**MCR 3U9** **Semester 1, 2016 - 2017**

**Grade 11 Pre-AP Functions**
**Unit 3 – Mid-unit Test – Make-up for TIPS Question**

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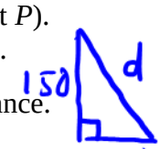
***Victim:*** Mr. Solutions Awe-inspiring work Mr. S.!!

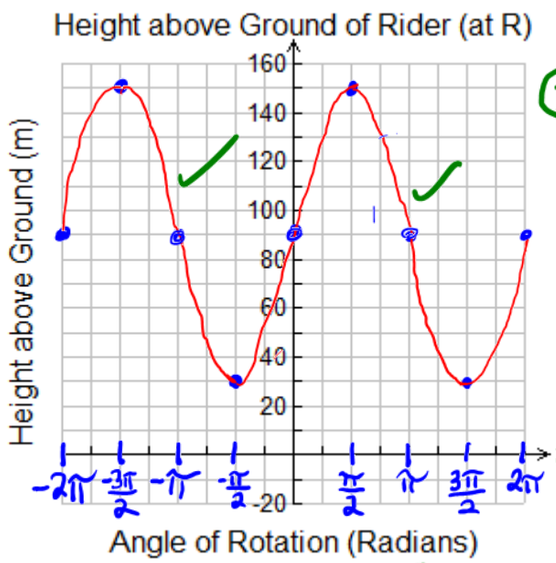
| TIPS |
|---|
| 18/18 |

**1.** The centre of a Ferris wheel is 90 m high and 120 m along the ground from the park entrance (at point *P*). As shown in the diagram below, the Ferris wheel has a radius of 60 m and it rotates counterclockwise.

A rider, who is located at point *R*, is $h(\theta)$ m above the ground and $d(\theta)$ m away from the park entrance.




**(a)** Sketch two cycles of the graph of *h*. Write two equations for *h*, one using "sin" and the other using "cos."

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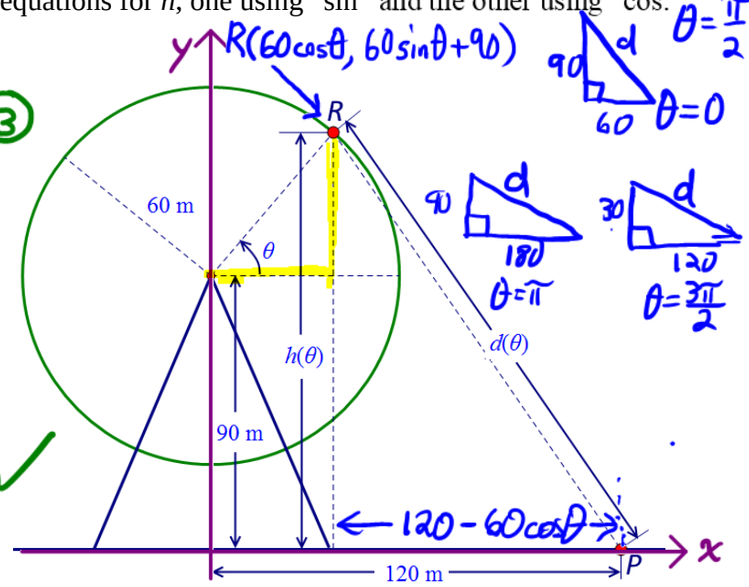








**Equations:**

1. $h(\theta) = 60\sin\theta + 90$

2. $h(\theta) = 60\cos\left(\theta - \frac{\pi}{2}\right) + 90$

3

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**(b)** Suppose that the Ferris wheel spins at a rate of 1 rad/min. Write an equation that describes the height of the rider as a function of time.

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$\omega = 1$ rad/min, $T = 2\pi$ min

$\therefore h(t) = 60\sin t + 90$, where $t$ represents time in minutes.

(or $h(t) = 60\sin\left(\frac{1}{60}t\right) + 90$, where $t$ is time in seconds)

**(c)** Would you expect *d* to change sinusoidally with $\theta$? Explain.

**Hint:** Consider what happens when $\theta = 0$, $\theta = \frac{\pi}{2}$, $\theta = \pi$ and $\theta = \frac{3\pi}{2}$.

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| $\theta$ | $d$ | |
|---|---|---|
| 0 | $\sqrt{60^2+90^2} \doteq 108.2$ | |
| | | +83.9 |
| $\frac{\pi}{2}$ | $\sqrt{150^2+120^2} \doteq 192.1$ | |
| | | +9.1 |
| $\pi$ | $\sqrt{90^2+180^2} \doteq 201.2$ | |
| | | −77.5 |
| $\frac{3\pi}{2}$ | $\sqrt{30^2+120^2} \doteq 123.7$ | |
| | | −15.5 |
| $2\pi$ | $\doteq 108.2$ | |

This does not have the symmetry of a sinusoidal function ∴ not sinusoidal

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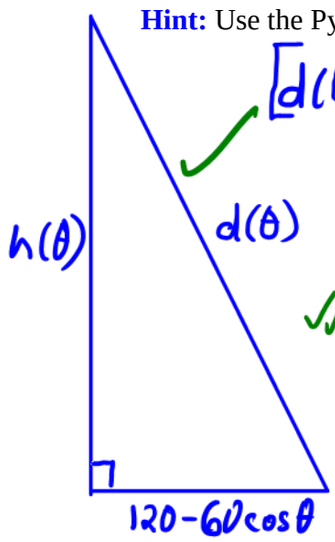
**(d)** Write a fully simplified expression for $d(\theta)$.

**Hint:** Use the Pythagorean Theorem and the Pythagorean identity.




$$[d(\theta)]^2 = (120 - 60\cos\theta)^2 + (60\sin\theta + 90)^2$$

$$= [60(2 - \cos\theta)]^2 + [30(2\sin\theta + 3)]^2$$

$$= 60^2(2 - \cos\theta)^2 + 30^2(2\sin\theta + 3)^2$$

$$= 3600(4 - 4\cos\theta + \cos^2\theta) + 900(4\sin^2\theta + 12\sin\theta + 9)$$

$$= 3600\cos^2\theta + 3600\sin^2\theta - 14400\cos\theta + 10800\sin\theta + 22500$$

$$= 10800\sin\theta - 14400\cos\theta + 3600(\cos^2\theta + \sin^2\theta) + 22500$$

$$= 10800\sin\theta - 14400\cos\theta + 26100$$

$$\therefore d(\theta) = \sqrt{10800\sin\theta - 14400\cos\theta + 26100}$$

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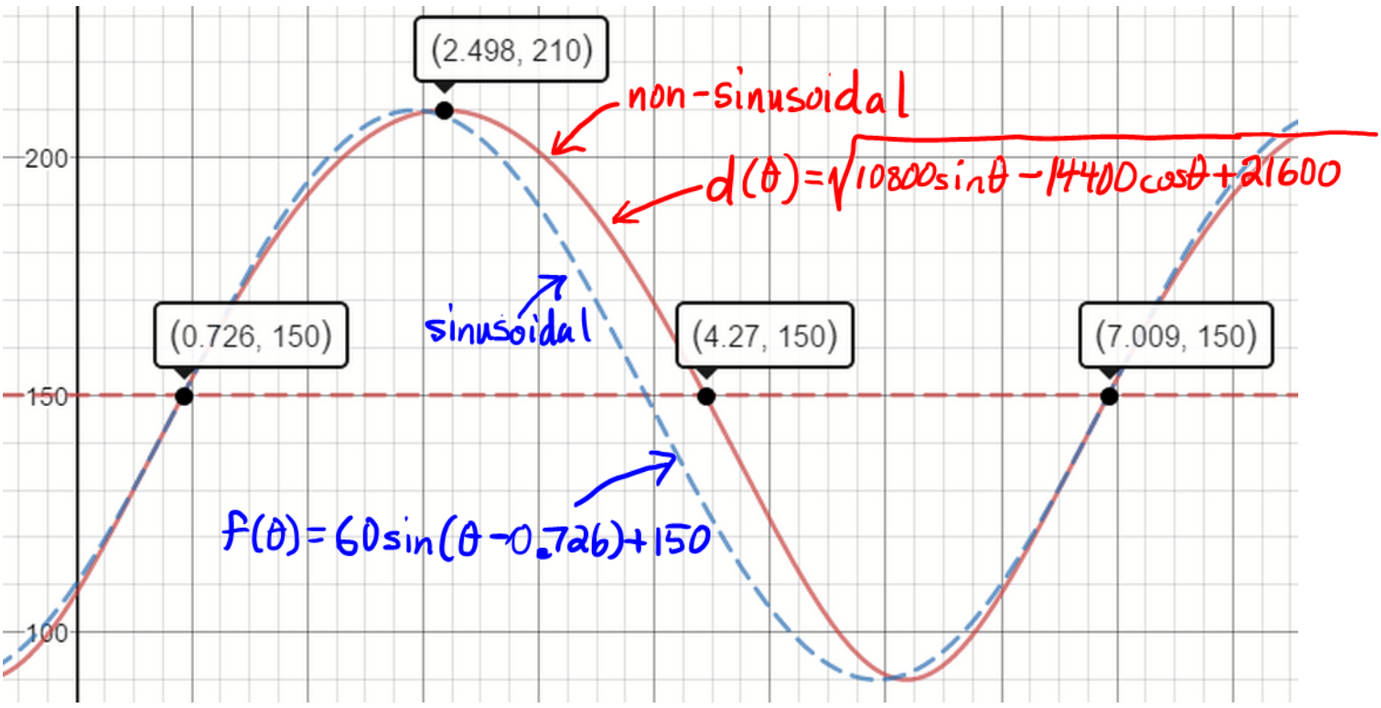
(2.498, 210)
non-sinusoidal
$d(\theta)=\sqrt{10800\sin\theta-14400\cos\theta+21600}$
200
(0.726, 150)
sinusoidal
(4.27, 150)
(7.009, 150)
150
$f(\theta)=60\sin(\theta-0.726)+150$
100