Grade 11 Pre-AP Functions

Quiz – Unit 2 – Trig Ratios, Radian Measure, Angular and Linear Velocity, Trig Functions

Mr. N. Nolfi You did it again Mr. N.!!

Victim: Mr. Solutions

KU	APP	TIPS	COM
10/10	10/10	10/10	10/10

Note: For all questions, assume that angles of rotation are in ***standard position***.

Modified True/False (6 KU)

State whether each statement is ***true*** or ***false***. If false, ***change*** the ***underlined part*** to make the statement true.

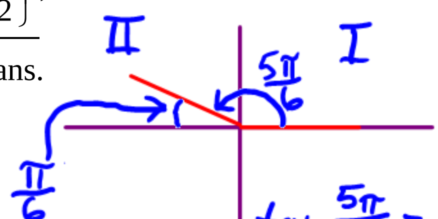
1. T/F F $\frac{\pi}{3}$ and $\underline{-\frac{\pi}{3}}$ are coterminal angles. **Change:** $\frac{7\pi}{3}$

2. T/F F The principal angle of $\frac{23\pi}{6}$ is $\underline{\frac{\pi}{6}}$. $\frac{23\pi}{6}=23(30^\circ)=690^\circ=360^\circ+330^\circ$ **Change:** $\frac{11\pi}{6}$

3. T/F F The related first quadrant angle of $\frac{23\pi}{6}$ is $\underline{\frac{\pi}{3}}$. **Change:** $\frac{\pi}{6}$

4. T/F F If the domain of the cosine function is restricted to $\underline{\left\{x\in\mathbb{R}:-\frac{\pi}{2}\le x\le\frac{\pi}{2}\right\}}$, then $\cos^{-1}$ is defined. **Change:** $\{x\in\mathbb{R}\mid 0\le x\le\pi\}$

5. T/F F $571^\circ \doteq \underline{9.97}\pi$ radians. **Change:** 9.97



6. T/F F $\tan\frac{5\pi}{6}=\underline{\sqrt{3}}$ **Change:** $-\frac{1}{\sqrt{3}}$

$\tan\frac{5\pi}{6}=-\tan\frac{\pi}{6}$ tan is negative in quadrant II

Multiple Choice (4 KU)

7. d Which of the following is an example of the power of trigonometry?

(a) Trigonometric ratios depend only the angles in a right triangle, not on the size of the triangle.
(b) It relates angles to side lengths.
(c) Angles are far easier to measure directly than distances.
(d) All of the above.

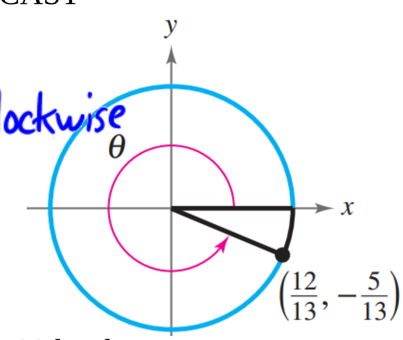
8. a For any angle of rotation θ in the third quadrant, $\tan\theta>0$ and $\cot\theta>0$. Why is this the case?

(a) For any point (x, y) in quadrant III, $x < 0$ and $y < 0$.
(b) ASTC
(c) For any point (x, y) in quadrant III, $x > 0$ and $y > 0$.
(d) CAST

9. c An angle of rotation θ is shown at the right. Which of the following is ***not true***?

(a) $\tan\theta=-\frac{5}{12}$
(b) $\cos\theta=\frac{12}{13}$
(c) $\theta\doteq-0.39479$
(d) $\sin\theta=-\frac{5}{13}$

Rotation is counterclockwise $\therefore \theta>0$ $\therefore$ (c) cannot be true

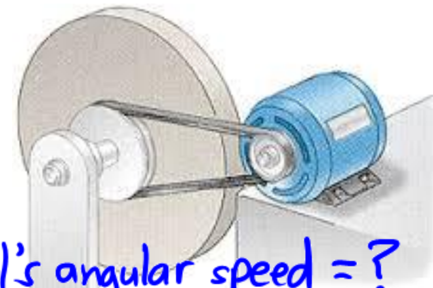


10. b A car's wheels have a radius of 40 cm. If the car moves at a speed of 100 km/h, what is the angular speed of the wheels?

$v=\omega r \therefore \omega=\frac{v}{r}=\frac{1}{r}(v)=\frac{1}{0.4}\left(\frac{250}{9}\right)$ 100 km/h $=\frac{250}{9}$ m/s

(a) $\frac{1250\pi}{9}$ rad/s
(b) $\frac{625}{9}$ rad/s
(c) $\frac{625}{9}$ rev/s
(d) $\frac{1250\pi}{9}$ rev/s

11. As shown in the diagram at the right, an electric motor is used to turn a grinding wheel. A drive belt is used to attach a pulley on the motor to a pulley on the grinding wheel. The pulley on the motor has a radius of 3 cm, the pulley on the grinding wheel has a radius of 9 cm and the motor spins at a rate of 3450 rpm (rotations per minute).



(a) Calculate the spin rate of the grinding wheel. **(5 APP)**

ω_m = motor's angular speed = 3450 rpm ω_g = grinding wheel's angular speed = ?

r_m = radius of motor's pulley = 3 cm r_g = radius of grinding wheel's pulley = 9 cm

Since the pulleys are attached by a drive belt, their linear velocities are equal.

$$\therefore \omega_m r_m = \omega_g r_g$$

See alternative solution on next page.

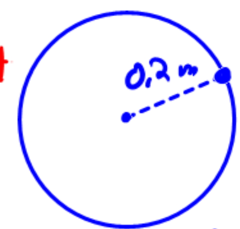
$$(3450\text{ rpm})(3\text{ cm}) = \omega_g(9\text{ cm})$$

$$\therefore \omega_g = \frac{(3450\text{ rpm})(3\text{ cm})}{9\text{ cm}} = \frac{3450}{3}\text{ rpm} = 1150\text{ rpm}$$

(b) Assuming that the grinding wheel has a radius of 20 cm, calculate the linear velocity of a point on the circumference of the wheel. **(5 APP)**

$$v = \frac{d}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = r\omega$$

← must be in rad/min to get cm/min



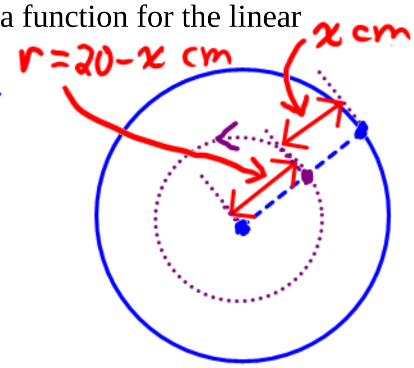
$$\therefore v = (20\text{ cm})(1150\text{ rot/min})$$

$$= (20\text{ cm})[1150(2\pi)\text{ rad/min}]$$

See alternative solution on next page.

$$= 46000\pi\text{ cm/min}\ \left(\frac{46000\pi}{60}\text{ cm/s} = \frac{2300\pi}{3}\text{ cm/s}\right)$$

(c) Still assuming that the grinding wheel has a radius of 20 cm, write an equation of a function for the linear velocity of a point on the grinding wheel, *x* cm from the circumference. **(7 TIPS)**



Let $v(x)$ represent the linear velocity of a point on the wheel, x cm from the circumference

$$\therefore v(x) = r\omega$$

$$= (20-x)(1150(2\pi)\text{ rad/min})$$

$$= 2300\pi(20-x)\text{ cm/min}$$

(d) Using your answer from (c), write a function for the distance travelled (in one hour) by a point on the grinding wheel, *x* cm from the circumference. **(3 TIPS)**

Let $d(x)$ represent the distance travelled by a point on the wheel, x cm from the circumference.

$$d(x) = v(x)(60\text{ min})$$

$$= 60[2300\pi(20-x)]\text{ cm}$$

$$= 138000\pi(20-x)\text{ cm}$$

Alternative Solutions

The solution given above for 11(a) relied on physical principles, specifically angular speed and linear speed. As shown below, 11(a) can also be solved using geometric principles.

11(a) Alternative Solution

$\because \quad r_g = 3r_m$

$\therefore \quad C_g = 3C_m$

$\because$ the pulleys are attached to each other by the drive belt, the grinding wheel makes one complete rotation for every 3 complete rotations of the motor

$$\therefore \quad \omega_g = \frac{\omega_m}{3} = \frac{3450 \text{ rpm}}{3} = 1150 \text{ rpm}$$

11(b) Alternative Solution

$$v = \frac{d}{t}$$

$$= \frac{r\theta}{t}$$

$$= \frac{(\cancel{20}^{1}\text{ cm})(\cancel{138000}^{46000}\pi \text{ rad})}{\cancel{60}_{\cancel{3}_{1}} \text{ min}}$$

$$= 46000\pi \text{ cm/min}$$

$$1150 \text{ rpm} = 1150(60) \text{ rot/h}$$

$$= 1150(60)(2\pi) \text{ rad/h}$$

$$= 138000\pi \text{ rad/h}$$

$\therefore$ the grinding stone rotates through $138\,000\pi$ rad in one hour