MCR 3U9 — **Semester 2, 2015 - 2016**

Grade 11 AP Mathematics
Unit 3 – Major Test – Polynomial Functions

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Victim: Mr. Solutions

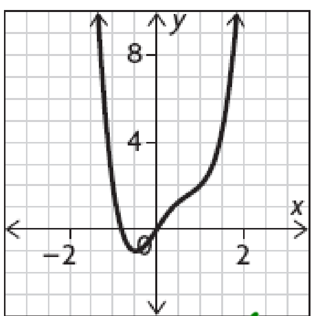
Another inspiring piece of work!!!

KU	APP	TIPS	COM
26/26	30/30	23/23	10/10

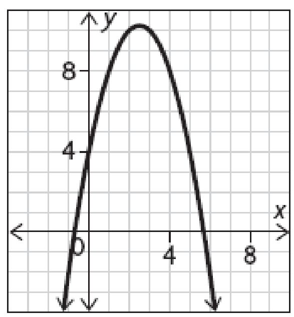
1. Use end behaviours, turning points and zeros to match each graph to the most likely polynomial equation. **(4 KU)**

(a) $y = x\left(2x^3 - 3x^2 + 3\right)$ (b) $y = x(x-1)(2x-1) - 2$ (c) $y = -x^4 + x^3 + x^2 - 2x + 7$ (d) $y = -x^2 + 6x + 5$

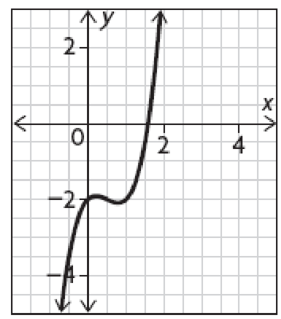
(e) $y = -x^2 + 5x + 4$ (f) $y = x^3 - x^2 + x - 2$ (g) $y = -7/40(4x+5)(5x-8)\left(x^2+1\right)$

A


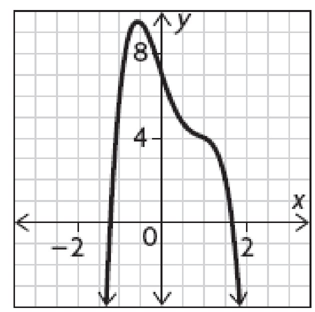
Equation: a ✓

B


Equation: e ✓

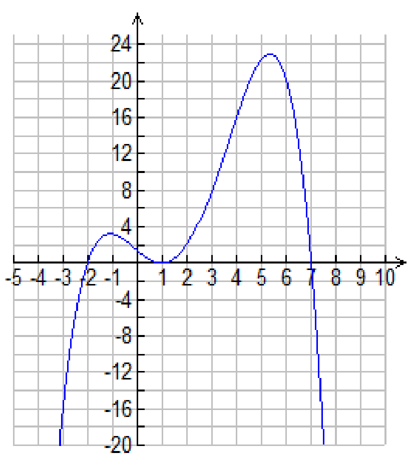
C


Equation: b ✓

D


Equation: c ✓

2. Given below is the graph of the polynomial function $p(x)$. Determine each of the following. **(12 KU)**



(a) ***End Behaviours***

As $x \to \infty$, $y \to$ $-\infty$ ✓

As $x \to -\infty$, $y \to$ $-\infty$ ✓

(b) ***Number of Turning Points*** (Mark the turning points on the graph)

3 ✓

(c) ***Zeros and Multiplicities***

Zero	Multiplicity
−2	1 ✓
1	2 ✓
7	1 ✓

(d) ***Intervals of Increase***

$(-\infty, -1.1)$ ✓
$(1, 5.3)$ ✓
approximately

(e) ***Intervals of Decrease***

$(-1.1, 1)$ ✓
$(5.3, \infty)$ ✓
approximately

(f) ***Possible Equation of p(x)***

$y = (x+2)(x-7)(x-1)^2$ ✓✓

3. Given the polynomial function $q(x) = -3x^5 - 9x^4 + 4x^2 + 7x - 13$, determine each of the following. **(10 KU)**

(a) ***End Behaviours***

As $x \to \infty$, $y \to$ $-\infty$ ✓

As $x \to -\infty$, $y \to$ ∞ ✓

(b) ***Number of Possible…***

Zeros: 1, 2, 3, 4 or 5 ✓

Turning Points: 0, 2, 4 ✓

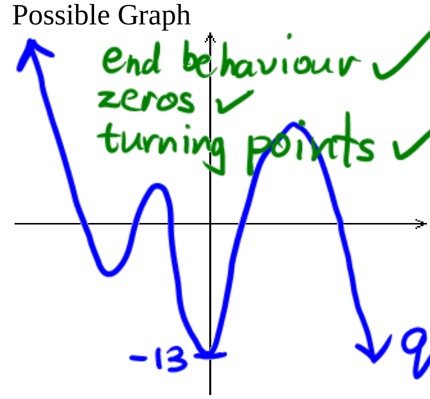
(e) ***The y-intercept of q(x)***

−13 ✓

(c) ***Absolute Max, Min or Neither? Why?***

Since q is a polynomial of odd degree, it cannot have absolute extreme points. ✓

(d) Possible Graph



end behaviour ✓
zeros ✓
turning points ✓

KU	APP	TIPS	COM
−0	−0	−0	−0

$\nearrow = -\frac{3}{2}[2(x+1)]^3 - 5 = -\frac{3}{2}(8)(x+1)^3 - 5 = -12(x+1)^3 - 5$

4. Sketch the graph of $g(x) = -\frac{3}{2}(2x+2)^3 - 5$ by applying transformations to the function $f(x) = x^3$. **(9 APP)**

(a) State the transformations required to obtain g from the base/parent/mother function $f(x) = x^3$.

Horizontal	*Vertical*
1. No horizontal stretch or compression ✓	1. Stretch vertically by a factor of −12 ✓
2. Translate one unit to the left.	2. Translate 5 units down

(b) Express the transformation in ***mapping notation***.

$(x, y) \to (x-1, -12y-5)$ ✓ ✓

(c) Apply the transformation to a few key points on the graph of the base function $f(x) = x^3$

Pre-image Point on $y = f(x)$	*Image Point on $y = g(x)$*
$(-2,-8) \longrightarrow$	$(-3, 91)$
$(-1,-1) \longrightarrow$	$(-2, 7)$
$(0,0) \longrightarrow$	$(-1,-5)$
$(1,1) \longrightarrow$	$(0,-17)$
$(2,8) \longrightarrow$	$(1,-101)$

✓✓

(d) Now sketch the graph of $g(x)$.

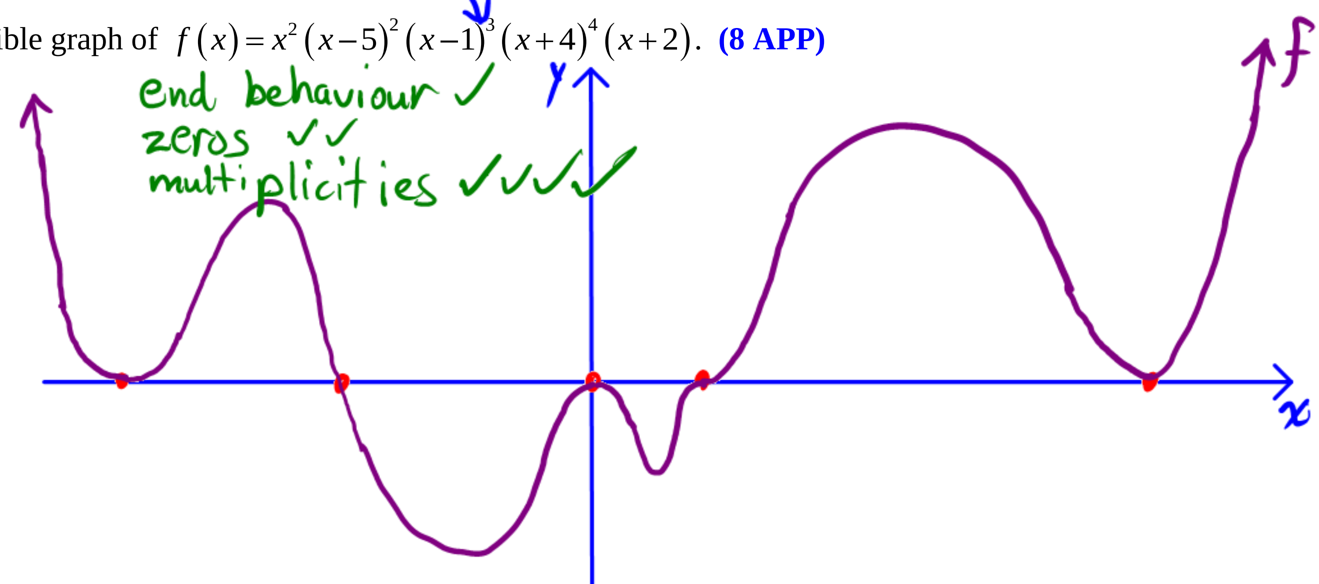
Rough Work

5. Sketch a possible graph of $f(x) = x^2(x-5)^2(x-1)^3(x+4)^4(x+2)$. **(8 APP)**

Zero	Mult.
−4	4
−2	1
0	2
1	3
5	2

Degree
$= 4+1+2+3+2$
$= 12$ ✓

end behaviour ✓
zeros ✓✓
multiplicities ✓✓✓✓



KU	APP	TIPS	COM
− 0	− 0	− 0	− 0

6. Consider the sixth-degree polynomial function $p(x) = -x^6 + 10x^4 - 9x^2$.

(a) ***Fully*** factor the polynomial. **(3 APP)**

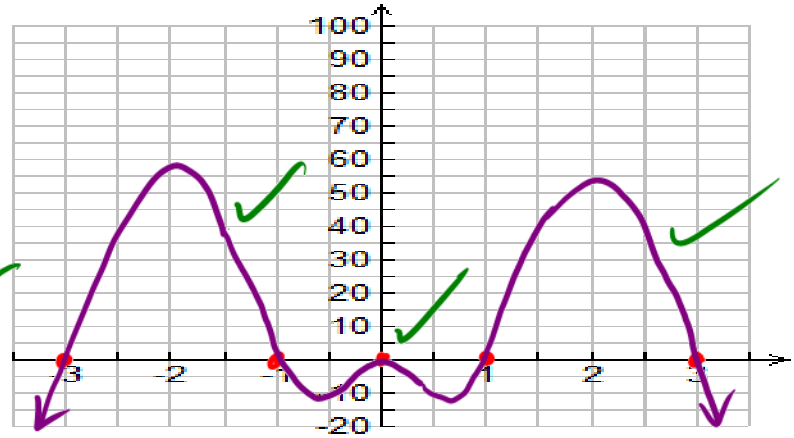
$p(x) = -x^6 + 10x^4 - 9x^2$

$= -x^2(x^4 + 10x^2 - 9)$

$= -x^2(x^2-1)(x^2-9)$

$= -x^2(x-1)(x+1)(x-3)(x+3)$

(b) Use the factored form of the polynomial to sketch the graph of $y = p(x)$. **(3 APP)**



(c) Use the factored form to solve the equation $-x^6 + 10x^4 - 9x^2 = 0$. **(3 APP)**

$-x^2(x-1)(x+1)(x-3)(x+3) = 0$

$\therefore x = 0$ or $x - 1 = 0$ or $x + 1 = 0$ or $x - 3 = 0$ or $x + 3 = 0$

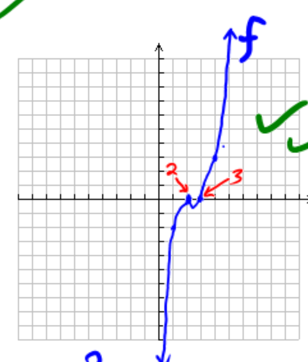
$\therefore x = 0, 1, -1, 3, -3$

(d) Use the factored form and the graph to solve the inequality $-x^6 + 10x^4 - 9x^2 \ge 0$. State the solution set using both set notation and interval notation. **(4 APP)**

$-x^2(x-1)(x+1)(x-3)(x+3) \ge 0$

From the graph it's clear that $p(x) \ge 0$ if $-3 \le x \le -1$, $x = 0$ or $1 \le x \le 3$. Therefore, the solution set is $\{x \in \mathbb{R} \mid -3 \le x \le -1, x = 0, 1 \le x \le 3\}$ or $[-3,-1] \cup \{0\} \cup [1,3]$

7. Solve the polynomial equation $x^3 - 7x^2 + 16x = 12$. Include a graph that clearly shows the solutions of the equation. **(10 TIPS)**



$x^3 - 7x^2 + 16x - 12 = 0$

Let $f(x) = x^3 - 7x^2 + 16x - 12$

Since $f(2) = 0$, $x - 2$ is a factor of $f(x)$

By long division, $f(x) = (x-2)(x^2 - 5x + 6)$

$$\begin{array}{r} x^2 - 5x + 6 \\ x - 2 \overline{) x^3 - 7x^2 + 16x - 12} \\ \underline{x^3 - 2x^2 } \\ -5x^2 + 16x \\ \underline{-5x^2 + 10x } \\ 6x - 12 \\ \underline{6x - 12} \\ 0 \end{array}$$

$\therefore (x-2)(x^2 - 5x + 6) = 0$

$\therefore (x-2)(x-2)(x-3) = 0$

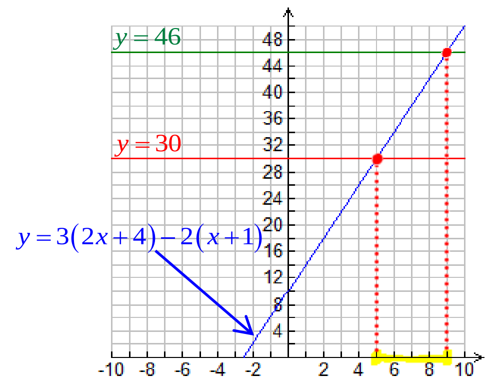
$\therefore (x-2)^2(x-3) = 0$

$\therefore (x-2)^2 = 0$ or $x - 3 = 0$

$\therefore x = 2$ or $x = 3$

KU	APP	TIPS	COM
–0	–0	–0	–0

8. Write an inequality that corresponds to the diagram given at the right. In addition, state the solution set using both set-builder notation and interval notation. (Do not solve the inequality. You should be able to see the solution just by looking at the graphs.) **(5 TIPS)**



$30 \le 3(2x+4) \le 46$

From the graph, it's clear that

$5 \le x \le 9$

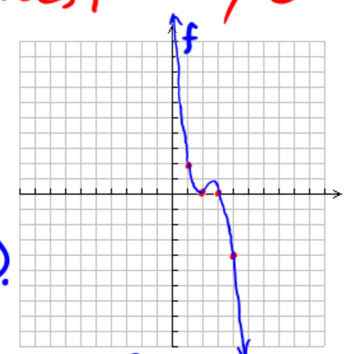
Solution Set: $\{x \in \mathbb{R} \mid 5 \le x \le 9\}$ or $[5, 9]$

odd degree, possibly 3

9. The polynomial function $f(x) = -x^n + kx^2 - (2k+2)x + 12$ has two turning points, no global extreme points and can be divided exactly by $x-3$. Determine the values of n and k as well as any zeros of f. Then sketch the graph of $y = f(x)$.

odd degree

Hint: There is no way to ***calculate*** the value of n. The best approach is to ***choose*** the simplest possible value of n. **(8 TIPS)**



Since $f(x)$ can be divided exactly by $x-3$, $f(3)=0$

$\therefore -3^n + k(3^2) - (2k+2)(3) + 12 = 0$

$\therefore -3^n + 9k - 6k - 6 + 12 = 0$

$\therefore 3k + 6 - 3^n = 0$

$$\begin{array}{r} -x^2 + 4x - 4 \\ x-3 \,\big)\, \overline{-x^3 + 7x^2 - 16x + 12} \\ \underline{-x^3 + 3x^2 } \\ 4x^2 - 16x \\ \underline{4x^2 - 12x } \\ -4x + 12 \\ \underline{-4x + 12} \\ 0 \end{array}$$

Since the polynomial has no global extreme points and an even number of turning points, the degree of f must be odd. Therefore, a good candidate for the value of n is 3. If $n=3$,

$3k + 6 - 3^3 = 0$.

$\therefore 3k - 21 = 0$

$\therefore k = 7$

$\therefore f(x) = -x^3 + 9x^2 - 16x + 12$

$= -(x-3)(x^2 - 4x + 4)$ by long division

$= -(x-3)(x-2)^2$

$\therefore$ zeros are $x = 3$ and $x = 2$

KU	APP	TIPS	COM
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