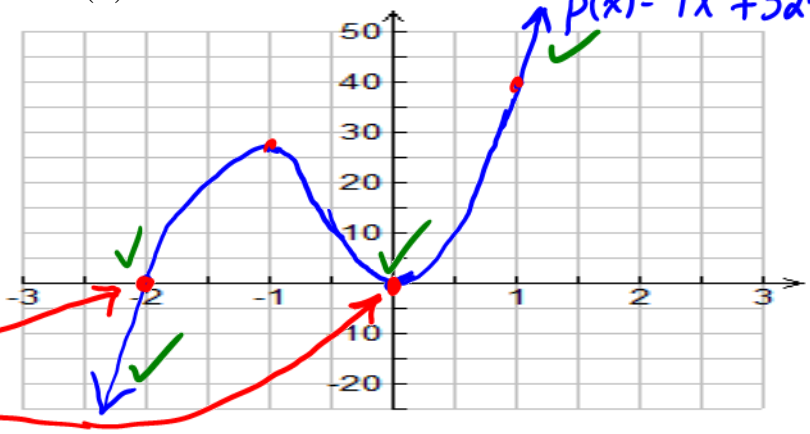
$= 4x^2(x^3+8)$

$= 4x^2(x+2)(x^2-2x+4)$

cannot factor further ($b^2-4ac<0$)

zero	multiplicity
−2	1
0	2

(b) Use the factored form of the polynomial to sketch the graph of $y = p(x)$. **(4 KU)**



(c) Use the factored form to solve the equation $4x^5 + 32x^2 = 0$. **(3 KU)**

$\therefore 4x^2(x+2)(x^2-2x+4) = 0$

$\therefore 4x^2 = 0$ or $x+2=0$ or $x^2-2x+4=0$

$b^2-4ac<0$ ∴ no real roots

$\therefore x = 0$ or $x = -2$

(d) Use the factored form and the graph to solve the inequality $4x^5 + 32x^2 \geq 0$. State the solution set using both set notation and interval notation. **(6 KU)**

$4x^2(x+2)(x^2-2x+4) \geq 0$

For all $x \in \mathbb{R}$, $4x^2 \geq 0$ and $x^2-2x+4 > 0$

∴ only $x+2$ affects the sign of $p(x)$

∴ $p(x) \geq 0$ when $x+2 > 0$

∴ $x \geq -2$

We can see this graphically by noting that the graph is at or above the x-axis only when $x \geq -2$.

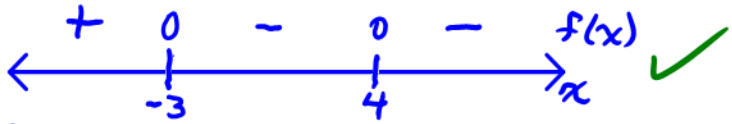
Solution Set

$\{x \in \mathbb{R} \mid x \geq -2\}$

$[-2, \infty)$

2. Solve the polynomial inequality $-2(x-4)^2(x+3) \leq 0$. State the solution set using both set notation and interval notation. In addition, include a graph that clearly shows the solution set. **(6 APP)**

Let $f(x) = -2(x-4)^2(x+3)$



$f(-4) = -2(-4-4)^2(-4+3) > 0$

$f(0) = -2(0-4)^2(0+3) < 0$

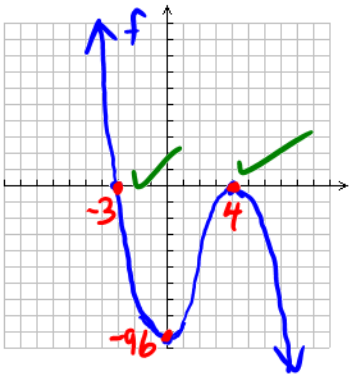
$f(5) = -2(5-4)^2(5+3) < 0$

} Test Points

Solution Set

$\{x \in \mathbb{R} \mid x \geq -3\}$

$[-3, \infty)$



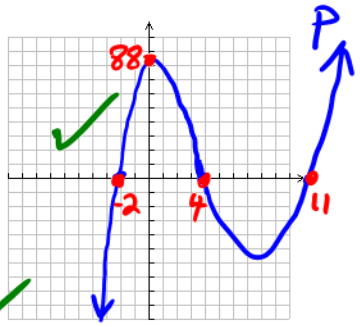
KU	APP	TIPS	COM
−0	−0	−0	−0

3. Solve the polynomial equation $x^3 - 13x^2 + 14x = -88$. Include a graph that clearly shows the solutions of the equation. **(10 APP)**

$x^3 - 13x^2 + 14x = -88$

$\therefore x^3 - 13x^2 + 14x + 88 = 0$

Let $p(x) = x^3 - 13x^2 + 14x + 88$



Since $p(-2) = 0$, $x-(-2) = x+2$ must be a factor of $p(x)$

By long division (as shown at the right),

$$x+2 \overline{)\, x^3 - 13x^2 + 14x + 88}$$

$x^2 - 15x + 44$

$x^3 + 2x^2$

$-15x^2 + 14x$

$-15x^2 - 30x$

$44x + 88$

$44x + 88$

0

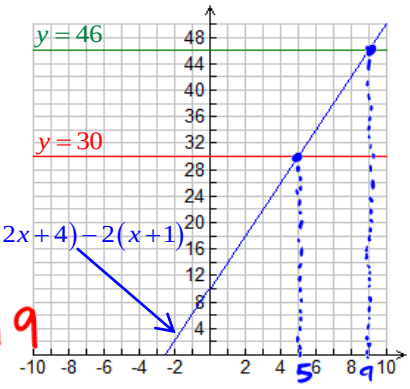
$p(x) = (x+2)(x^2 - 15x + 44)$

$= (x+2)(x-4)(x-11)$

If $p(x) = 0$, then $x+2=0$, $x-4=0$ or $x-11=0$

$\therefore x = -2$, $x = 4$ or $x = 11$

4. Write an inequality that corresponds to the diagram given at the right. In addition, state the solution set using both set notation and interval notation. (Do not solve the inequality. You should be able to see the solution just by looking at the graphs.) **(5 TIPS)**



$30 \le 3(2x+4) - 2(x+1) \le 46$

y is between 30 and 46 when x is between 5 and 9

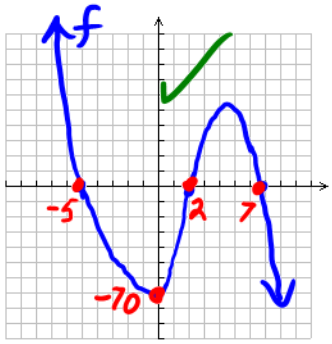
$\{x \in \mathbb{R} \mid 5 \le x \le 9\}$

$[5, 9]$

degree must be odd

$f(2) = 0$

5. The polynomial function $f(x) = -x^n + kx^2 + (8k-1)x - 70$ has two turning points, no global extreme points and when it is divided by $x-2$, the remainder is zero. Determine the values of n and k as well as any other zeros of f. Then sketch the graph of $y = f(x)$. **Hint:** There is no way to ***calculate*** the value of n. The best approach is to choose the simplest possible value of n. **(8 TIPS)**



Since f has no global extreme points, its degree must be odd. The simplest choice for n, therefore, is 3.

Also, since a remainder of zero is obtained when $x-2$ is divided into f, $f(2) = 0$ (remainder theorem).

$\therefore -2^3 + k(2)^2 + (8k-1)(2) - 70 = 0$

$\therefore 20k - 80 = 0$

$\therefore k = 4$

$$x-2 \overline{)\, -x^3 + 4x^2 + 31x - 70}$$

$-x^2 + 2x + 35$

$-x^3 + 2x^2$

$2x^2 + 31x$

$2x^2 - 4x$

$35x - 70$

$35x - 70$

0

By long division,

$-x^3 + 4x^2 + 31x - 70 = (x-2)(-x^2 + 2x + 35)$

$= -(x-2)(x^2 - 2x - 35)$

$= -(x-2)(x-7)(x+5)$

Graph extends from II to IV

$\therefore$ zeros are $2, 7, -5$

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0