

M. Solutions

Well done M. S.!

KU	APP	TIPS	COM
21/20	20/20	12/12	10/10

Part 1: Modified True/False (6 KU)

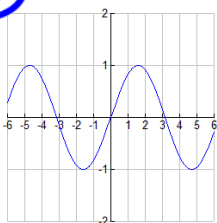
State whether each statement is **true** or **false**. If false, **change** the **underlined part** to make the statement true.1. T/F **F** $f(2+7) = f(2) + f(7)$ for all functions f . Change: $f(9)$ ✓2. T/F **F** If $g(3) = -1$ then $(-1, 3)$ lies of the graph of g . Change: $(3, -1)$ ✓3. T/F **F** All functions are relations and all relations are functions. Change: but not all ✓4. T/F **F** The function $h(t) = -4.9t^2 + 14t + 2$ describes the height, in metres, of a ball thrown vertically upward, t seconds after it was thrown. Then $h(0)$ means the height of the ball when it hits the ground. Change: the height at time 0 ✓5. T/F **F** The symbol $f(u)$ is read " f of u ." Change: " f of u " ✓

Part 2: Problems

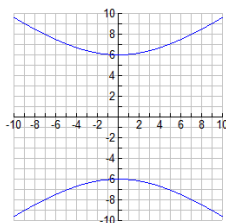
6. Circle the relations that are functions. (5 KU)

One mark deducted for each incorrect identification

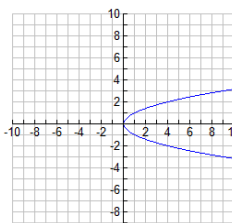
(a)



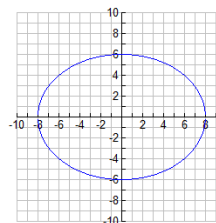
(b)



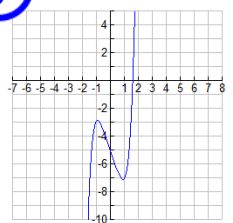
(c)



(d)



(e)



(f) $x^2 - y^2 = 0$

(g) $\{(1,1), (1,2), (1,3), (1,5)\}$

(h)

$\{(0,2), (1,2), (2,2)\}$

(i) $|y| = 16 - x^2$

(j)

$y = x^3 + 2x^2 - 3x$

7. State the domain and range of $f(x) = \frac{x^3 + 5x^2 - 66x}{x+11}$. (2 KU)Domain = $\{x \in \mathbb{R} \mid x \neq -11\}$ ✓
See page 3 for detailed analysis of #7Range = $\{y \in \mathbb{R} \mid y \geq -9\}$ ✓
(bonus +1, KU reduced to 20)

8. A kayak-rental company charges \$40 per kayak rental and averages 400 rentals per day. According to marketing studies of the kayak-rental industry, for every \$5 increase in price, a typical company can expect to lose 10 rentals per day. How much should the company charge to maximize revenue? (10 APP)

Let x represent the number of \$5 price increases and let $R(x)$ represent the revenue obtained when x \$5 price increases have been made.

Revenue = (# kayaks rented)(price per rental)

$\therefore R(x) = (400 - 10x)(40 + 5x)$ ✓

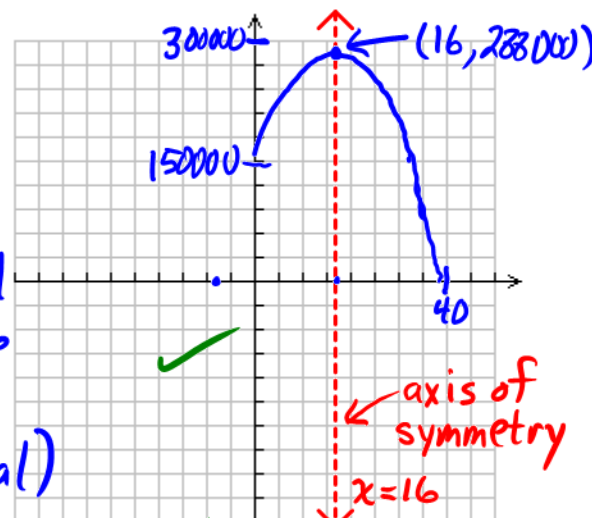
$\therefore R(x) = 10(40 - x)(5)(8 + x)$

$\therefore R(x) = 50(40 - x)(8 + x)$

$\therefore$ the zeros of R are -8 and 40 ✓

Since R is a quadratic function,

its axis of symmetry has equation $x = \frac{-8+40}{2} = 16$ ✓

The max revenue is found at the vertex, which lies on the axis of symmetry. Therefore, to maximize the company should charge $40 + 5(16) = \$120$ per rental. ✓

KU	APP	TIPS	COM
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9. Consider the function f defined by the equation $f(x) = |x - 10|$. The function g is obtained by performing the following transformations to f :

Horizontal Transformations	Vertical Transformations
1. Reflect in the y-axis	1. Reflect in the x-axis
2. Stretch by a factor of 2	2. Stretch by a factor of 3
3. Translate 8 units to the right	3. Translate 15 units up

(a) Write the transformation using mapping notation. (4 KU)

$(x, y) \rightarrow (-2x + 8, -3y + 15)$

(b) Write the transformation using function notation. (4 KU)

$g(x) = -3f(-\frac{1}{2}(x - 8)) + 15$

(c) Apply the transformation to five key points on f . (5 APP)

Pre-Image	Image
$(10, 0)$	$(-12, 15)$
$(14, 4)$	$(-20, 3)$
$(20, 10)$	$(-32, -15)$
$(6, 4)$	$(-4, 3)$
$(0, 10)$	$(8, -15)$

(d) On the given grid, sketch the graphs of both f and g . (5 APP)

(e) Find the equation of g by using your answer to (b) as well as your graph. (4 TIPS)

$g(x) = -3f(-\frac{1}{2}(x - 8)) + 15$

$= -3|-\frac{1}{2}(x - 8) - 10| + 15$

$= -3|-\frac{1}{2}x + 4 - 10| + 15$

$= -3|-\frac{1}{2}x - 6| + 15$

$= -3|-\frac{1}{2}(x + 12)| + 15$

$= -3|-\frac{1}{2}|x + 12| + 15$

$= -\frac{3}{2}|x + 12| + 15$

optional! Relies on knowing $|xy| = |x||y|$

agrees with graph

10. The graph of the function f is shown at the right. A function g is defined as follows:

$g(x) = 3f(\sqrt{x})$ (8 TIPS)

$\sqrt{x}$ must be squared to obtain x

(a) Express the transformation from f to g using mapping notation.

$(x, y) \rightarrow (x^2, 3y)$

(b) Using the grid at the right, sketch the graph of g .

Hint: Use points on f whose x-co-ordinates range from 1 to 5. Estimate the y-co-ordinates of these points.

pre-image (on f)	image (on g)
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$(1, 0.9)$ $(1, 2.7)$

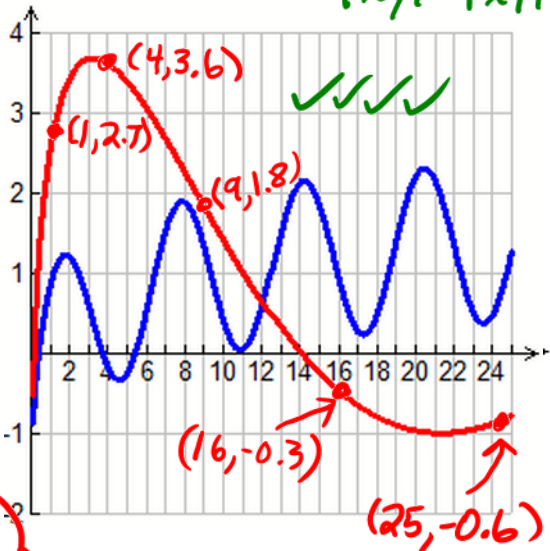
$(2, 1.2)$ $(4, 3.6)$

$(3, 0.6)$ $(9, 1.8)$

$(4, -0.1)$ $(16, -0.3)$

$(5, -0.2)$ $(25, -0.6)$

y-co-ordinates are approximate



KU	APP	TIPS	COM
- 0	- 0	- 0	- 0

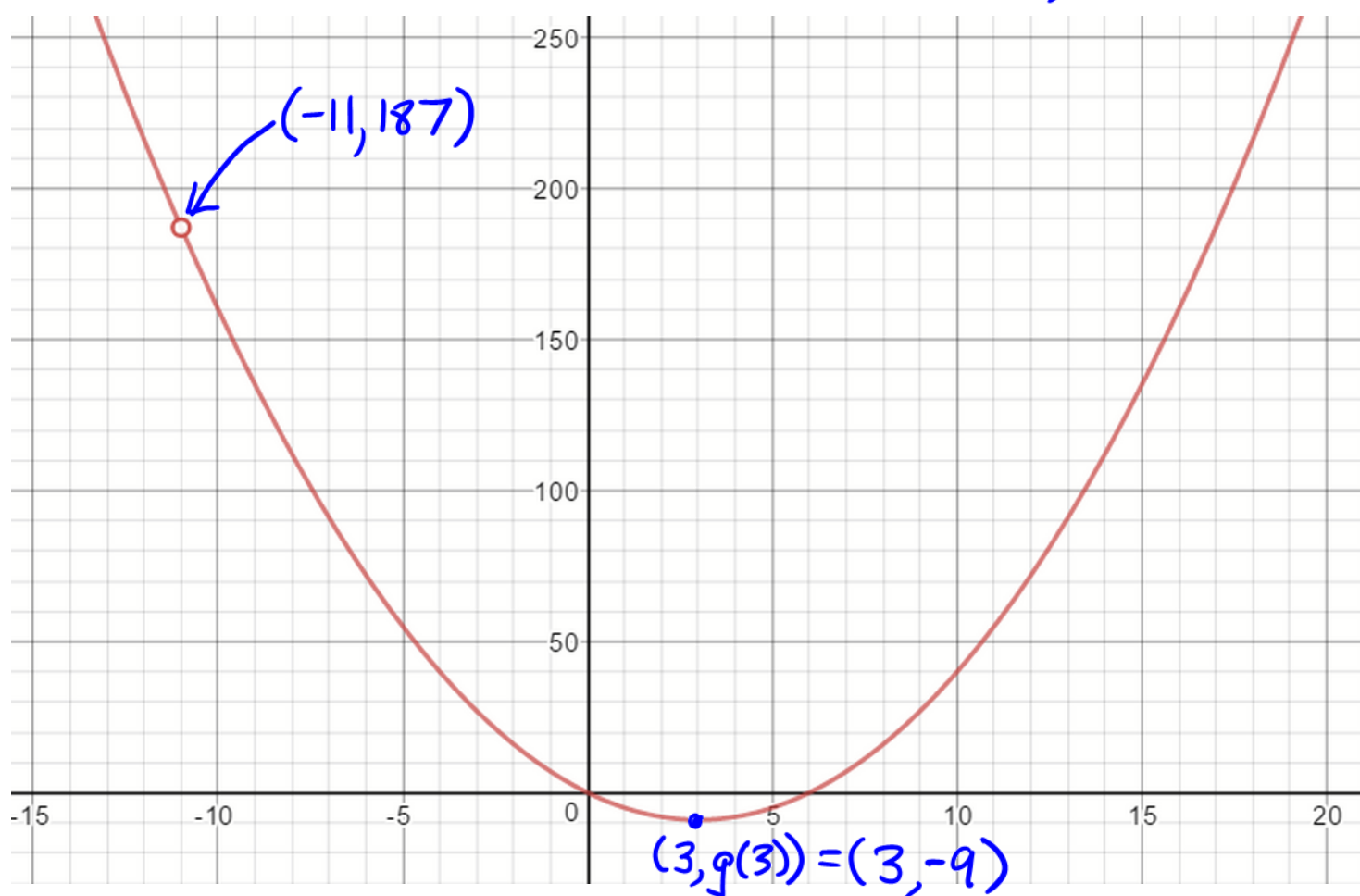
$$f(x) = \frac{x^3 + 5x^2 - 66x}{x+11}$$

$$= \frac{x(x^2 + 5x - 66)}{x+11}$$

$$= \frac{x(\cancel{x+11})(x-6)}{\cancel{x+11}}$$

$$= x(x-6), x \neq -11$$

Therefore, the graph of f is a parabola with a hole at $x = -11$. If we let $g(x) = x(x-6)$, $g(-11) = 187$. Thus, the co-ordinates of the hole are $(-11, 187)$.



$\therefore$ domain of $f = \{x \in \mathbb{R} \mid x \neq -11\}$ and the range of $f = \{y \in \mathbb{R} \mid y \geq -9 \text{ and } y \neq 187\}$