MCR3U9 **Semester 1, 2016 - 2017**

Grade 11 Pre-AP Functions

Major Test – Unit 1 – Introduction to Functions

Mr. N. Nolfi

Victim: Mr. Solutions

Inspiring work Mr. S.!!

KU	APP	TIPS	COM
28/28	20/20	18/18	10/10

1. The graph of the function f is given below. **(10 KU)**

(a) Evaluate each of the following:

$f(-4)$ is undefined ✓

$f(0) =$ 1 ✓

$f(2) =$ 0 ✓

$f(4) =$ 2 ✓

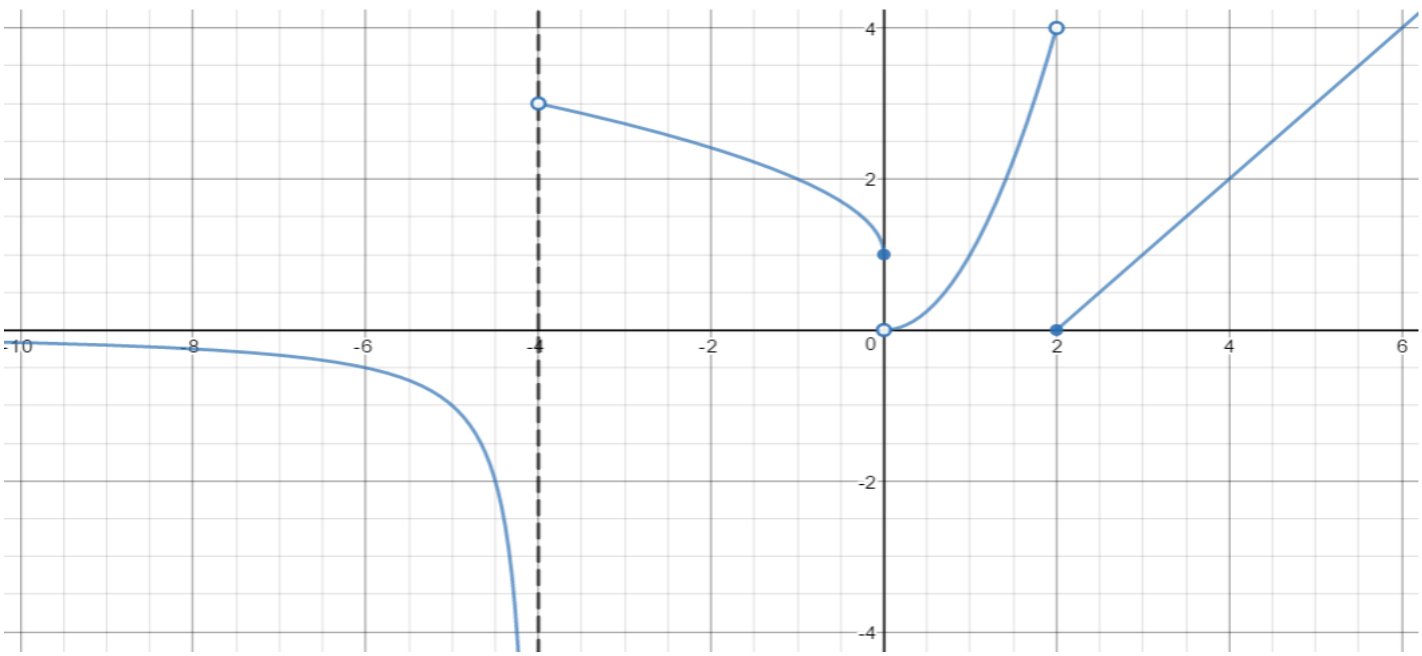
(b) State the equation of each asymptote of f.

$x = -4$ ✓

$y = 0$ ✓

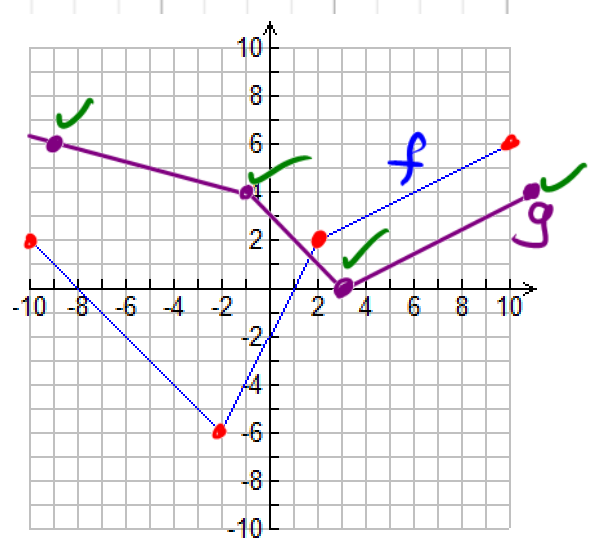
(c) Write an equation of f.

$$f(x) = \begin{cases} \dfrac{1}{x+4}, & x < -4 \\ \sqrt{-x} + 1, & -4 < x \le 0 \\ x^2, & 0 < x < 2 \\ x - 2, & x \ge 2 \end{cases}$$



2. The graph of f is shown at the right. The function g is defined by the equation $g(x) = \frac{1}{2} f(-(x-1)) + 3$. Using the same grid, sketch the graph of g. **(8 KU)**

$(x, y) \rightarrow (-x+1, \frac{1}{2}y + 3)$



Pre-image	Image
(-10, 2)	(11, 4)
(-2, -6)	(3, 0)
(2, 2)	(-1, 4)
(10, 6)	(-9, 6)

3. The function $f(x) = 2\sqrt{x}$ has undergone the following transformations:

Horizontal Transformations	***Vertical Transformations***
1. Reflect in the *y*-axis	1. Reflect in the *x*-axis
2. Compress by a factor of $\frac{1}{2}$	2. Stretch by a factor of 4
3. Translate 3 units to the right	3. Translate 5 units down

(a) Write the transformation using mapping notation. **(2 KU)**

$(x, y) \to \left(-\frac{1}{2}x+3,\ -4y-5\right)$

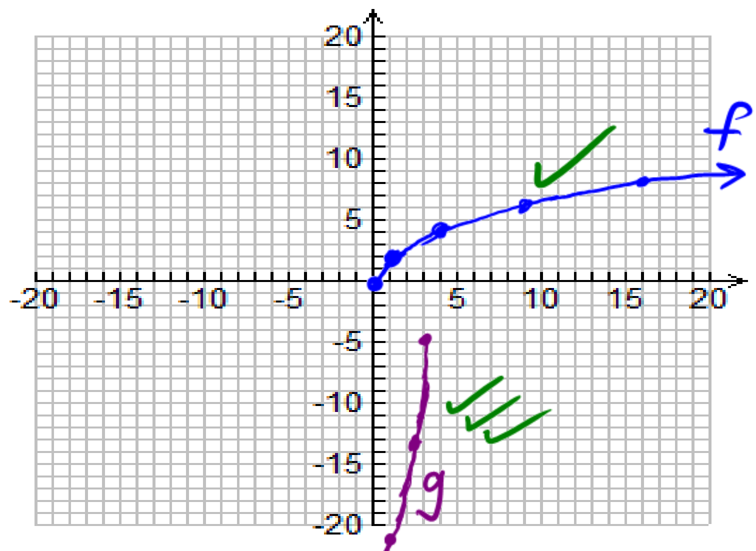
(b) Write the transformation using function notation. **(2 KU)**

$g(x) = -4f(-2(x-3)) - 5$

(c) Apply the transformation to four key points on *f*. **(2 KU)**

Pre-Image	Image
$(0,0)$	$(3,-5)$
$(1,2)$	$\left(\frac{5}{2}, -13\right)$
$(4,4)$	$(1, -21)$
$(9,6)$	$\left(-\frac{3}{2}, -29\right)$

(d) On the given grid, sketch the graphs of both *f* and *g*. **(4 KU)**



4. A farmer has 120 m of fencing to enclose a rectangular pig pen and divide it into three sections. Find the dimensions of the pen that maximize its area. **(10 APP)**

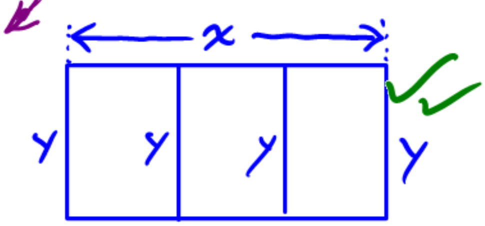
Let A represent the area of the pen

Then $A = xy$

$= (60-2y)y$

$= 2(30-y)y$

$= 2y(30-y)$



Length of fence $= 120$

$\therefore 2x+4y = 120$

$\therefore 2(x+2y) = 120$

$\therefore x+2y = 60$

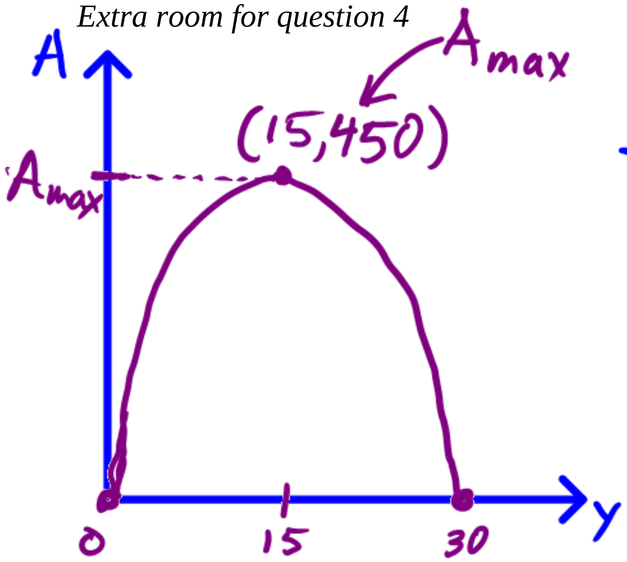
$\therefore x = 60-2y$

$\therefore$ the area of the pen as a function of y is

$A(y) = 2y(30-y)$

(see next page $\longrightarrow$)

Extra room for question 4



The zeros of the quadratic function A are 0 and 30.
Therefore, the area is maximized when $y = \frac{0+30}{2} = 15$.
$\therefore x = 60 - 2(15) = 30$.
$\therefore$ the area is maximized when the pen has dimensions 15 m × 30 m

5. The student council is organizing the annual white-water-rafting trip. In the past, the price per student was set to \$220 and an average of 30 students chose to go on the trip. This year, the student council wants to decrease the price. Based on a student survey, they estimate that for every \$3 decrease in the price, 2 more students would choose to go on the trip. How much should the student council charge to maximize revenue? **(10 APP)**

Let x represent the number of \$3 price decreases and $R(x)$ represent the revenue obtained for x \$3 price decreases.

revenue = (price per student)(# students)

$$\therefore R(x) = (220 - 3x)(30 + 2x)$$

The zeros of R are -15 and $\frac{220}{3}$.

Therefore, the axis of symmetry has equation

$$x = \frac{-15 + \frac{220}{3}}{2} = \frac{\frac{175}{3}}{2} = \frac{175}{6}$$

Since the vertex of the quadratic function $R(x)$ lies on the axis of symmetry, the maximum revenue is obtained if $x = \frac{175}{6}$.

Therefore, the student council should charge

$$220 - \cancel{3}^{1}\left(\frac{175}{\cancel{6}_{2}}\right) \text{ dollars} = \$132.50.$$

6. A one-to-one function f has domain $\{x \in R | x \geq -4\}$ and range $\{y \in R | y < -1\}$. Determine the domain and range of the function g defined by the equation $g(x) = 3f^{-1}(2x+2)+4$. **(8 TIPS)**

f^{-1} must have domain $\{x \in \mathbb{R} | x < -1\}$
and range $\{y \in \mathbb{R} | y \geq -4\}$

$g(x) = 3f^{-1}(2x+2)+4 = 3f^{-1}(2(x+1))+4$

In mapping notation this can be expressed as

Because $\frac{1}{2}x-1$ is an increasing function

$$(x,y) \xrightarrow{f^{-1}} \left(\tfrac{1}{2}x-1, \overset{g}{3y+4}\right)$$

$\therefore$ upper limit of domain of g is $\frac{1}{2}(-1)-1 = -\frac{3}{2}$
and the lower limit of the range of g is $3(-4)+4 = -8$

Because $3y+4$ is an increasing function.

$\therefore$ domain of g is $\{x \in \mathbb{R} | x < -\frac{3}{2}\}$ and range of g is $\{y \in \mathbb{R} | y \geq -8\}$

7. Let $f(x) = -\frac{1}{x}$. **(10 TIPS)**

(a) Prove that $f(x) = f^{-1}(x)$.

Apply transformation
$(x,y) \to (y,x)$
$x = f(y)$
$\therefore x = -\frac{1}{y}$
$\therefore y = -\frac{1}{x}$

$\therefore f^{-1}(x) = -\frac{1}{x}$
$\therefore f^{-1}(x) = f(x)$

(b) Let the function g be defined by the equation $g(x) = af(bx)$, where a and b represent any ***non-zero*** real numbers. Prove that $g(x) = g^{-1}(x)$.

$g(x) = af(bx)$
$= a\left(-\frac{1}{bx}\right)$
$= -\frac{a}{bx}$

To find $g^{-1}(x)$, apply
$(x,y) \to (y,x)$:

$x = g(y)$
$\therefore x = -\frac{a}{by}$
$\therefore y = -\frac{a}{bx}$
$\therefore g^{-1}(x) = -\frac{a}{bx}$
$\therefore g^{-1}(x) = g(x)$.