MCR 3U9 **Semester 1, 2016 - 2017**

Grade 11 Pre-AP Functions

Unit 3 – Mid-unit Test (Radian Measure, Trig Ratios, Transformations, Modelling)

Mr. N. Nolfi

Victim: Mr. Solutions   Well done Mr. S.!!

KU	APP	TIPS	COM
17/17	17/17	18/18	10/10

- Unless otherwise noted, ***radian measure must be used***. COM marks will be deducted for using degree measure.
- Exact values must be given for trigonometric ratios of all special angles and all angles related to them.
- Up to 10 COM marks may be deducted for poor mathematical form, inappropriate use of terminology, etc.

Part 1: Modified True/False (6 KU)

✓ = $\frac{1}{2}$ mark

$\frac{23\pi}{3} = \frac{18\pi}{3} + \frac{5\pi}{3} = 6\pi + \frac{5\pi}{3}$ $3(2\pi)$ 3 full revs.   principal angle

State whether each statement is ***true*** or ***false***. If false, ***change*** the ***underlined part*** to make the statement true.

1. T/F T ✓ $\frac{\pi}{4}$ and $\underline{-\frac{7\pi}{4}}$ are coterminal angles.   **Change:** ——— ✓

2. T/F F ✓ The principal angle of $\frac{23\pi}{3}$ is $\underline{-\frac{\pi}{3}}$.   **Change:** $\frac{5\pi}{3}$ ✓

3. T/F F ✓ The related first quadrant angle of $\frac{21\pi}{8}$ is $\underline{\frac{\pi}{8}}$.   **Change:** $\frac{3\pi}{8}$ ✓

4. T/F F ✓ If the domain of the sine function is restricted to $\underline{\{x \in \mathbb{R} : 0 \le x \le \pi\}}$, then $\sin^{-1}$ is defined.   **Change:** $\{x \in \mathbb{R} \mid -\frac{\pi}{2} \le x \le \frac{\pi}{2}\}$ ✓

5. T/F F ✓ $477° \doteq \underline{\cancel{8.325}}$ radians.   **Change:** 8.325 ✓

6. T/F F ✓ $\tan\frac{5\pi}{6} = \underline{\sqrt{3}}$ **Change:** $-\frac{1}{\sqrt{3}}$ ✓

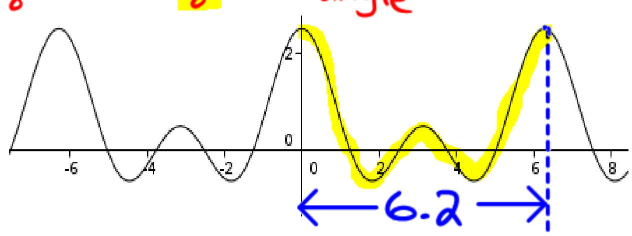
$\frac{23\pi}{3}$ $\frac{5\pi}{3}$ $\frac{3\pi}{8}$ $\frac{21\pi}{8}$

$\frac{21\pi}{8} = \frac{16\pi}{8} + \frac{3\pi}{8} = 2\pi + \frac{5\pi}{8}$ ← principal angle $\pi - \frac{5\pi}{8} = \frac{3\pi}{8}$

Part 2: Multiple Choice (6 KU)

Identify the choice that ***best*** answers the question.

7. b ✓ The graph of a periodic function is shown at the right. What is the approximate ***period*** of the function?



(a) 3.1   **(b)** 6.2   **(c)** 12.4   **(d)** 2.5

8. c ✓ Which of the following ***is most unlikely*** to produce a periodic graph?

(a) Little Sudhanshu's height above the floor as he jumps up and down in a playpen.

(b) The height above the floor of Ashutosh's (naughty former student) mother's hand as she "disciplines" him.

(c) The height above the ground of Arnavi's airplane as it gradually descends toward a runway for a landing.

(d) Sakshi's height above the ground as she rides an extremely fast Ferris wheel.

9. d ✓ Which of the following is an example of the power of trigonometry?

(a) Trigonometric ratios depend only the angles in a right triangle, not on the size of the triangle.

(b) It relates angles to side lengths.

(c) Angles are far easier to measure directly than distances.

(d) All the above.

KU	APP	TIPS	COM
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10. d Let $f(x)=\tan x$ and $g(x)=A\tan(\omega(x-p))+d$. Knowing that the period of f is π, we can deduce that the period of g must be $\dfrac{\pi}{\omega}$. Why is this true?

B.S.
~~(a)~~ This information is found in Mr. Nolfi's notes as well as the textbook. It therefore MUST be true!
~~(b)~~ It is true because period is calculated by dividing π by ω.
~~(c)~~ To obtain the graph of g, the graph of f must be stretched or compressed horizontally by the factor ω, which means that the period of f is also stretched or compressed horizontally by the same factor. ✗
(d) Same as (c) but the stretch or compression factor is ω^{-1}. ✓

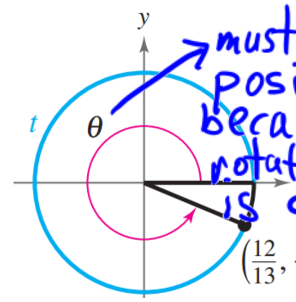
11. C For any angle of rotation θ in the third quadrant, $\cos\theta<0$ and $\sec\theta<0$. Why is this the case?

~~(a)~~ For any point (x, y) in quadrant III, $x > 0$ and $r < 0$. ✗
~~(b)~~ ASTC
(c) For any point (x, y) in quadrant III, $x < 0$ and $r > 0$.
~~(d)~~ CAST

B.S.

12. An angle of rotation in a ***unit circle*** is shown. Which of the following is ***not true***?
c

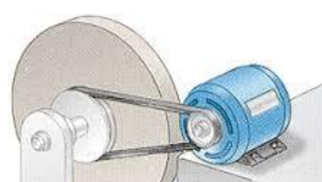
(a) $\tan\theta=-\dfrac{5}{12}$ ✓ (b) $\cos\theta=\dfrac{12}{13}$ ✓ (c) $\theta \doteq -0.39479$ ✓ (d) $\sin\theta=-\dfrac{5}{13}$ ✓



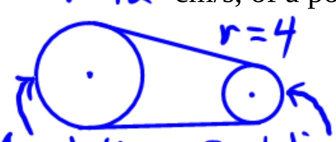
must be positive because rotation is ccw

Part 3: Written Responses

13. As shown in the diagram at the right, an electric motor is used to turn a grinding wheel. The pulley on the motor has a radius of 4 cm, the pulley on the grinding wheel has a radius of 12 cm and the motor spins at a rate of 3600 RPM.



(a) If the grinding wheel has a radius of 25 cm, calculate the linear velocity, in cm/s, of a point on the circumference of the wheel. **(5 KU)**



$12:4=3:1$

The motor pulley must make 3 complete rotations for the grinding wheel to rotate exactly once

$\therefore \omega_{\text{grinding wheel}} = 1200 \text{ RPM}$ ✓

$= (1200 \text{ RPM})(2\pi \text{ rad/rot})$ ✓

$= 2400\pi \text{ rad/min} = 40\pi \text{ rad/s}$ ✓

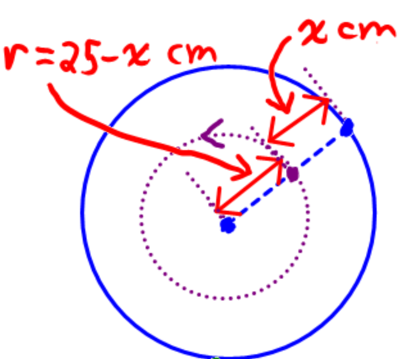
$\boxed{v=\frac{d}{t}=\frac{r\theta}{t}=r\left(\frac{\theta}{t}\right)=r\omega}$

$\therefore v = r\omega = (25 \text{ cm})(40\pi \text{ rad/s})$ ✓ $= 1000\pi \text{ cm/s}$ ✓

Check
$C=2\pi(25)=50\pi$
40π rad/s $=20$ rot/s
$50\pi(20)$ $=1000\pi$

(b) Still assuming that the grinding wheel has a radius of 25 cm, write an equation of a function for the linear velocity, in cm/s, of a point on the grinding wheel x cm from the circumference. **(5 APP)**

Let $v(x)$ represent the linear velocity of a point on the wheel, x cm from the circumference.



$\therefore v(x) = r\omega$

in cm ✓ from (a)

$= (25-x)(40\pi \text{ rad/s})$ ✓

$= 40\pi(25-x) \text{ cm/s}$ ✓

✓✓

KU	APP	TIPS	COM
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14. Suppose that $g(x) = -2\csc\left(\frac{1}{4}(x+\pi/2)\right)$. **(12 APP)**

(a) State the transformations required to obtain g from the base/parent/mother function $f(x) = \csc x$. 3

Horizontal	*Vertical*
1. Stretch horizontally by a factor of 4 ✓	**1.** Stretch vertically by a factor of -2 ✓ (includes reflection
2. Translate $\frac{\pi}{2}$ units left ✓	**2.** in x-axis)

(b) Express the transformation in ***mapping notation***.

$(x,y) \to \left(4x - \frac{\pi}{2}, -2y\right)$ 2

(c) Apply the transformation to a few key points on the graph of the base function $f(x) = \csc x$ 2

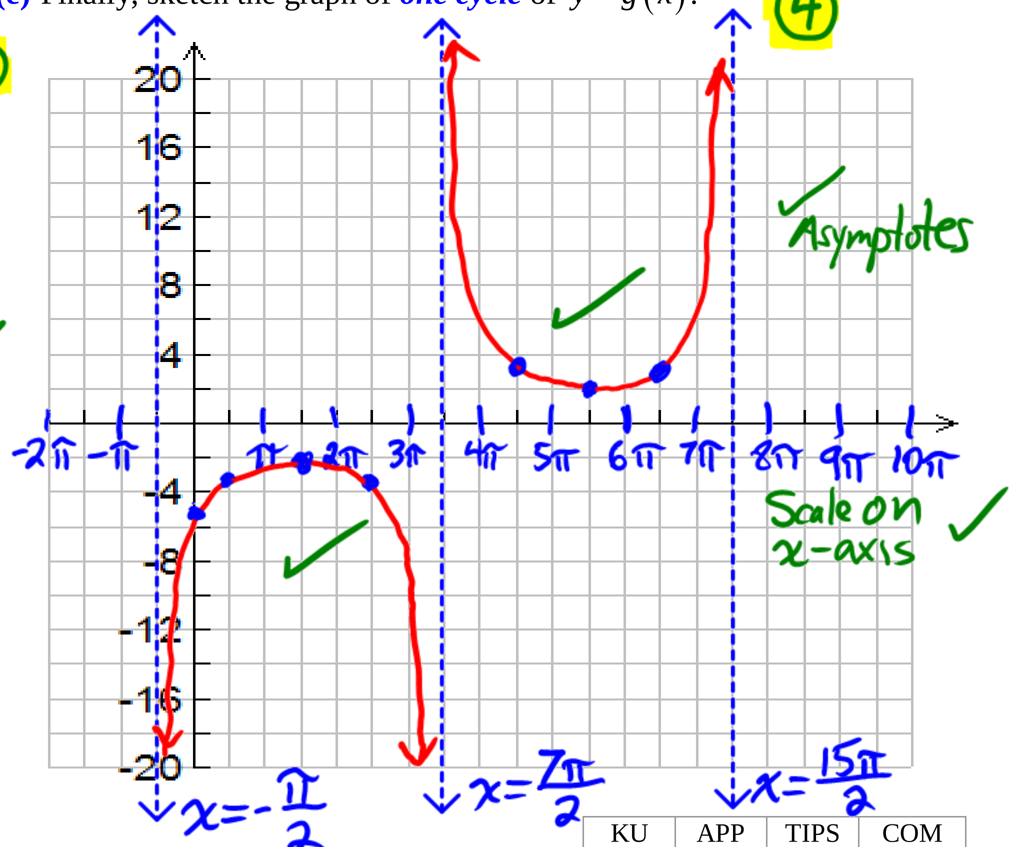
Pre-image Point on $y = f(x)$	*Image Point on* $y = g(x)$
$\left(\frac{\pi}{4}, \sqrt{2}\right)$	$\left(\frac{\pi}{2}, -2\sqrt{2}\right)$
$\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{3\pi}{2}, -2\right)$
$\left(\frac{3\pi}{4}, \sqrt{2}\right)$	$\left(\frac{5\pi}{2}, -2\sqrt{2}\right)$ ✓
$\left(\frac{5\pi}{4}, -\sqrt{2}\right)$	$\left(\frac{9\pi}{2}, 2\sqrt{2}\right)$
$\left(\frac{3\pi}{2}, -1\right)$	$\left(\frac{11\pi}{2}, 2\right)$
$\left(\frac{7\pi}{4}, -\sqrt{2}\right)$	$\left(\frac{13\pi}{2}, 2\sqrt{2}\right)$

(d) Apply the transformation to the given asymptotes of $f(x) = \csc x$. 1

Pre-image Asymptote of $y = f(x)$	*Image Asymptote of* $y = g(x)$
$x = 0$	$x = -\frac{\pi}{2}$
$x = \pi$	$x = \frac{7\pi}{2}$
$x = 2\pi$	$x = \frac{15\pi}{2}$

✓

(e) Finally, sketch the graph of ***one cycle*** of $y = g(x)$. 4



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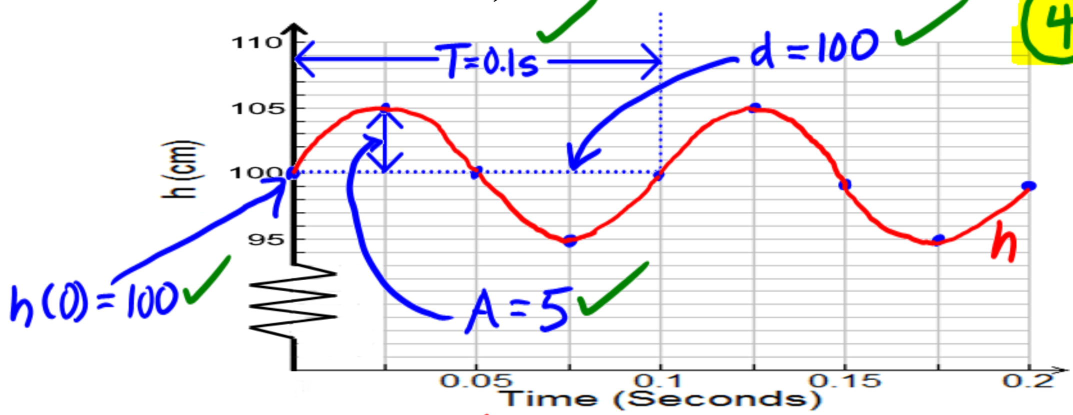
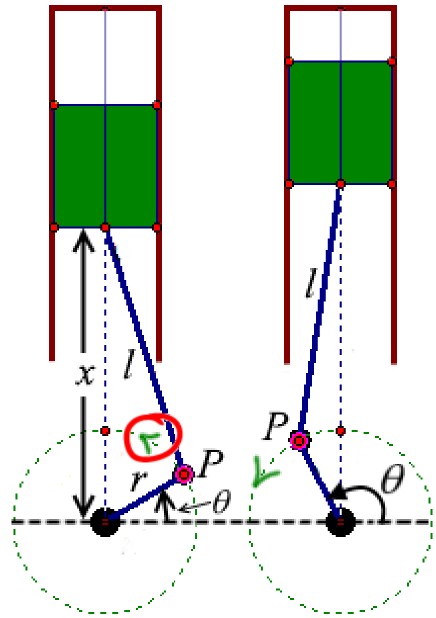
15. A piston is connected to a crankshaft by means of a connecting rod. As it moves up and down in a cylinder, it causes the crankshaft to rotate. Let l represent the length of the connecting rod, r represent the turning radius of the crankshaft, x represent the distance from the centre of rotation of the crankshaft to the bottom of the piston and θ represent the angle of rotation through which the crankshaft rotates. **(18 TIPS)**

(a) Suppose that a crankshaft spins at a rate of 600 RPM and the turning radius of the crankshaft is 5 cm. Let h represent the distance above the ground of the point P. If the centre of rotation of the crankshaft is 1 m above the ground, sketch two cycles of the graph of h, in cm, versus time, in seconds. Assume that at time $t = 0$ s, $h = 100$ cm.



4



sin For cos, $p = -0.025$

(b) Write ***two different equations***, one using "cos" and the other using "sin," of a function that models the value of h, in cm, versus time, in seconds. $T = 0.1 \rightarrow \omega = \frac{2\pi}{T} = 20\pi$ (Check: Horizontal comp. by factor $\frac{1}{20\pi}$)

4

$A = 5$, $p = 0$, $d = 100$ Let $h(t)$ represent height of point P above ground

$h(t) = 5\sin(20\pi t) + 100$, $h(t) = 5\cos(20\pi(t - 0.025)) + 100$

(c) What would happen to the graph above if Victor (former student) came along and revved up the engine to 3600 RPM?

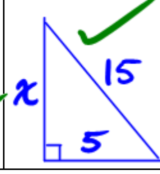
3600 RPM = 6(600 RPM)

2

$\therefore$ there are 6 cycles in 0.1s instead of 1

$\therefore$ there would be a horizontal compression by a factor of $\frac{1}{6}$

(d) If the connecting rod has a length of 15 cm, what is the value of x when θ is any integer multiple of π? (i.e. 0, π, 2π, 3π,…) When $\theta = k\pi$, $k \in \mathbb{Z}$, the angle between the crankshaft arm and the vertical is $\frac{\pi}{2}$.



$\therefore x^2 = 15^2 - 5^2$ (Pyth. Thm.)

$\therefore x = \sqrt{200}$

$= 10\sqrt{2}$

3

(e) Does x change sinusoidally over time? Explain. Assume $r = 5$, $l = 15$

$x_{min} = 10$, $x_{max} = 20$, $x_{avg} = 15$

θ (rad)	t (s)	x (cm)
0	0	$10\sqrt{2}$
$\frac{\pi}{2}$	0.025	20
π	0.05	$10\sqrt{2}$
$\frac{3\pi}{2}$	0.075	10
2π	0.1	$10\sqrt{2}$

5

If x changed sinusoidally, then for all values of θ that are multiples of π, or equivalently, all values of t that are multiples of 0.05 s, $x = \frac{x_{min} + x_{max}}{2} = \frac{10+20}{2} = 15$. But this is not the case! At such values of θ or t, $x = 10\sqrt{2} \doteq 14.1$.

Therefore, x DOES NOT change sinusoidally over time.

KU	APP	TIPS	COM
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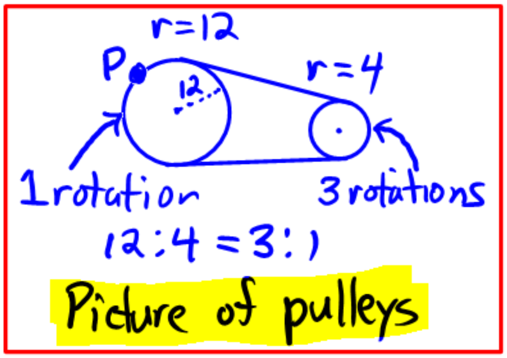
Alternative Solution to #13



$r_{\text{grinding wheel}} = 25 \text{ cm}$

$\omega_{\text{grinding wheel}} = \dfrac{3600 \text{ RPM}}{3}$

$= 1200 \text{ RPM}$

$= \dfrac{1200}{60} \text{ rot/s}$

$= 20 \text{ rot/s}$

(a) Linear velocity, in cm/s, of a point P on circumference of grinding wheel

$$= \frac{d \leftarrow \text{cm}}{t \leftarrow \text{s}}$$

$$= \frac{\text{distance travelled in cm}}{1 \text{ s}}$$

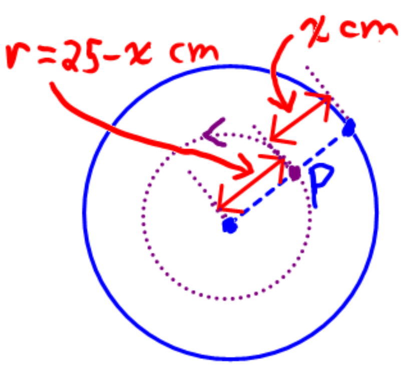
$$= (\text{dist travelled in one rotation})(\#\text{ rot/s})$$

$$= C_{\text{grinding wheel}} (\omega \text{ in rot/s})$$

$$= 2\pi(25 \text{ cm})(20 \text{ rot/s})$$

$$= 1000\pi \text{ cm/s}$$

(b) Same as (a) but $r = 25 - x$



$$\therefore\ v(x) = 2\pi\underbrace{(25-x)}_{\text{cm}}\underbrace{(20)}_{\text{rot/s}}$$

$$\therefore\ v(x) = 40\pi(25-x)$$