Grade 11 Pre-AP Mathematics

Trigonometric Identities, Solving Trigonometric Equations

Mr. N. Nolfi

Victim: Mr. Solutions What a difference intense preparation makes Mr. S.!

KU	APP	TIPS	COM
24/24	20/20	20/20	10/10

Modified True/False (5 KU)

State whether each statement is ***true*** or ***false***. If false, ***change*** the ***underlined part*** to make the statement true.

✓ = ½ mark

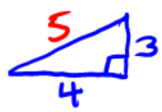
1. T/F F ✓ The equation $\cos 3x = 1$ has $\underline{6}$ solutions in $[0, 2\pi]$. **Change:** 4 ✓

2. T/F F ✓ $\sec 3x$ is equivalent to $\underline{\dfrac{1}{3\cos x}}$. **Change:** $\dfrac{1}{\cos 3x}$ ✓

3. T/F F ✓ The equation $\csc 3x = \dfrac{1}{2}$ has $\underline{6}$ solutions in $[0, 2\pi]$. **Change:** 0 ✓

4. T/F F ✓ If $\tan\theta = \dfrac{3}{4}$ then $\underline{\cos\theta = 4}$. **Change:** $\cos\theta = \frac{4}{5}$ or $\cos\theta = \frac{-4}{5}$ ✓

$3 \leftarrow y$, $4 \leftarrow x$



5. T/F F ✓ For $f(x) = \cos x$, $f(x+y) = \underline{f(x) + f(y)}$. **Change:** $f(x)f(y) - f(\frac{\pi}{2} - x)f(\frac{\pi}{2} - y)$ ✓

$\cos x \cos y - \cos(\frac{\pi}{2} - x)\cos(\frac{\pi}{2} - y)$

$\cos x \cos y - \sin x \sin y$

Problems

6. Use the three methods indicated below to demonstrate that the equation $\sin(\theta + \frac{3\pi}{2}) = -\cos\theta$ ***is*** an identity.

(a) Compound-angle identity **(3 KU)**

$\sin(\theta + \frac{3\pi}{2})$

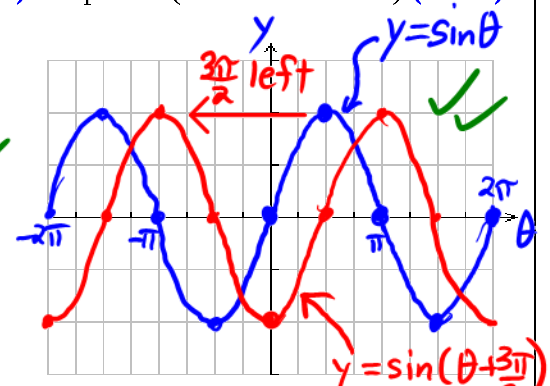
$= \sin\theta \cos\frac{3\pi}{2} + \cos\theta \sin\frac{3\pi}{2}$ ✓

$= \sin\theta(0) + \cos\theta(-1)$ ✓

$= -\cos\theta$ ✓

$\therefore \sin(\theta + \frac{3\pi}{2}) = -\cos\theta$ is an identity

(b) Graphical (Transformations) **(3 KU)**

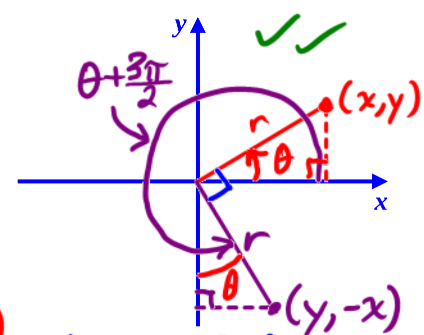


✓✓

When the graph of $y = \sin\theta$ is translated $\frac{3\pi}{2}$ to the left, the graph of $y = -\cos\theta$ is obtained ✓

$\therefore \sin(\theta + \frac{3\pi}{2}) = -\cos\theta$ is an identity

(c) Angles of Rotation **(3 KU)**



✓✓

$\sin(\theta + \frac{3\pi}{2})$

$= \dfrac{-x}{r}$

$= -\dfrac{x}{r}$ ✓

$= -\cos\theta$

$\therefore \sin(\theta + \frac{3\pi}{2}) = -\cos\theta$ is an identity

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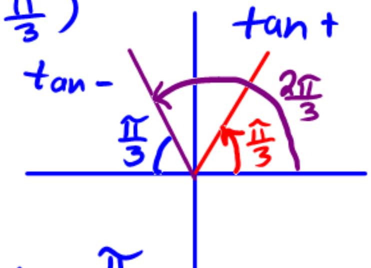
$\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$	$\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$	$\cot(x+y)=\dfrac{\cot x\cot y-1}{\cot x+\cot y}$	$\cot(x-y)=\dfrac{\cot x\cot y+1}{\cot y-\cot x}$

7. Use a counterexample to demonstrate that the equation $\tan(x+y)=\tan x+\tan y$ ***is not*** an identity. Choose your counterexample in such a way that a calculator is not required to evaluate any of the trig ratios.

Hint: x and y can represent the same value. **(5 KU)**

Let $x=y=\frac{\pi}{3}$. Then,

$$\begin{aligned} L.S. &= \tan\left(\tfrac{\pi}{3}+\tfrac{\pi}{3}\right)\\ &= \tan\tfrac{2\pi}{3}\\ &= -\tan\tfrac{\pi}{3}\\ &= -\sqrt{3}\end{aligned}$$



$$\begin{aligned} R.S. &= \tan\tfrac{\pi}{3}+\tan\tfrac{\pi}{3}\\ &= \sqrt{3}+\sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

$\therefore$ L.S. $\neq$ R.S.

$\therefore$ the equation is not an identity

8. Evaluate $\tan 195°$ without using a calculator. Exact values are required!

Hint: Use the identity for $\tan(x-y)$. **(5 KU)**

$$\begin{aligned} \tan 195° &= \tan(225°-30°)\\ &= \frac{\tan 225°-\tan 30°}{1+\tan 225°\tan 30°}\\ &= \frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}\end{aligned}$$

full marks for gettin to this point

$$= \frac{\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)}{\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)}$$

$$= \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}+1}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

9. Write $\tan 3\theta$ entirely in terms of $\tan\theta$. **(10 APP)**

$$\boxed{\begin{aligned}\tan 2\theta &= \tan(\theta+\theta)\\ &= \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}\\ &= \frac{2\tan\theta}{1-\tan^2\theta}\end{aligned}}$$

$$\begin{aligned} \tan 3\theta &= \tan(2\theta+\theta)\\ &= \frac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}\\ &= \frac{\frac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\tan\theta}\\ &= \frac{\left(\frac{2\tan\theta+\tan\theta(1-\tan^2\theta)}{1-\tan^2\theta}\right)}{\left(\frac{1-\tan^2\theta-2\tan^2\theta}{1-\tan^2\theta}\right)}\\ &= \left(\frac{2\tan\theta+\tan\theta-\tan^3\theta}{1-\tan^2\theta}\right)\left(\frac{1-\tan^2\theta}{1-3\tan^2\theta}\right)\\ &= \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\end{aligned}$$

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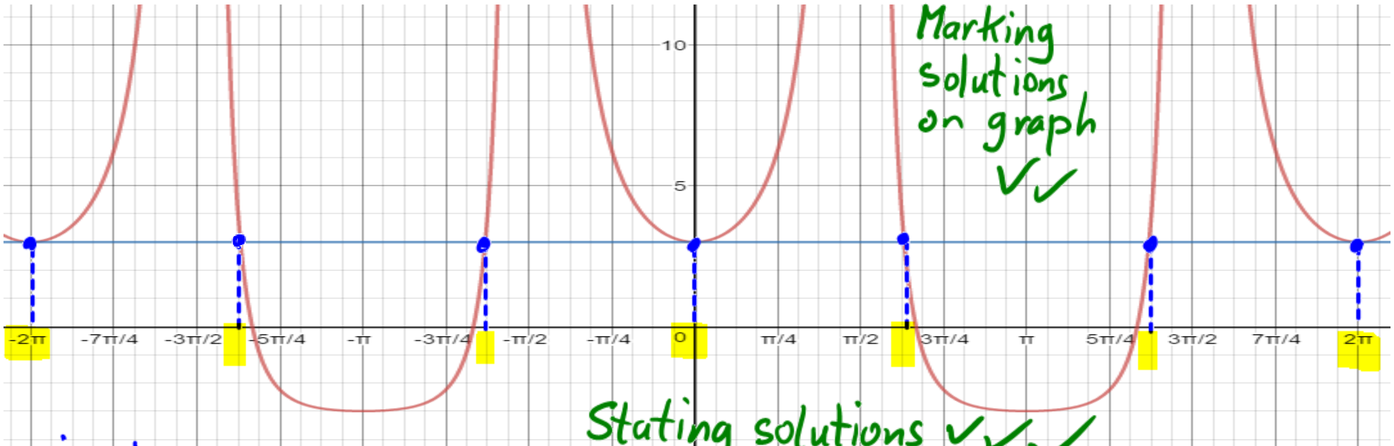
10. The following question deals with solving trigonometric equations both graphically and algebraically.

(a) Shown below are the graphs of $y=(\sec x+2)(2\sec x-1)$ and $y=3$. State ***approximate*** solutions to the equation $(\sec x+2)(2\sec x-1)=3$ for $x\in[-2\pi,2\pi]$. ***Mark*** the solutions on the graph. **(5 APP)**



Stating solutions ✓✓✓

Approximate Solutions: $-2\pi,\ -\frac{11\pi}{8}\doteq-4.3197,\ -\frac{5\pi}{8}\doteq-1.9635,\ 0,\ \frac{5\pi}{8}\doteq1.9635,\ \frac{11\pi}{8}\doteq4.3197,\ 2\pi$

(b) Use an algebraic method to solve the equation $(\sec x+2)(2\sec x-1)=3$, where $x\in[-\pi,\pi]$. Verify that your solutions agree with those that you obtained in (a). **(5 APP)**

$2\sec^2 x+3\sec x-2=3$ ✓

$\therefore 2\sec^2 x+3\sec x-5=0$

$\therefore (2\sec x+5)(\sec x-1)=0$ ✓

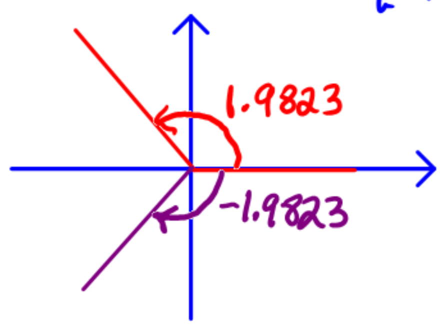
$\therefore 2\sec x+5=0$ or $\sec x-1=0$

$\therefore \sec x=-\frac{5}{2}$ or $\sec x=1$ ✓

$\therefore \cos x=-\frac{2}{5}$ or $\cos x=1$

$\therefore x\doteq1.9823$ or $x\doteq-1.9823$ ✓

or $x=0$ (3 solutions in $[-\pi,\pi]$)



11. Prove that the equation $\dfrac{\cos x}{1-\tan x}+\dfrac{\sin x}{1-\cot x}=\sin x+\cos x$ is an identity. **(7 TIPS)**

Proof:

$$L.S.=\frac{\cos x}{1-\frac{\sin x}{\cos x}}+\frac{\sin x}{1-\frac{\cos x}{\sin x}}$$ ✓

$$=\frac{\cos x}{\left(\frac{\cos x-\sin x}{\cos x}\right)}+\frac{\sin x}{\left(\frac{\sin x-\cos x}{\sin x}\right)}$$ ✓

$$=\left(\frac{\cos x}{1}\right)\left(\frac{\cos x}{\cos x-\sin x}\right)+\left(\frac{\sin x}{1}\right)\left(\frac{\sin x}{\sin x-\cos x}\right)$$

$$=\frac{\cos^2 x}{\cos x-\sin x}+\frac{\sin^2 x}{\sin x-\cos x}$$ ✓

$$=\frac{\cos^2 x}{\cos x-\sin x}+\frac{(-1)\sin^2 x}{\cos x-\sin x}$$ ✓

$$=\frac{\cos^2 x-\sin^2 x}{\cos x-\sin x}$$ ✓

$$=\frac{(\cos x-\sin x)(\cos x+\sin x)}{\cos x-\sin x}$$ ✓

$$=\cos x+\sin x=R.S.$$ ✓

$\therefore$ the given equation is an identity

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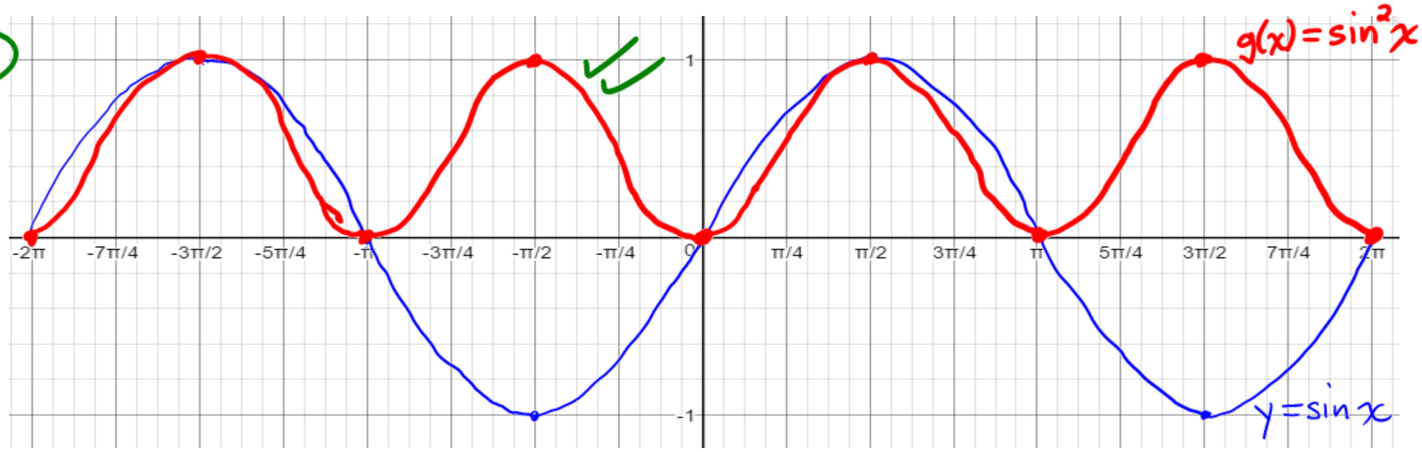
12. Consider the functions $f(x)=\dfrac{\tan^2 x}{\sec^2 x+\sin^2 x+\cos^2 x-1}$ and $g(x)=\sin^2 x$. **(13 TIPS)**

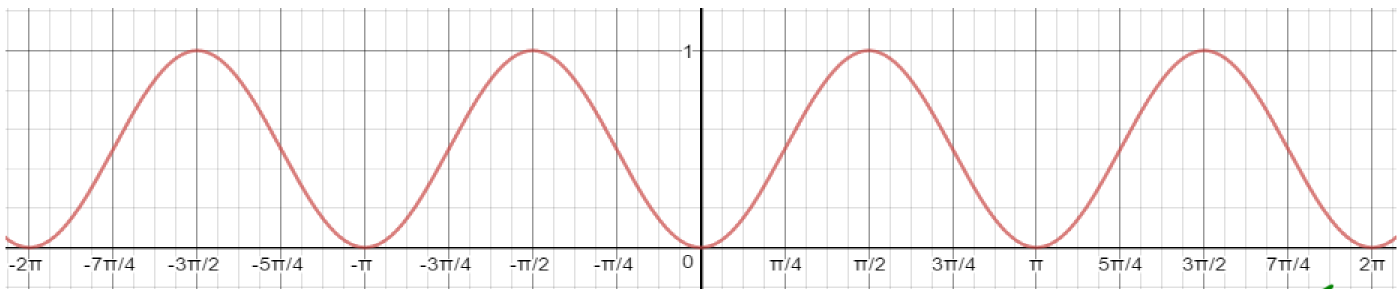
(a) Use the grid to sketch the graph of g. **Hint:** First sketch the graph of $y=\sin x$. Then "square" it.

2



(b) Shown below is the graph of f. What identity is suggested by this graph? Explain.

2



The given graph of f looks just like the graph of g in (a). This suggests that $\dfrac{\tan^2 x}{\sec^2 x+\sin^2 x+\cos^2 x-1}=\sin^2 x$ is an identity.

(c) Prove that the equation that you wrote in (b) is an identity.

3

$$L.S.=\frac{\tan^2 x}{\sec^2 x+1-1}$$
$$=\frac{\tan^2 x}{\sec^2 x}$$
$$=\left(\frac{\tan^2 x}{1}\right)\left(\frac{\cos^2 x}{1}\right)$$
$$=\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{\cos^2 x}{1}\right)$$
$$=\sin^2 x$$
$$=R.S.$$

∴ the equation in (b) is an identity.

(d) Finally, solve the equation $\dfrac{\tan^2 x}{\sec^2 x+\sin^2 x+\cos^2 x-1}=\cos^2 x$ for $0\le x\le 2\pi$.

3

Since the equation in (b) is an identity, this equation can be rewritten as $\sin^2 x=\cos^2 x$.

$\therefore \dfrac{\sin^2 x}{\cos^2 x}=1$

$\therefore \tan^2 x=1$

$\therefore \tan x=\pm 1$

$\therefore x=\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$

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