MCR 3U9 | **Semester 2, 2015 - 2016**

Grade 11 Pre-AP Functions

Unit 1 – Major Test 1 – Functions, Relations and Transformations

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Victim: Mr. Solutions

Inspiring work Mr. S!!

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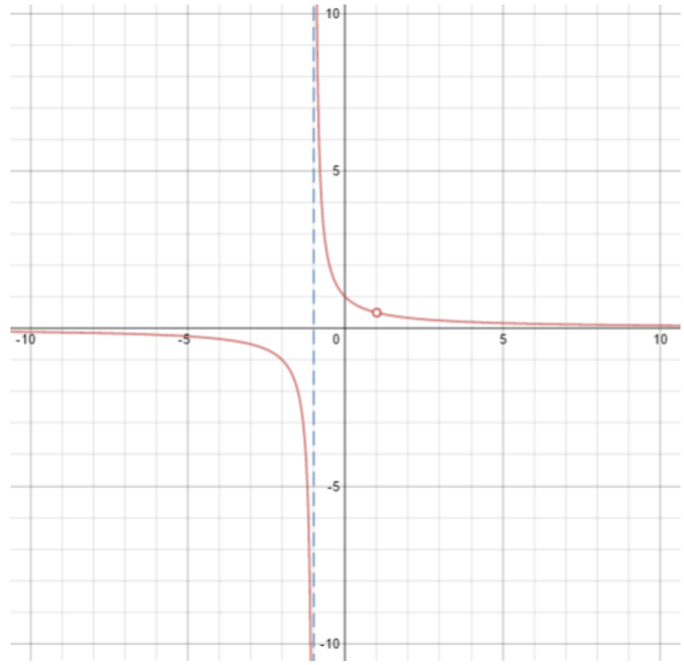
1. Carefully study the graph of the function *f* at the right and then answer the questions given below. **(15 KU)**



(a) State the domain and range of *f*.

Domain = { $x \in \mathbb{R} \mid x \neq -1, x \neq 1$ }

Range = { $y \in \mathbb{R} \mid y \neq 0, y \neq \frac{1}{2}$ }

(b) Evaluate each of the following.

$f(0) = 1$ $f(a) = 2 \quad \therefore a = -\frac{1}{2}$

$f(1) =$ undefined (hole) $f(-1) =$ undefined

$f(2) = \frac{1}{3}$ $f(3-5) = f(-2) = -1$

(c) State the equation of the vertical asymptote.

$x = -1$

(d) Suggest a possible equation for *f*.

$$f(x) = \frac{x-1}{x^2-1}$$

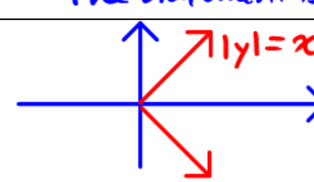
$$= \frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1}, \; x \neq 1$$

(e) Use your equation from (d) to explain why *f* has a vertical asymptote and a hole.

Hole: Common factor $x-1$ in numerator and denominator divides out but $f(1)$ is undefined

Asymptote: As x approaches -1, $\frac{1}{x+1}$ gets larger and larger. If $x > -1$, $\frac{1}{x+1}$ is positive. If $x < -1$, $\frac{1}{x+1}$ is negative.

2. State whether each of the following is true or false. Provide an explanation to support each response. **(8 TIPS)**

Statement	*True or False?*	*Explanation*
For all functions *f* and all real numbers *u* and *c*, $f(uc) = f(u)f(c)$	F	Consider $f(x) = x+1$, $u=1$, $c=1$ L.S. $= f(uc) = f(1(1)) = f(1) = 2$ R.S. $= f(u)f(c) = f(1)f(1) = 2(2) = 4$ Since L.S ≠ R.S., we have found a counterexample that shows that the statement is false.
The equation $\lvert y \rvert = x$ is that of a function.	F	x: 0, 1, 1, 2, 2; y: 0, 1, −1, 2, −2 For all $x > 0$, there are two values of y for each value of x.  $\lvert y \rvert = x$

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Statement	*True or False?*	*Explanation*
If $f(x)=x$ then $f^{-1}(x)=\frac{1}{x}$.	F	$f(x)=x$ is invariant under a reflection in the line $y=x$. $\therefore f^{-1}(x)=x$ NOT $\frac{1}{x}$ ($\frac{1}{x}$ is the reciprocal function not the inverse function)
Suppose that $g(x)=-3f(2(x-8))+6$. To obtain the graph of *g*, the following transformations must be performed to *f*: • Vertical stretch by a factor of -3 followed by a shift up by 6 units • Horizontal ~~compression~~ stretch by a factor of 2, followed by a shift 8 units left.	F	To determine the nature of the horizontal transformations, f's input must be transformed to g's input. $f \quad 2(x-8) \rightarrow \div 2 \text{ (or } \times\frac{1}{2}) \rightarrow +8 \rightarrow x \quad g$ $\therefore$ there should be a horizontal compression by a factor of $\frac{1}{2}$ followed by a shift 8 units right

3. The transformation expressed in mapping notation below is applied to $f(x)=x^3+1$ to produce the function *g*. Complete the table for this transformation. **(15 APP)**

Transformation

Mapping Notation

$$(x,y)\rightarrow\left(4x+2,-\frac{1}{2}y+1\right)$$

Function Notation

$$g(x)=-\frac{1}{2}f\left(\frac{1}{4}(x-2)\right)+1$$

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Verbal

Horizontal

1. Stretch by a factor of 4
2. Translate 2 to the right

Vertical

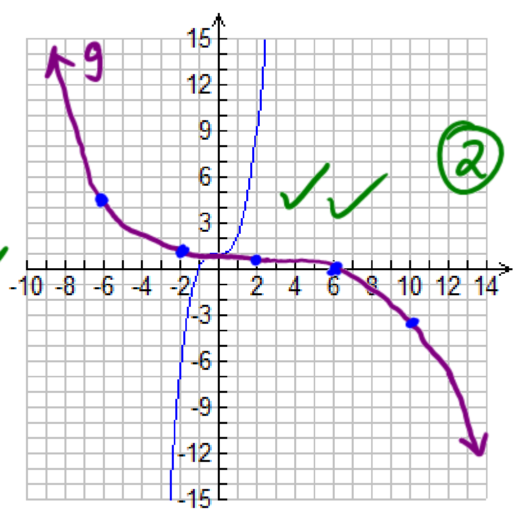
1. Compress by a factor of $-\frac{1}{2}$ (includes reflection in x-axis)
2. Translate up one unit

4

Images of Five Key Points 2

Pre-Image	Image
$(0,1)$	$(2,\frac{1}{2})$
$(1,2)$	$(6,0)$
$(-1,0)$	$(-2,1)$
$(2,9)$	$(10,-\frac{7}{2})$
$(-2,-7)$	$(-6,\frac{9}{2})$

Graph of y = g(x)
(The graph of y = f(x) is given)



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Equation of the Image Function y = g(x) 3

$$\begin{aligned}
g(x)&=-\frac{1}{2}f\left(\frac{1}{4}(x-2)\right)+1\\
&=-\frac{1}{2}\left[\left(\frac{1}{4}(x-2)\right)^3+1\right]+1\\
&=-\frac{1}{2}\left[\left(\frac{1}{4}\right)^3(x-2)^3+1\right]+1\\
&=-\frac{1}{2}\left[\frac{1}{64}(x-2)^3+1\right]+1\\
&=-\frac{1}{128}(x-2)^3-\frac{1}{2}+1\\
&=-\frac{1}{128}(x-2)^3+\frac{1}{2}
\end{aligned}$$

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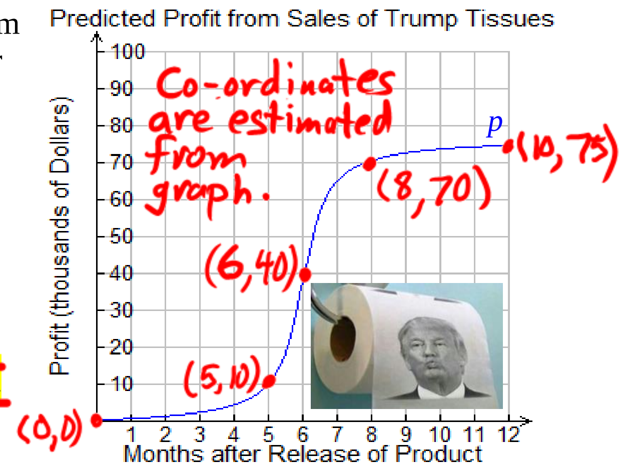
4. The graph of the function p at the right shows the ***predicted profit*** from sales of Donald Trump Bathroom Tissues for the first 12 months after introduction of the product.



Due to manufacturing problems, the introduction of the product was ***delayed by two months***. In spite of the delay, the ***actual profit*** turned out to be \$10,000 more than 1.2 times the predicted profit.

Express this using mapping notation, function notation and graphically. For the purposes of function notation, let p represent the predicted-profit function and let a represent the actual-profit function. **(10 APP)**

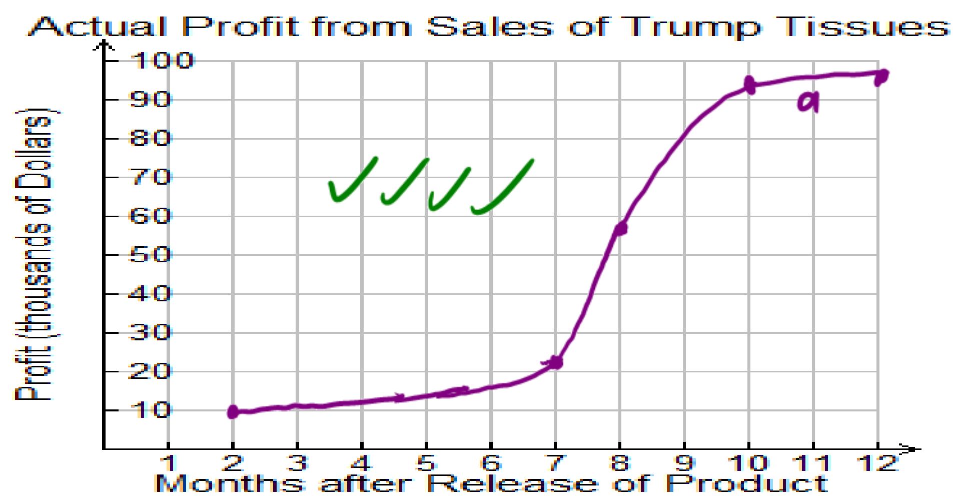
Mapping Notation

$(x,y)\to(x+2,\ 1.2y+10)$

Function Notation

$a(x)=1.2p(x-2)+10$

Graph

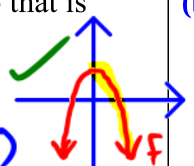


5. Consider the function $f(x)=-2x^2+5$. **(10 KU)**

(a) Restrict the domain of f to a "piece" to that is one-to-one.

Restricted Domain $=\{x\in\mathbb{R}\mid x\geq 0\}$

1 (highlighted piece of graph)



(b) Sketch the graphs of both f, restricted to the domain that you chose in (a), and its inverse function for this domain.

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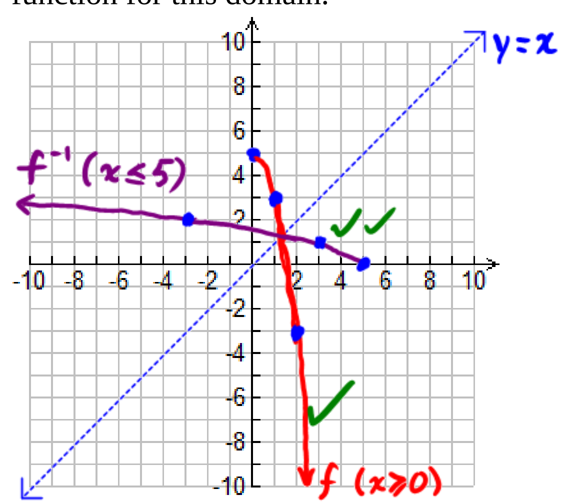

(c) Find f^{-1} algebraically for the one-to-one "piece" of f that you chose in (a). Use your graph to check your answer.

$y=-2x^2+5$

To find the inverse, apply the transformation $(x,y)\to(y,x)$.

$x=-2y^2+5$

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$\therefore\ -2y^2=x-5$

$\therefore\ y^2=\dfrac{x-5}{-2}=\dfrac{5-x}{2}$

$\therefore\ y=\pm\sqrt{\dfrac{5-x}{2}}$

For f, $x\geq 0$. Therefore, for f^{-1}, $y\geq 0$.

$\therefore f^{-1}(x)=\sqrt{\dfrac{5-x}{2}}$

(d) State the domain and range of f^{-1}.

Domain $=\{x\in\mathbb{R}\mid x\leq 5\}$

Range $=\{y\in\mathbb{R}\mid y\geq 0\}$

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6. Smartphone "finger" stands normally sell for $20 apiece. At this price, 300,000 are expected to be sold.

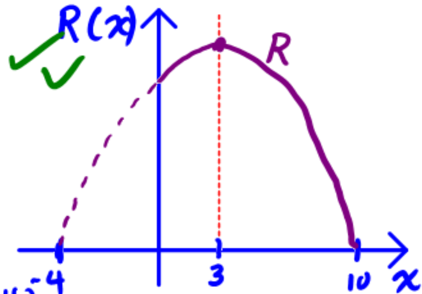
(a) For every 5-dollar increase in price, 30,000 fewer are sold. What selling price will produce the maximum revenue and what will the maximum revenue be? **(6 TIPS)**

Let x represent the number of 5-dollar increases in price

$\therefore$ # sold $= \underline{300000 - 30000x}$ and price in \$ $= \underline{20+5x}$

Let $R(x)$ represent the revenue, in dollars, for x \$5 price increases

$$\therefore R(x) = (300000-30000x)(20+5x)$$
$$= [30000(10-x)][5(4+x)]$$
$$= 150000(10-x)(4+x)$$



Since the zeros of R are -4 and 10, the equation of the axis of symmetry is $x = \frac{10+(-4)}{2} = 3$.

This means that the co-ordinates of the vertex are $(3, R(3))$.

$$R(3) = \underbrace{[300000-30000(3)]}_{\text{\# sold}}\underbrace{[20+5(3)]}_{\text{price}} = 210000(35) = 7350000.$$

Therefore, the maximum revenue is \$7,350,000, which, according to the model, is obtained if the price is set to \$35 $(20+5(3))$.

(b) Explain the ***meaning*** of the ***inverse*** of the revenue function that you obtained in part (a). **(4 COM)**

The revenue function takes the # of \$5 price increases as input and produces revenue as output.

The inverse of the revenue function, R^{-1}, does the opposite. It takes revenue as input and produces the # of \$5 price increases as output.

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