MCR 3U9	**Semester 2, 2015 - 2016**

Grade 11 Pre-AP Functions
Diagnostic Test

Mr. N. Nolfi
Victim: Mr. Solutions

Your responses are both precise and elegant Mr. S.!!

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1. Complete the following statements by filling in the blanks with logical answers that relate to what we have learned in the review unit of this course. **[6]**

✓ = ½ mark

(a) Many students find mathematics difficult because they see it as a massive collection of complicated, incomprehensible rules that are used to manipulate myriad meaningless symbols. Gladly, there are simple strategies that students can apply that will help them develop a mindset that makes mathematics much easier to understand. Mr. Nolfi described three of these strategies in Unit 0. List the three strategies in the space provided below.

(i) Focus on important ideas rather than blindly memorized facts ✓

(ii) View mathematical relationships from different perspectives. ✓

(iii) Don't just scratch the surface! Learn in depth! ✓

As examples of one of these strategies, Mr. Nolfi pointed out that for equations of lines, we only need to remember slope = slope ✓, for the midpoint of a line segment, we only need to remember the average of a and b is $\frac{a+b}{2}$ ✓ and for the length of a line segment, we only need to remember the Pythagorean theorem ✓.

(b) ***Linear relationships*** model quantities that change at a constant ✓ rate ✓. For example, if a car moves with a constant ✓ velocity, the relationship between ***distance travelled*** and ***time elapsed*** is ***linear***. Given a ***table of values***, it is possible to spot linear relationships because the ***first differences*** are always constant ✓. ***Quadratic relationships***, on the other hand, model certain quantities that do not change at a constant rate. For example, if a cannonball is fired vertically into the air, then the relationship between its height ✓ and ***time elapsed*** is ***quadratic***. Given a ***table of values***, it is possible to spot quadratic relationships because the ***first differences*** always change linearly ✓ and the ***second differences*** are constant ✓.

(c) Miley C. and Justin B. collaborated to ***solve*** the equation $x^2+4x-5=27$. Together, they wrote the "solution" shown below. Is it correct? If so, explain why. If not, write a correct solution. **[5]**

$x^2+4x-5=27$

$\therefore (x+5)(x-1)=27$ *

✗ $\therefore x+5=27$ or $x-1=27$ ✗

$\therefore x=22$ or $x=28$

This is a non sequitur. The only valid conclusions that can be drawn from * are $x+5=\frac{27}{x-1}$ and $x-1=\frac{27}{x+5}$. ✓

Correct Approach

$x^2+4x-5=27$

$\therefore x^2+4x-32=0$ ✓

$\therefore (x+8)(x-4)=0$ ✓

$\therefore x+8=0$ or $x-4=0$ ✓

$\therefore x=-8$ or $x=4$ ✓

2. Solve. Show all steps.

(a) Solve the following linear equation. **[5]**

$$\frac{10}{1}\left[\frac{3}{5}(x-1)+2x\right]=\left[-4-\frac{7}{10}x\right]\left(\frac{10}{1}\right)$$

$\therefore 6(x-1)+20x=-40-7x$

$\therefore 6x-6+20x=-40-7x$

$\therefore 26x-6=-40-7x$

$\therefore 33x=-34$

$\therefore x=\frac{-34}{33}$

(b) Solve the following quadratic equation. **[7]**

$$2(3x-1)(x+1)=-x(2x-5)+3$$

$\therefore (6x-2)(x+1)=-2x^2+5x+3$

$\therefore 6x^2+4x-2=-2x^2+5x+3$

$\therefore 8x^2-x-5=0$

$b^2-4ac=161$ $\therefore$ doesn't factor

$\therefore x=\dfrac{1\pm\sqrt{(-1)^2-4(8)(-5)}}{2(8)}$

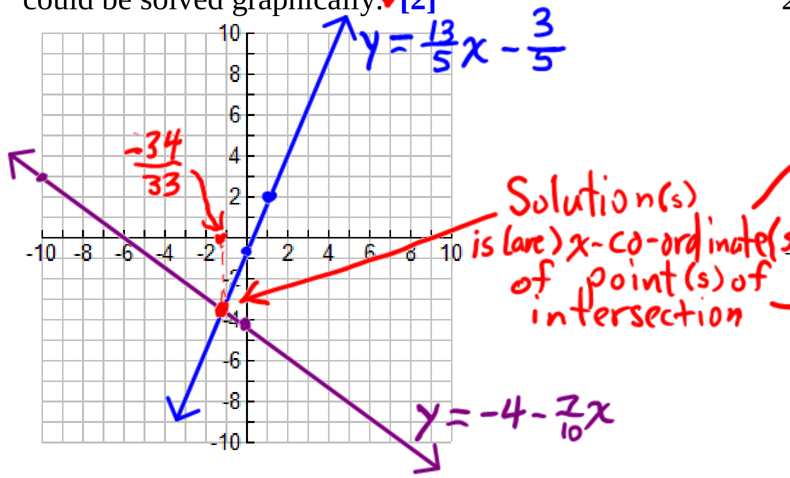
$=\dfrac{1\pm\sqrt{161}}{16}$

Rough Work:

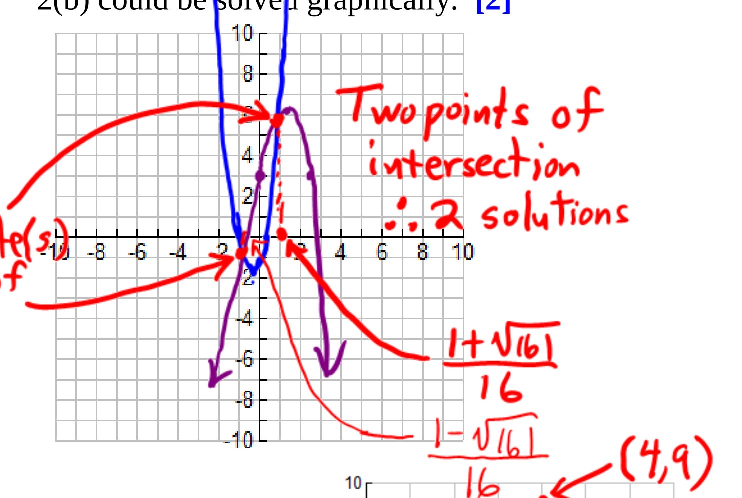
$\frac{3}{5}(x-1)+2x$

$=\frac{13}{5}x-\frac{3}{5}$

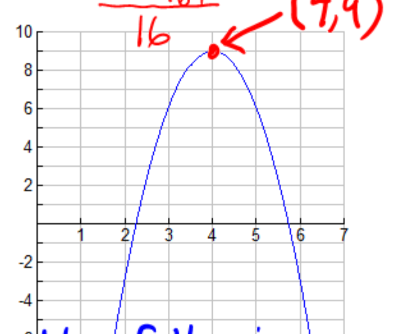
(c) Sketch a graph that shows how the equation in 2(a) could be solved graphically. **[2]**



(d) Sketch a graph that shows how the equation in 2(b) could be solved graphically. **[2]**



3. Which of the following could be the equation of the graph shown at the right? Explain. **[3]**



(a) $y=-3(x+4)^2+9$ **(b)** $y=-3(x+9)^2+4$

(c) $y=-3(x-4)^2+9$ **(d)** $y=-3(x-9)^2+4$

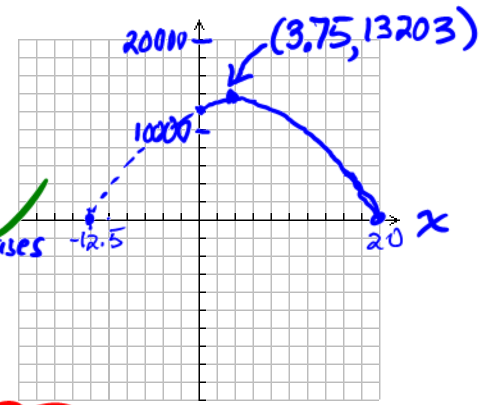
The correct equation is $y=-3(x-4)^2+9$

By taking the graph of $y=x^2$ and performing the following transformations, the given graph is obtained

- stretch vertically by a factor of -3 (stretch by factor of 3, reflect in x-axis)
- shift 4 units right, 9 units up

These transformations correspond to equation (c)

4. A transit company charges \$1.25 per ride and averages 10,000 riders per day. The company needs to increase revenue but found that for each \$0.10 increase in fare the company would lose 500 riders. What should the company charge to maximize revenues? **[8]**



Let x represent the number of \$0.10 fare increases and r represent the revenue obtained for x fare increases

fare $= 1.25 + 0.1x$, # passengers $= 10000 - 500x$

$$\therefore r = (1.25 + 0.1x)(10000 - 500x)$$
$$= 0.1(12.5 + x)(500)(20 - x)$$
$$= 50(12.5 + x)(20 - x)$$

Rough Work:
For $x = 3.75$,
$r = 50(12.5 + 3.75)(20 - 3.75)$
$\doteq 13203$

zeros of r: -12.5, 20

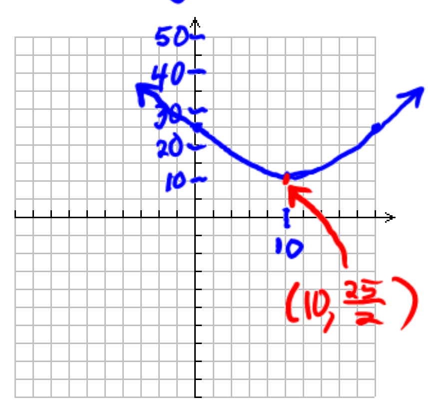
axis of symmetry: $x = \dfrac{-12.5 + 20}{2} = 3.75$

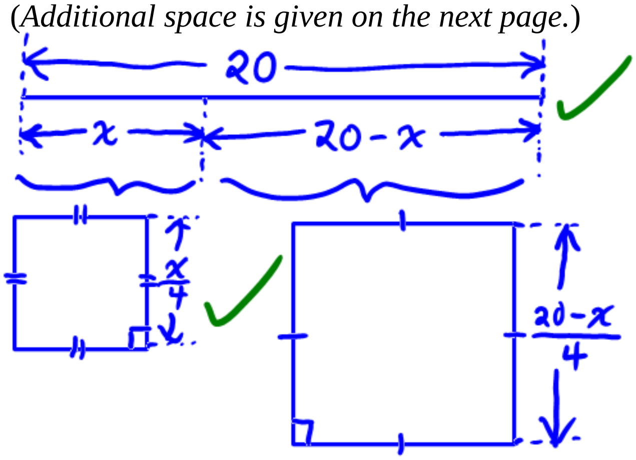
Since the vertex of a parabola lies on the axis of symmetry, revenue is maximized when $x = 3.75$.

Therefore, revenue is maximized when the fare is $1.25 + 0.1(3.75) = 1.625$.

The transit company should set the fare to \$1.62 to maximize revenue.
(scientific method of rounding used here)

5. A piece of wire 20 m long is cut into two pieces and each piece is bent to form a square. Determine the length of the two pieces so that the sum of the areas of the two squares is a minimum. **[8]**
(*Additional space is given on the next page.*)





Let x represent the distance shown in the diagram and let A represent the sum of the areas of the two squares.

$$\therefore A = \left(\frac{x}{4}\right)^2 + \left(\frac{20 - x}{4}\right)^2$$
$$= \frac{x^2}{16} + \frac{(20 - x)^2}{16}$$
$$= \frac{1}{16}x^2 + \frac{1}{16}(400 - 40x + x^2)$$
$$= \frac{1}{16}x^2 + \frac{1}{16}x^2 + 25 - \frac{5}{2}x$$
$$= \frac{1}{8}x^2 - \frac{5}{2}x + 25$$

Additional Space for # 5

$$A=\frac{1}{8}x^2-\frac{5}{2}x+25$$
$$=\frac{1}{8}(x^2-20x)+25$$
$$=\frac{1}{8}(x^2-20x+10^2-10^2)+25$$
$$=\frac{1}{8}\left[(x-10)^2-100\right]+25$$
$$=\frac{1}{8}(x-10)^2-\frac{25}{2}+\frac{50}{2}$$

$$\therefore A=\frac{1}{8}(x-10)^2+\frac{25}{2}$$

$\therefore$ the co-ordinates of the vertex are $\left(10,\frac{25}{2}\right)$

$\therefore$ when the wire is cut in half, the minimal area of $\frac{25}{2}\,\text{m}^2$ is produced.

6. Given that the points $(1,11)$ and $(-2,-34)$ lie on the graph of $y=ax^2+bx+6$, find the values of a and b. **[8]**

Since the given points lie on the parabola, their co-ordinates must satisfy the given equation.

$\therefore\ 11=a(1^2)+b(1)+6$ and $-34=a(-2)^2+b(-2)+6$

$\therefore\ a+b=5$ ① and $4a-2b=-40$ ②

From ①, $b=5-a$. ③

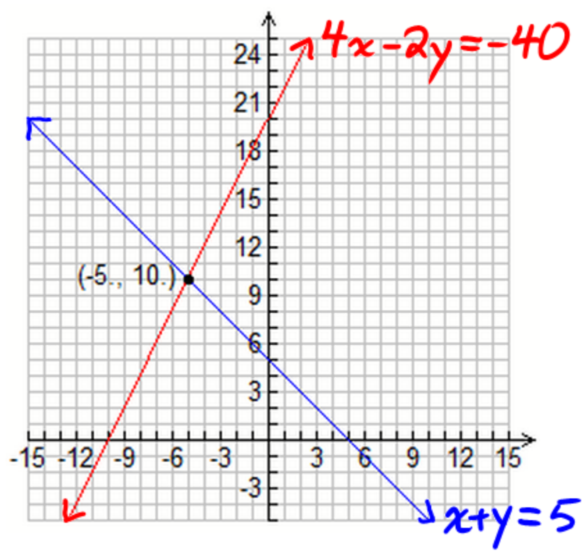
Substituting in ②,

$4a-2(5-a)=-40$

$\therefore\ 4a-10+2a=-40$

$\therefore\ 6a=-30$

$\therefore\ a=-5$



Substituting in ③,

$b=5-(-5)=10.$

$\therefore\ a=-5,\ b=10$ and $y=-5x^2+10x+6$