MCR 3U9 **Semester 2, 2016 - 2017**

Grade 11 Pre-AP Functions
Diagnostic Test

Mr. N. Nolfi
***Victim*:** Mr. Solutions Another brilliant work of art Mr. S.!!

$\frac{57}{57}$

1. Complete the following statements by filling in the blanks with logical answers that relate to what we have learned in the review unit of this course. **(13)**

(a) Many students find mathematics difficult because they see it as a massive collection of complicated, incomprehensible rules that are used to manipulate myriad meaningless symbols. Gladly, there are simple strategies that students can apply that will help them develop a mindset that makes mathematics much easier to understand. Mr. Nolfi described three of these strategies in Unit 0. List the three strategies in the space provided below.

(i) Focus on important ideas rather than blindly memorized facts. ✓

(ii) View mathematical relationships from different perspectives. ✓

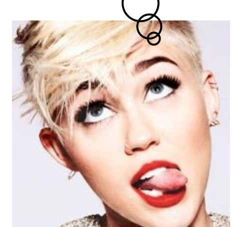
(iii) Don't just scratch the surface! Learn in depth! ✓

As examples of one of these strategies, Mr. Nolfi pointed out that for equations of lines, we only need to remember Slope = slope ✓, for the midpoint of a line segment, we only need to remember the average of a and b is $\frac{a+b}{2}$ ✓ and for the length of a line segment, we only need to remember the Pythagorean theorem ✓.

(b) To maximize my learning potential in mathematics, I must learn to distinguish between exercises ✓ and problems ✓ so that I can devote more time to problem solving ✓, which is the ultimate goal of studying mathematics. In addition, I must become very well acquainted with George Polya's four steps of problem solving, which are as follows:

1. Understand the problem ✓
2. Devise a strategy ✓
3. Carry out the strategy ✓
4. Check the solution ✓

Boy that was so easy!
Finally, the whole world will realize what a great genius I truly am!



2. When Miley C. was asked to ***factor*** the expression x^2+4x-5, she wrote the "solution" shown below. Is it correct? Explain. **(2)**

$\boxed{x^2+4x-5=(x-1)(x+5)}$

$\therefore x-1=0$ or $x+5=0$

$\therefore x=1$ or $x=-5$

Miley is NOT correct.
If she had stopped at this step, she would have received full marks. ✓

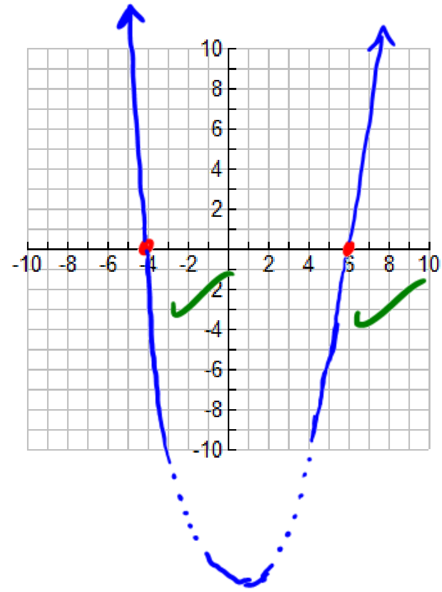
These steps would be performed to SOLVE the equation $x^2+4x-5=0$. ✓

3. Feeling that she was on a roll, Miley decided to tackle another question. Here is the "solution" that she offered. Is Miley's "solution" correct? If not, provide a ***correct solution*** along with a ***graph*** that clearly shows the ***roots*** of the ***equation***. **(6)**

$$5x^2 - 10x - 120 = 0$$
$$\therefore \cancel{5x^2 - 10x = 24}$$
$$\therefore \cancel{5x(x-2) = 24}$$
$$\therefore \cancel{5x = 24 \text{ or } x - 2 = 24}$$
$$\therefore \cancel{x = \frac{24}{5} \text{ or } x = 26}$$

Correct Solution

$$5x^2 - 10x - 120 = 0$$
$$\therefore 5(x^2 - 2x - 24) = 0$$
$$\therefore 5(x-6)(x+4) = 0$$
$$\therefore x - 6 = 0 \text{ or } x + 4 = 0$$
$$\therefore x = 6 \text{ or } x = -4$$



4. Solve. Show all steps.

(a) Solve the following linear equation. **(5)**

$$\left(\frac{14}{1}\right)\left(-\frac{3}{7}\right)(x+2) = \left[-4 - \frac{11}{14}x\right]\left(\frac{14}{1}\right)$$
$$\therefore -6(x+2) = -56 - 11x$$
$$\therefore -6x - 12 = -56 - 11x$$
$$\therefore 5x = -44$$
$$\therefore x = \frac{-44}{5}$$

(b) Solve the following quadratic equation. **(7)**

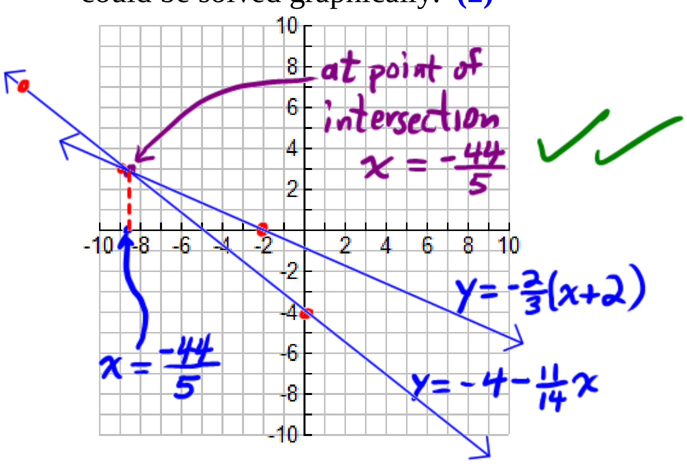
$$(3z-5)(z+4) = -3z(2z-5)$$
$$\therefore 3z^2 + 7z - 20 = -6z^2 + 15z$$
$$\therefore 9z^2 - 8z - 20 = 0$$
$$\therefore 9z^2 - 18z + 10z - 20 = 0$$
$$\therefore 9z(z-2) + 10(z-2) = 0$$
$$\therefore (z-2)(9z+10) = 0$$
$$\therefore z - 2 = 0 \text{ or } 9z + 10 = 0$$
$$\therefore z = 2 \text{ or } z = \frac{-10}{9}$$

180 → 10, 18

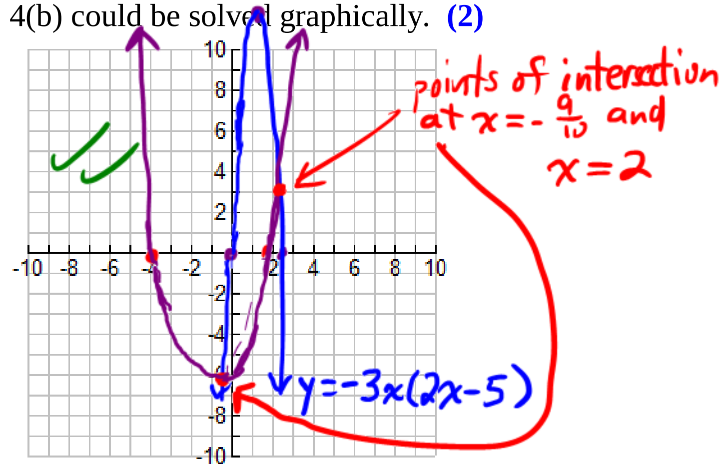
$9(-20) = -180$

$-18 + 10 = -8, \quad -18(10) = -180$

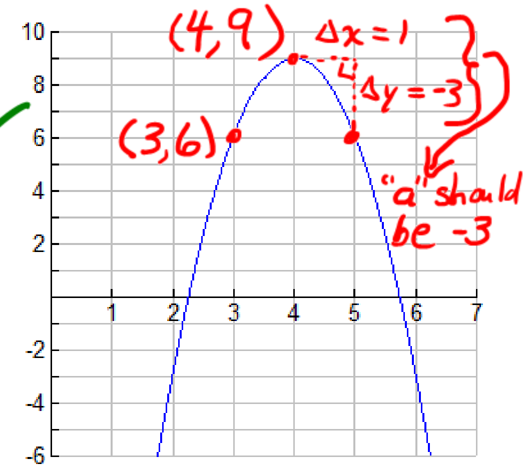
(c) Sketch a graph that shows how the equation in 4(a) could be solved graphically. **(2)**



(d) Sketch a graph that shows how the equation in 4(b) could be solved graphically. **(2)**



5. State an equation of the graph shown at the right. Justify your answer. **(4)**



Equation in vertex form: $y=a(x-h)^2+k$

The vertex of the given graph is (4,9).

Therefore, the equation of the graph must take the form $y=a(x-4)^2+9$.

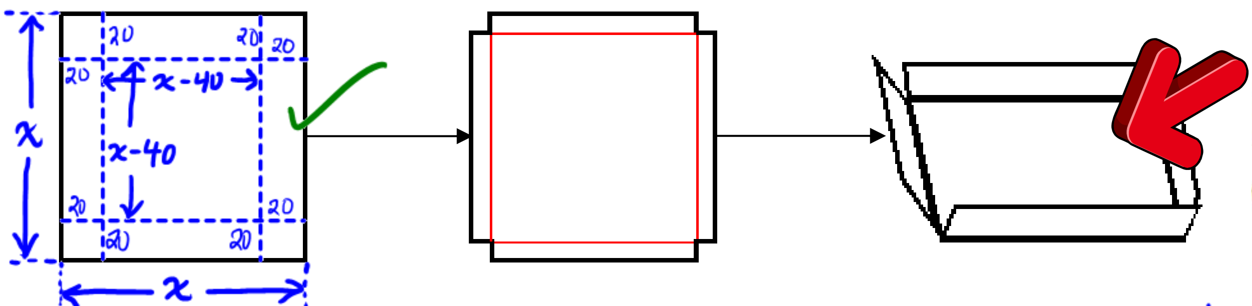
Since (3,6) lies on the graph,

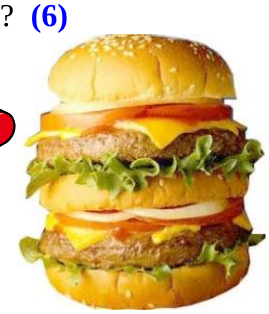
$$6=a(3-4)^2+9$$

$$\therefore -3=a$$

$\therefore$ an equation of the graph is $\boxed{y=-3(x-4)^2+9}$

6. McDonald's Canada® has hired Sukhman J. (aka *M. Bouche de Moteur*) to design a box for their new ultra-sized burger. The box must have a volume of 8000 cm^3, a depth of 20 cm and it must be manufactured from a ***square*** sheet of cardboard. What should be the dimensions of the square sheet of cardboard? **(6)**





Let x represent the length and width (in cm) of the square sheet of cardboard and let V represent the volume of the box (in cm^3).

Then $V=lwh=(x-40)(x-40)(20)=20(x-40)^2$

Since the volume must be 8000,

$$20(x-40)^2=8000$$

$$\therefore (x-40)^2=400$$

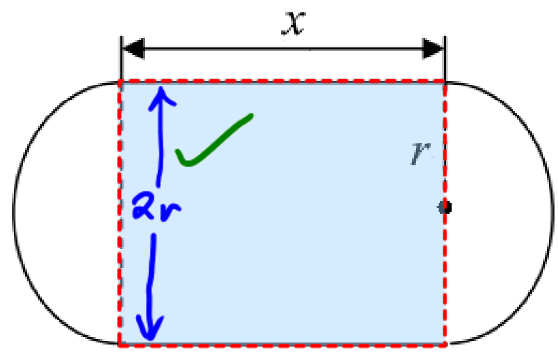
$$\therefore x-40=\pm 20$$

$$\therefore x=60 \text{ or } \cancel{x=20}$$

Dimensions of base of box would be -20×-20, which is impossible.

The square sheet of cardboard should be 60 cm by 60 cm.

7. A running track consists of a rectangle with a semicircle at each end. If the perimeter of the track is 400 m, find the dimensions that yield the greatest possible area for the rectangular region. **(10)**



$$2x + 2\pi r = 400$$
$$\therefore 2(x + \pi r) = 400$$
$$\therefore x + \pi r = 200$$

Maximize area of rectangular region
i.e. maximize $A = x(2r) = 2rx$

Since $x + \pi r = 200$,
$$x = 200 - \pi r$$

Therefore,
$$A = 2r(200 - \pi r)$$

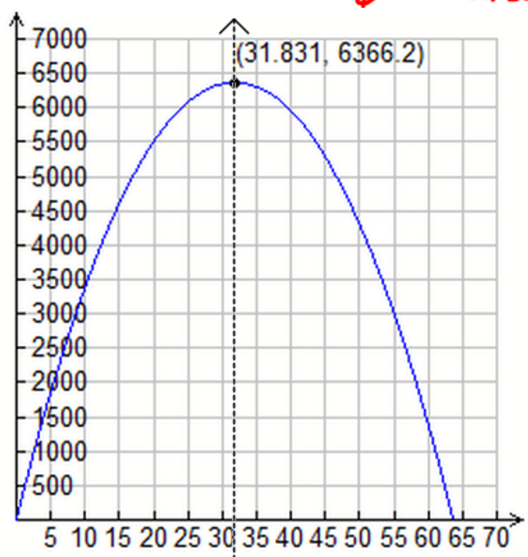
This is a quadratic function that opens downward, which means that the max area is found at the vertex.

The zeros of A are 0 and $\frac{200}{\pi}$.
Thus, the r-co-ordinate of the vertex is $\frac{200}{2\pi} = \frac{100}{\pi}$. $\left(\therefore 2r = \frac{200}{\pi}\right)$

$$\therefore x = 200 - \pi r$$
$$= 200 - \pi\left(\frac{100}{\pi}\right)$$
$$= 200 - 100 = 100$$

The dimensions of the rectangular region should be $100\,\text{m} \times \frac{200}{\pi}\,\text{m}$.

approximate maximum area



$\frac{100}{\pi}$

$x = \frac{100}{\pi}$ $\frac{200}{\pi}$

FUN PUZZLE: ATTEMPT ONLY IF YOU HAVE COMPLETED ALL THE OTHER QUESTIONS!

An eccentric old king wants to give his throne to one of his two sons. He decides that a horse race will be run and the son who owns the slower horse will become king. The sons, each fearing that the other will cheat by having his horse run less fast than it is capable, ask the court fool for his advice. With only two words, the fool tells them how to make sure that the race will be fair. What are the two words?

Switch horses! +1 Bonus