MCR 3U9	Semester 1, 2016 - 2017

Grade 11 AP Mathematics
Unit 4 – Major Test – Polynomial and Rational Functions

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Victim: Mr. Solutions

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/26	/30	/23	/10

1. Use end behaviours, turning points and zeros to match each graph to the most likely equation. **(6 KU)**

~~(a)~~ $y = x^4 + 5x^3 + 4x$ ~~(b)~~ $y = x^3 - x^2 + x - 2$ ~~(c)~~ $y = -x^4 + x^3 + x^2 - 2x + 7$ ~~(d)~~ $y = -x^2 + 6x + 5$ no parabolas

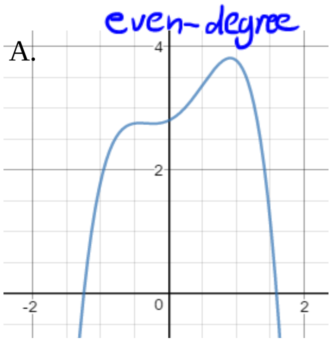
~~(e)~~ $y = x\left(x^3 - 2x^2 + 3\right)$ ~~(f)~~ $y = x(x-1)(2x-1) - 2$ ~~(g)~~ $y = -0.07(4x+5)(5x-8)\left(x^2+1\right)$

0 is a zero
-1 " " "

$5(-8)(1) = -40$

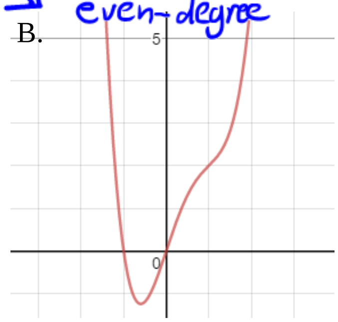
(h) $y = \dfrac{x-1}{x+1}$ **(i)** $y = \dfrac{x+1}{x-1}$ **(j)** $y = \dfrac{-x^2 + 4x - 5}{x-2}$ ~~(k)~~ $y = \dfrac{x-2}{-x^2 + 4x - 5}$

A. even-degree



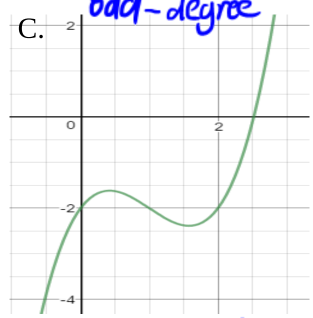
Equation: g

B. even-degree



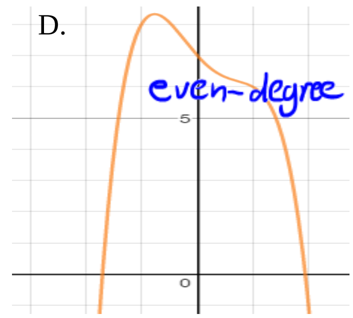
Equation: e

C. odd-degree



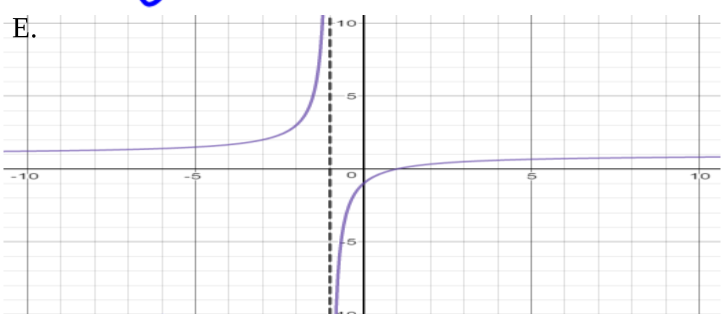
Equation: b or f

D. even-degree



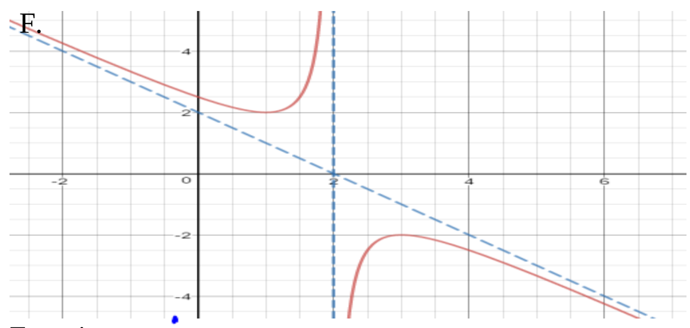
Equation: c

E.



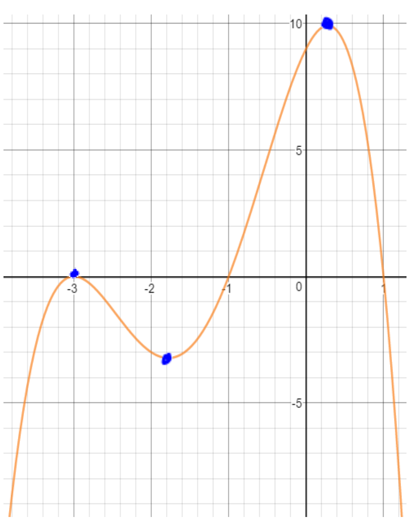
Equation: h

F.



Equation: j

2. Given below is the graph of the polynomial function $p(x)$. Determine each of the following. **(12 KU)**



(a) ***End Behaviours***

As $x \to \infty$, $y \to$ $-\infty$

As $x \to -\infty$, $y \to$ $-\infty$

(b) ***Number of Turning Points*** (Mark the turning points on the graph)

3

(c) ***Zeros and Multiplicities***

Zero	Multiplicity
-3	2 (even)
-1	1 (odd)
1	1 (odd)

(d) ***Intervals of Increase***

$\left(-\infty, -3\right)$

$\left(-\frac{7}{4}, \frac{1}{4}\right)$

(e) ***Intervals of Decrease***

$\left(-3, -\frac{7}{4}\right)$

$\left(\frac{1}{4}, \infty\right)$

(f) ***Possible Equation of p(x)***

$p(x) = a(x+3)^2(x+1)(x-1)$ for some $a \in \mathbb{R}$

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3. Sketch the graph of $g(x) = -\frac{1}{2}\left(\frac{1}{4}(x+1)\right)^3 - 3$ by applying transformations to the function $f(x) = x^3$. **(9 APP)**

(a) State the transformations required to obtain g from the base/parent/mother function $f(x) = x^3$.

Horizontal	*Vertical*
1. Stretch by a factor of 4	**1.** Compress by a factor of $\frac{1}{2}$ and reflect in x-axis
2. Translate one unit to the left	**2.** Translate three units down.

(b) Express the transformation in ***mapping notation***.

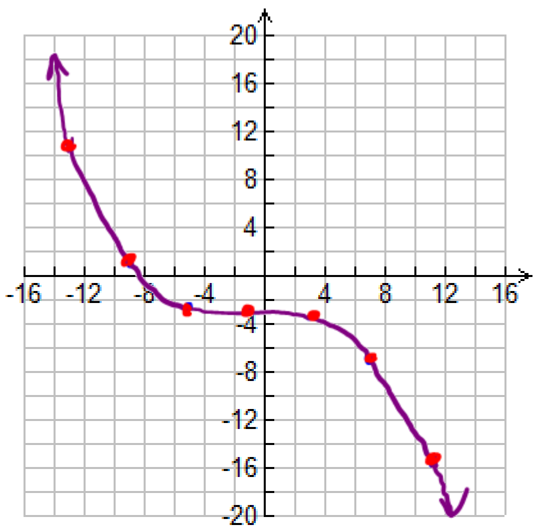
$(x, y) \rightarrow \left(4x-1, -\frac{1}{2}y-3\right)$

(c) Apply the transformation to a few key points on the graph of the base function $f(x) = x^3$

Pre-image Point on $y = f(x)$	*Image Point on $y = g(x)$*
$(-3, -27)$	$(-13, \frac{31}{2})$
$(-2, -8)$	$(-9, 1)$
$(-1, -1)$	$(-5, -\frac{5}{2})$
$(0, 0)$	$(-1, -3)$
$(1, 1)$	$(3, -\frac{7}{2})$
$(2, 8)$	$(7, -7)$
$(3, 27)$	$(11, -\frac{31}{2})$

Rough Work

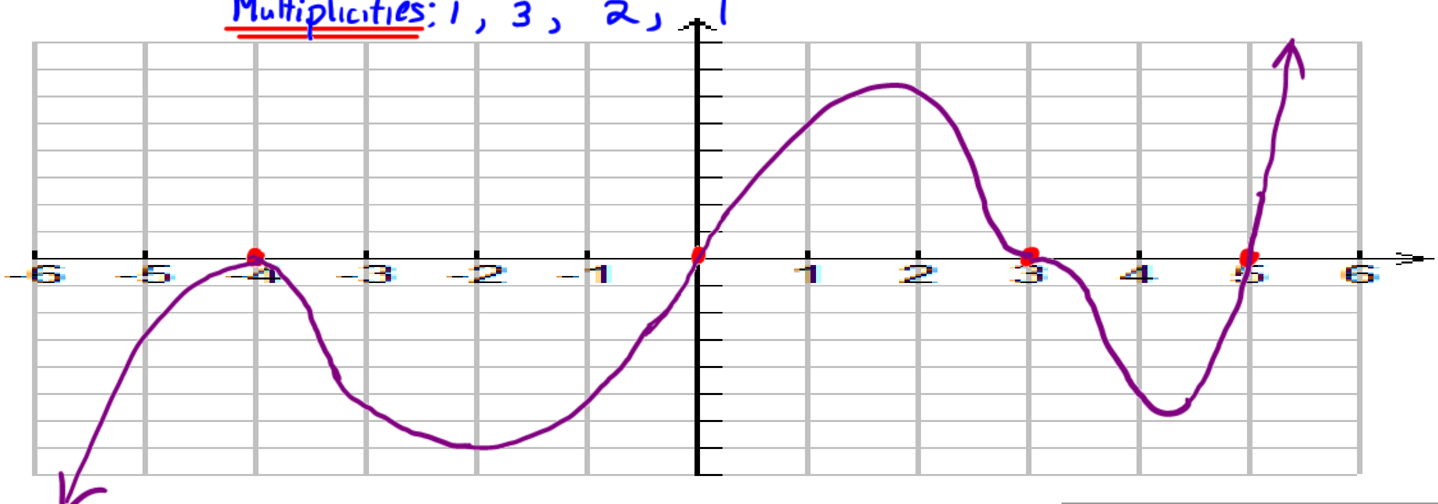
(d) Now sketch the graph of $g(x)$.



4. Sketch a possible graph of $f(x) = x(x-3)^3(x+4)^2(x-5)$. **(8 APP)**

Degree $= 1+3+2+1 = 7$ = odd

Multiplicities: 1, 3, 2, 1



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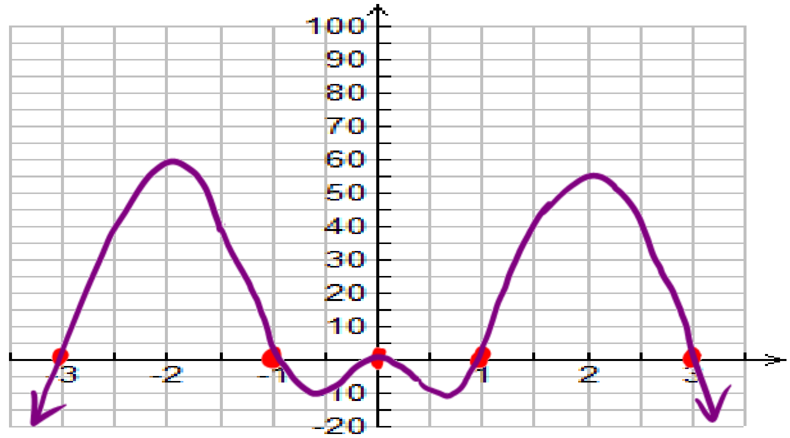
5. Consider the sixth-degree polynomial function $p(x) = -x^6 + 10x^4 - 9x^2$.

(a) ***Fully*** factor the polynomial. **(3 APP)**

$p(x) = -x^6 + 10x^4 - 9x^2$
$= -x^2(x^4 - 10x^2 + 9)$
$= -x^2(x^2 - 9)(x^2 - 1)$
$= -x^2(x-3)(x+3)(x-1)(x+1)$

(b) Use the factored form of the polynomial to sketch the graph of $y = p(x)$. **(3 APP)**



(c) Use the factored form to solve the equation $-x^6 + 10x^4 - 9x^2 = 0$. **(3 APP)**

$-x^2(x-3)(x+3)(x-1)(x+1) = 0$

$\therefore -x^2 = 0$ or $x - 3 = 0$ or $x + 3 = 0$ or $x - 1 = 0$ or $x + 1 = 0$

$\therefore x = 0$ or $x = \pm 3$ or $x = \pm 1$

(d) Use the factored form and the graph to solve the inequality $-x^6 + 10x^4 - 9x^2 \geq 0$. State the solution set using both set notation and interval notation. **(4 APP)**

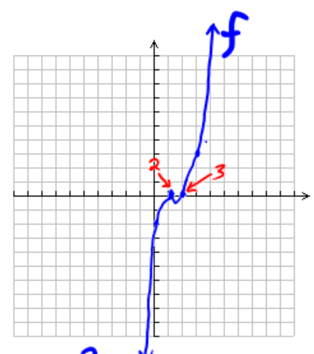
$-x^6 + 10x^4 - 9x^2 \geq 0$ wherever the graph of p is at or above the x-axis

$\therefore$ the solution set is

$\{x \in \mathbb{R} \mid -3 \leq x \leq -1$ or $x = 0$ or $1 \leq x \leq 3\}$

or $[-3, -1] \cup \{0\} \cup [1, 3]$

6. Solve the polynomial equation $x^3 - 7x^2 + 16x = 12$. Include a graph that clearly shows the solutions of the equation. **(10 TIPS)**



$x^3 - 7x^2 + 16x - 12 = 0$

Let $f(x) = x^3 - 7x^2 + 16x - 12$

Since $f(2) = 0$, $x - 2$ is a factor of $f(x)$

By long division, $f(x) = (x-2)(x^2 - 5x + 6)$

$$\begin{array}{r} x^2 - 5x + 6 \\ x-2 \overline{) x^3 - 7x^2 + 16x - 12} \\ \underline{x^3 - 2x^2} \qquad\qquad \\ -5x^2 + 16x \qquad \\ \underline{-5x^2 + 10x} \qquad \\ 6x - 12 \\ \underline{6x - 12} \\ 0 \end{array}$$

$\therefore (x-2)(x^2 - 5x + 6) = 0$

$\therefore (x-2)(x-2)(x-3) = 0$

$\therefore (x-2)^2(x-3) = 0$

$\therefore (x-2)^2 = 0$ or $x - 3 = 0$

$\therefore x = 2$ or $x = 3$

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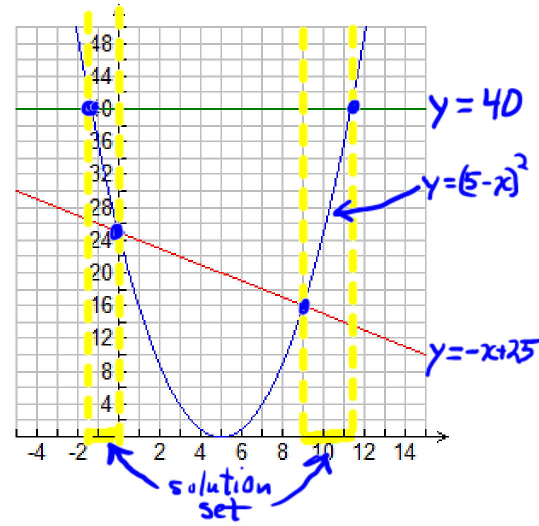
7. Shown at the right are the graphs of $y=(5-x)^2$, $y=-x+25$ and $y=40$. Write an inequality that corresponds to the given diagram. In addition, state the solution set using both set-builder notation and interval notation. (Do not solve the inequality. You should be able to see the solution just by looking at the graphs.) **(5 TIPS)**



$-x+25 \le (5-x)^2 \le 40$

(a) $-x+25 \le (5-x)^2$ when $x \le 0$ or $x \ge 9$

(b) $(5-x)^2 \le 40$ when $-\sqrt{40} \le 5-x \le \sqrt{40}$

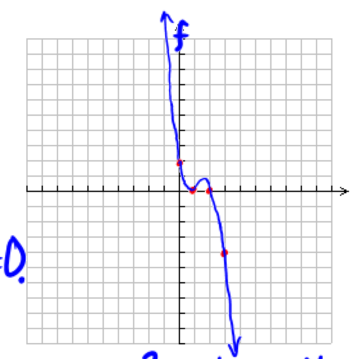
$2\sqrt{10} \ge x-5 \ge -2\sqrt{10}$

$5+2\sqrt{10} \ge x \ge 5-2\sqrt{10}$

The graph of $y=(5-x)^2$ lies "between" the other two graphs when $5-2\sqrt{10} \le x \le 0$ or $9 \le x \le 5+2\sqrt{10}$

Solution Set: $\{x \in \mathbb{R} \mid 5-2\sqrt{10} \le x \le 0 \text{ or } 9 \le x \le 5+2\sqrt{10}\} = [5-2\sqrt{10}, 0] \cup [9, 5+2\sqrt{10}]$

8. The polynomial function $f(x)=-x^n+kx^2-(2k+2)x+12$ has two turning points, no global extreme points and can be divided by $x-3$ with no remainder. Determine the values of n and k as well as any zeros of f. Then sketch the graph of $y=f(x)$. **Hint:** There is no way to ***calculate*** the value of n. The best approach is to ***choose*** the simplest possible value of n. **(8 TIPS)**



Since $f(x)$ can be divided exactly by $x-3$, $f(3)=0$

$\therefore -3^n+k(3^2)-(2k+2)(3)+12=0$

$\therefore -3^n+9k-6k-6+12=0$

$\therefore 3k+6-3^n=0$

$$\begin{array}{r} -x^2+4x-4 \\ x-3\,\big)\,\overline{-x^3+7x^2-16x+12} \\ \underline{-x^3+3x^2} \\ 4x^2-16x \\ \underline{4x^2-12x} \\ -4x+12 \\ \underline{-4x+12} \\ 0 \end{array}$$

Since the polynomial has no global extreme points and an even number of turning points, the degree of f must be odd. Therefore, a good candidate for the value of n is 3. If $n=3$,

$3k+6-3^3=0$.

$\therefore 3k-21=0$

$\therefore k=7$

$\therefore f(x) = -x^3+9x^2-16x+12$

$= -(x-3)(x^2-4x+4)$ by long division

$= -(x-3)(x-2)^2$

$\therefore$ zeros are $x=3$ and $x=2$

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