MCR 3U9	Grade 11 Pre-AP Functions Unit 2 – Major Test up to Part 1 of Identities	Semester 2, 2015 - 2016

Mr. N. Nolfi

Well done Mr. S.!!

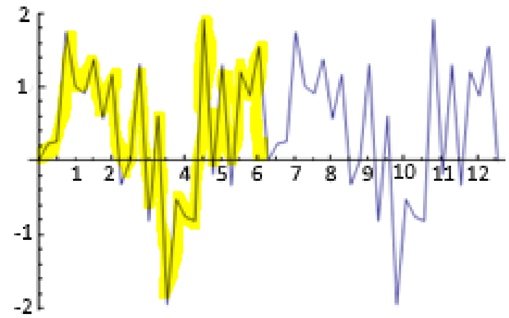
Victim: Mr. Solutions

KU	APP	TIPS	COM
11 /11	14 /14	16 /16	13 /13

- Unless otherwise noted, ***radian measure must be used***. COM marks will be deducted for using degree measure.
- Up to 10 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

Part 1: Multiple Choice (11 KU)

Identify the choice that ***best*** answers the question.



1. b The graph of a periodic function is shown at the right. What is the approximate ***period*** of the function?

(a) 3.5 **(b)** 6.3 **(c)** 12.6 **(d)** 7

2. c Which of the following ***is most unlikely*** to produce a periodic graph?

(a) Little Prabhnoor's height above the floor as he jumps up and down in a playpen.

(b) Saaya's height above the ground as she rides an extremely fast Ferris wheel.

(c) The height above the ground of Haris' airplane as it takes off for a long trip across the country.

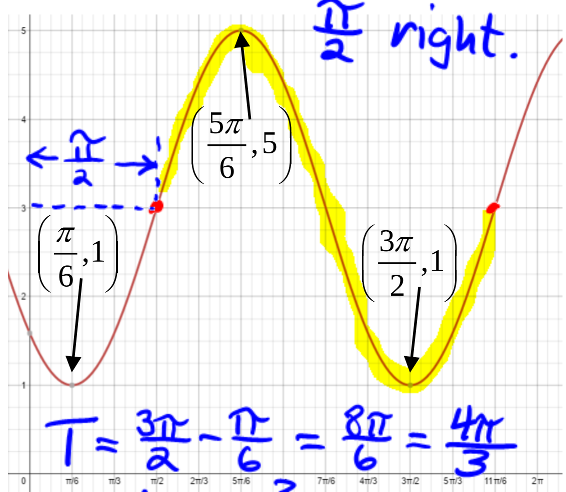
(d) The height above the floor of Ashutosh's (naughty former student) mother's hand as she "disciplines" him.

3. a Which of the following is a ***correct*** equation for the graph at the right?

Base function translated $\frac{\pi}{2}$ right.

(a) $f(x)=2\sin\left(\frac{3}{2}\left(x-\frac{\pi}{2}\right)\right)+3$ ~~**(b)**~~ $f(x)=2\sin\left(\frac{3}{2}\left(x+\frac{\pi}{2}\right)\right)+3$

~~**(c)**~~ $f(x)=2\sin\left(\frac{2}{3}\left(x-\frac{\pi}{2}\right)\right)+3$ ~~**(d)**~~ $f(x)=2\sin\left(\frac{2}{3}\left(x+\frac{\pi}{2}\right)\right)+3$



$T=\frac{3\pi}{2}-\frac{\pi}{6}=\frac{8\pi}{6}=\frac{4\pi}{3}$

$\therefore \omega=\frac{3}{2}$

4. b The function shown at the right has domain and range

~~**(a)**~~ $D=\{x\in\mathbb{R}:1\geq x\geq 5\}$, $R=\mathbb{R}$ **(b)** $D=\mathbb{R}$, $R=\{y\in\mathbb{R}:1\leq y\leq 5\}$

~~**(c)**~~ $D=\{x\in\mathbb{R}:1\leq x\leq 5\}$, $R=\mathbb{R}$ ~~**(d)**~~ $D=\mathbb{R}$, $R=\{y\in\mathbb{R}:1\geq y\geq 5\}$

5. b Which of the following ***does not*** make sense?

✓**(a)** A sinusoidal function has a period of π. **(b)** A sinusoidal function has an amplitude of $-1/1000$.

✓**(c)** A sinusoidal function has an amplitude of 3. ✓**(d)** A sinusoidal function has an angular frequency of π.

6. c Tiffany jumps up and down on a trampoline. Her height in metres above the ground after *t* seconds is given by the function $h(t)=1.5\sin(2\pi t)+1$. What does the "2π" in the equation represent?

~~**(a)**~~ Tiffany's maximum displacement from the average. 1.5 ~~**(b)**~~ Tiffany's average height above the ground. 1

(c) How long, in seconds, it takes Tiffany to go up and down 2π times. ~~**(d)**~~ How long, in seconds, it takes Tiffany to go up and down once.

$T=\frac{1}{\omega}(2\pi)=\frac{1}{2\pi}\left(\frac{2\pi}{1}\right)=1$

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7. C A sinusoidal function has an amplitude of 0.75 units, a period of 8π and a maximum at $(0,-3)$. Which of the following ***is not*** a possible equation of the function?

$f(0)=\frac{3}{4}\cos 0-\frac{15}{4}=-3$

(a) $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}x\right)-\frac{15}{4}$ **(b)** $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}(x-8\pi)\right)-\frac{15}{4}$ $f(0)=\frac{3}{4}\cos(-2\pi)-\frac{15}{4}=-3$

(c) $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}x\right)-\frac{15}{4}$ **(d)** $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}(x+2\pi)\right)-\frac{15}{4}$ $f(0)=\frac{3}{4}\sin\frac{\pi}{2}-\frac{15}{4}=-3$

$f(0)=-\frac{15}{4}$

8. C Let $f(x)=\sin x$ and $g(x)=A\sin(\omega(x-p))+d$. Knowing that the period of *f* is 2π, we can deduce that the period of *g* must be $\frac{2\pi}{\omega}$. Why is this true?

~~**(a)**~~ This information is found in Mr. Nolfi's notes as well as the textbook. Everyone knows that neither source can ever be wrong. Textbooks and teachers are right about everything!

~~**(b)**~~ It is true because period is calculated by dividing 2π by ω.

(c) To obtain the graph of *g*, the graph of *f* must be stretched or compressed horizontally by the factor $1/\omega$, which means that the period of *f* is also stretched or compressed horizontally by the same factor. ✓

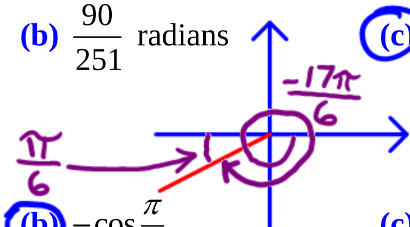
~~**(d)**~~ To obtain the graph of *g*, the graph of *f* must be stretched or compressed horizontally by the factor ω, which means that the period of *f* is also stretched or compressed horizontally by the same factor.

9. c $502°$ is equal to

$502\left(\frac{\pi}{180}\right)=\frac{502\pi}{180}=\frac{251\pi}{90}$

(a) $\frac{90\pi}{251}$ radians **(b)** $\frac{90}{251}$ radians **(c)** $\frac{251\pi}{90}$ radians **(d)** $\frac{251}{90}$ radians

10. b $\cos\frac{-17\pi}{6}$ is equal to



(a) $\cos\frac{\pi}{6}$ **(b)** $-\cos\frac{\pi}{6}$ **(c)** $\cos\frac{\pi}{3}$ **(d)** $-\cos\frac{\pi}{3}$

11. C Consider two ***coterminal*** angles *x* and *y*, and their ***principal*** angle θ. If $-4\pi<x<-2\pi$, $2\pi<y<4\pi$, $\boxed{\cos\theta=-\frac{1}{2} \text{ and } \sin\theta=\frac{\sqrt{3}}{2}}$, then

$\theta=\frac{2\pi}{3}$

(a) $x=\frac{-7\pi}{3}$, $y=\frac{11\pi}{3}$ **(b)** $x=\frac{-8\pi}{3}$, $y=\frac{10\pi}{3}$ **(c)** $x=\frac{-10\pi}{3}$, $y=\frac{8\pi}{3}$ **(d)** $x=\frac{-11\pi}{3}$, $y=\frac{7\pi}{3}$

Part 2: Written Responses

12. For ***negative two marks***, evaluate $\sin\frac{-13\pi}{3}$. For ***three communication marks***, explain why both the tangent and cotangent of any angle in quadrant III must be positive. (Use a diagram to illustrate your answer.) For ***one bonus mark***, write ***one sentence*** that describes the purpose of mathematical modelling. **(3 COM)**

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14. Prove that the equation $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{\cos\theta} - \tan\theta$ is an identity. **(6 APP)**

$$\begin{aligned}
R.S. &= \frac{1}{\cos\theta} - \tan\theta \\
&= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \quad \text{(quotient identity)} \\
&= \frac{1-\sin\theta}{\cos\theta} \\
&= \frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta\,(1+\sin\theta)} \\
&= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} \\
&= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} \quad \text{(Pythagorean identity)} \\
&= \frac{\cos\theta}{1+\sin\theta} \\
&= L.S.
\end{aligned}$$

Since L.S. = R.S., the given equation is an identity.

15. Prove that the equation $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 4\tan x \sec x$ is an identity. **(8 APP)**

$$\begin{aligned}
L.S. &= \frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)} - \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)} \quad \text{(express with common denominator)} \\
&= \frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{1-\sin^2 x} \\
&= \frac{4\sin x}{\cos^2 x} \quad \text{(Pythagorean identity)} \\
&= \left(\frac{4}{1}\right)\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right) \\
&= 4\tan x \sec x \\
&= R.S.
\end{aligned}$$

Since L.S. = R.S., the given equation is an identity.

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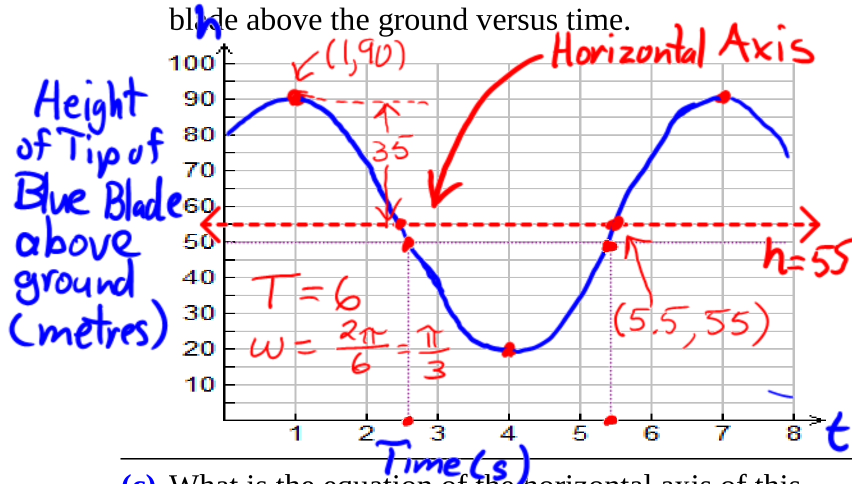
16. A certain type of wind turbine has three blades, one that is red, another that is green and yet another that is blue. An observer notices that at $t = 1$ s, the tip of the blue blade is 90 m above the ground. Then, over a period of 6 seconds, the tip of the blue blade moves from 90 m above the ground down to 20 m above the ground and back up to 90 m. **(16 TIPS)**

(a) Sketch the graph of height of the tip of the blue blade above the ground versus time.



(b) Write ***two different equations***, one using "cos" and the other using "sin," of a sinusoidal function that models the height of the tip of the blue blade above the ground versus time.

$h(t) = 35\cos\left(\frac{\pi}{3}(t-1)\right) + 55$

$h(t) = 35\sin\left(\frac{\pi}{3}(t-5.5)\right) + 55$

$h(t) = -35\sin\left(\frac{\pi}{3}(t-2.5)\right) + 55$

$h(t) = 35\sin\left(\frac{\pi}{3}(t+0.5)\right) + 55$

$\boxed{h(t) = -35\cos\left(\frac{\pi}{3}(t-4)\right) + 55}$

(c) What is the equation of the horizontal axis of this sinusoidal function? What does the horizontal axis represent in this context?

$h = 55$

The average height above the ground of the tip of the blue blade is 55 m.

(d) How high above the ground is the tip of the blue blade after 4 seconds?

$h(4) = -35\cos\left(\frac{\pi}{3}(4-4)\right) + 55$

$= -35\cos 0 + 55$

$= -35(1) + 55$

$= 20$ (agrees with graph)

At 4s, the tip of the blue blade is 20 m above the ground.

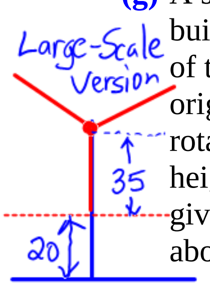
(e) At approximately what time(s) during the 6-second period is the tip of the blue blade 50 m above the ground?

This happens at approximately 2.6 s and 5.4 s. (estimated from graph)

(f) What is the length of one of the blades? Explain.

The diameter of the spinning part of the turbine is 70 m. Therefore, the length of one blade must be 35 m.

(g) A small 5m-diameter version of the wind turbine is built for demonstration purposes. The small version of the wind turbine is designed to ***scale*** with the original, but rotates more rapidly, completing a full rotation in 2 s. Write an equation to model the height above the ground of the tip of the blue blade given that at $t = 1$ s, the tip is at its maximum height above the ground.

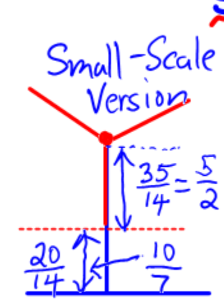


Scale: $70:5 = 14:1$, $T = 2 \rightarrow \omega = \pi$

$\therefore$ blade length $= \frac{35}{14} = \frac{5}{2} = A$

average height above ground $= \frac{55}{14} = d$

$h(t) = \frac{5}{2}\cos(\pi(t-1)) + \frac{55}{14}$



(h) Would it be safe for a 2-m tall individual to stand underneath the spinning blades of the ***small*** wind turbine? Explain.

Min height above ground of the tip of any blade is $\frac{10}{7}$ m (see diagram bottom left corner). Since this is smaller than 2 m, a 2-m tall person standing underneath the spinning blades would probably be killed!

(Alternate method of finding min height: $h_{min} = \frac{5}{2}(-1) + \frac{55}{14} = \frac{55}{14} - \frac{35}{14} = \frac{20}{14} = \frac{10}{7}$)

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