Grade 11 Pre-AP Functions
Minor Test – Unit 1 – Function Concepts, Notation, Perspectives, Applications, Transformations

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Victim: _Mr. Solutions_ *Exquisite display of mathematical intuition* Mr. S. !!!

KU	APP	TIPS	COM
20/20	20/20	17/17	10/10

Part 1: Modified True/False (6 KU)

State whether each statement is *true* or *false*. If false, *change* the *underlined part* to make the statement true.

1. T/F __F__ $f(ab) = f(a)f(b)$ for *all* functions f
   and all real numbers a and b.   {✓ = ½ mark}
   Change: ___$f(ab)$ or $f(ba)$___

2. T/F __F__ If $g(a) = b$ then (b,a) lies of the graph of g.   **Change:** ___(a,b)___
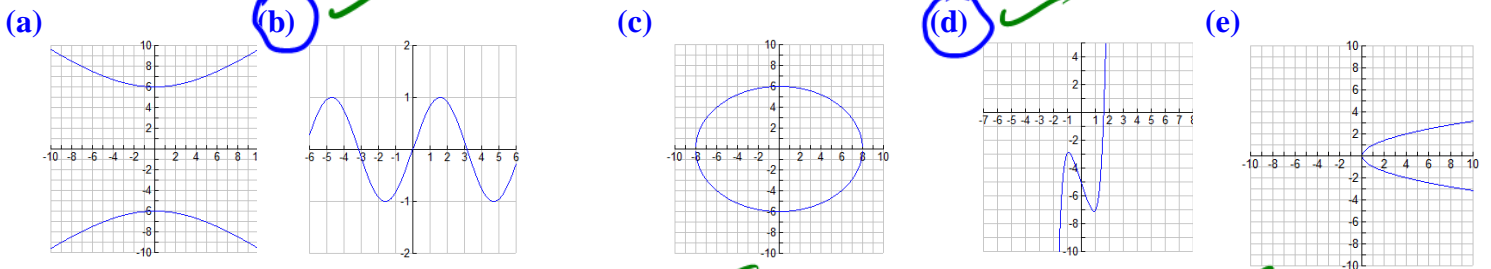
3. T/F __F__ All functions are relations <u>and all</u> relations are functions.   **Change:** ___but not all___

4. T/F __F__ $g(x) = -f(x) - 12$. The graph of g is obtained by
   shifting f <u>twelve units down then reflecting in the x-axis</u>.   **Change:** reflect in x-axis, then shift down 12

5. T/F __F__ If $f(u) = u$ then $f^{-1}(u) = \dfrac{1}{u}$.   **Change:** $f^{-1}(u) = u$
   → read this "f of u" NOT "$f \cdot u$"

Part 2: Problems

6. Circle the relations that are functions. (4 KU)
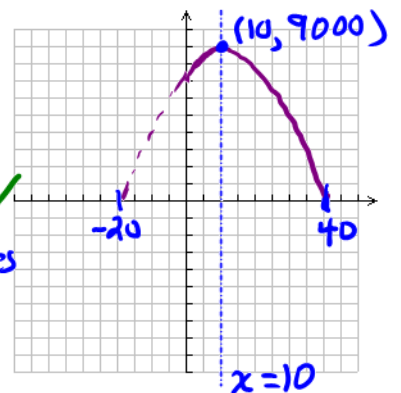
(a)   (b) ✓   (c)   (d) ✓   (e)

(f) $x^2 - y^2 = 0$ (g) $\{(1,1),(1,2),(1,3),(1,5)\}$ (h) $\{(0,2),(1,2),(2,2)\}$ ✓   (i) $|y| = 16 - x^2$ (j) $y = x^3 + 2x^2 - 3x$ ✓

7. State the domain and range of $f(x) = \dfrac{x^2 + 5x - 66}{x+11}$. (2 KU) $f(x) = \dfrac{(x+11)(x-6)}{x+11} = x - 6, \; x \neq -11$

   Domain = $\{x \in \mathbb{R} \mid x \neq -11\}$ ✓   Range = $\{y \in \mathbb{R} \mid y \neq -17\}$ ✓

8. A bowling-shoe-rental company charges \$2 per shoe rental and averages 4000 rentals per day. According to marketing studies of the bowling-shoe-rental industry, for every \$0.10 increase in price, a typical company can expect to lose 100 rentals per day. How much should the company charge to maximize revenue? (8 APP)

Let x represent the # of \$0.10 increases in price ✓
Let $R(x)$ represent the revenue (in \$) for x price increases
Then $R(x) = (4000 - 100x)(0.1x + 2)$ ✓
   (# tickets sold)   (price)
zeros: $-20, 40$ ✓
Since R is a quadratic function and it opens downward, the maximum revenue occurs at the average of the zeros, that is,
   at $x = \dfrac{-20 + 40}{2} = 10$. ✓
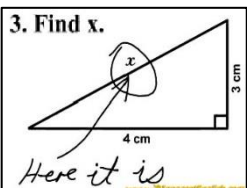
Thus, to maximize revenue, the price should be set to $0.1(10) + 2 = 3$ dollars. ✓

3. Find x.
   HA HA!   Here it is

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10. Let $f(x) = |x+2|$. The function g is obtained by performing the following transformations to f:

Horizontal Transformations	Vertical Transformations
1. Reflect in the y-axis	1. Reflect in the x-axis
2. Stretch by a factor of 2	2. Stretch by a factor of 3
3. Translate 4 units to the right	3. Translate 30 units up

(a) Write the transformation using mapping notation. **(4 KU)**
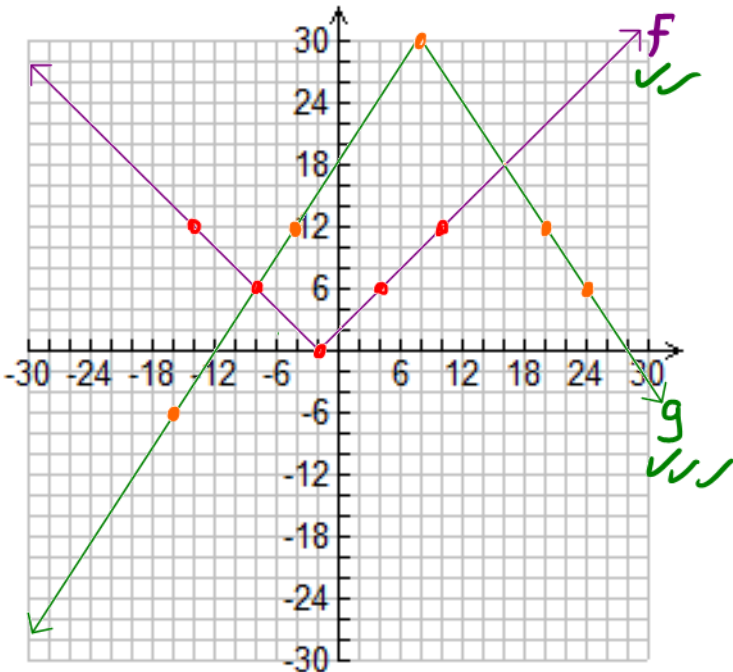
$(x, y) \rightarrow (-2x+4, -3y+30)$

(b) Write the transformation using function notation. **(4 KU)**

$g(x) = -3f\left(-\frac{1}{2}(x-4)\right) + 30$

(c) Apply the transformation to five key points on f. **(5 APP)** $(-2x+4, -3y+30)$

Pre-Image	Image
$(-2, 0)$	$(8, 30)$ ✓
$(4, 6)$	$(-4, 12)$ ✓
$(10, 12)$	$(-16, -6)$ ✓
$(-8, 6)$	$(20, 12)$ ✓
$(-14, 12)$	$(24, 6)$ ✓

(d) On the given grid, sketch the graphs of both f and g. **(5 APP)**

(e) Find the equation of g by using your answer to (b) as well as your graph. **(4 TIPS)**

$g(x) = -3f\left(-\frac{1}{2}(x-4)\right) + 30$

$\qquad = -3\left|-\frac{1}{2}(x-4)+2\right| + 30$ ✓✓

$\qquad = -3\left|-\frac{1}{2}x+2+2\right| + 30$

$\rightarrow = -3\left|-\frac{1}{2}x+4\right| + 30$ ✓

$\left. \begin{array}{l} = -3\left|-\frac{1}{2}(x-8)\right| + 30 \\ = -3\left|-\frac{1}{2}\right|\left|x-8\right| + 30 \\ = -\frac{3}{2}\left|x-8\right| + 30 \end{array} \right\}$ optional

④①

11. Suppose that $f(x) = x^2 - 3x - 10$. **(13 TIPS)** $= (x-5)(x+2)$

③ **(a)** Use the zeros of f to sketch its graph on the provided grid.

could be obtained from graph using property $|ab| = |a||b|$

① **(b)** Restrict the domain of f in such a way that f^{-1} is defined.

$D = \left\{ x \in \mathbb{R} \mid x \geq \frac{3}{2} \right\}$

② **(c)** Sketch the graph of f^{-1} for the restricted domain in (b).

(d) Find the equation of f^{-1}.

$f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{49}{4}$ ✓ (Obvious because of (a))

⑤ To form f^{-1}, perform transf. $(x,y) \rightarrow (y,x)$:

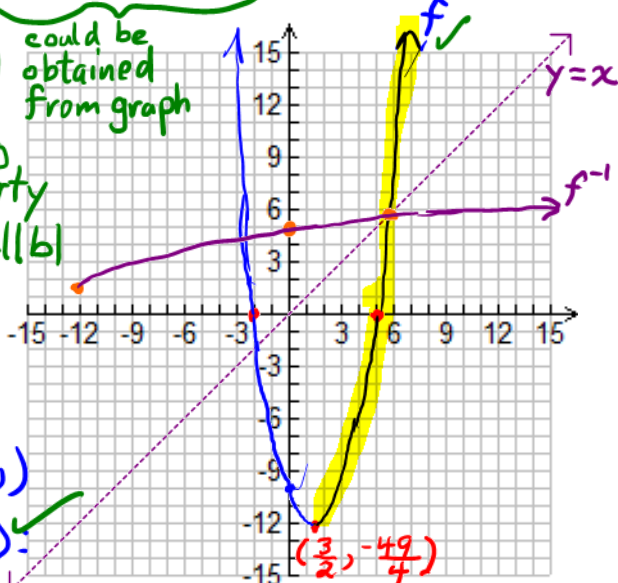$x = \left(y - \frac{3}{2}\right)^2 - \frac{49}{4}$ — because of the restricted domain of f ("upper half" only)

$\therefore y = \frac{3}{2} \pm \sqrt{x + \frac{49}{4}}$ ✓✓ ← also obvious

$\therefore y = \frac{3}{2} + \sqrt{x + \frac{49}{4}} = \sqrt{x + \frac{49}{4}} + \frac{3}{2}$ ✓

from graph: $\sqrt{x}$ translated $\frac{49}{4}$ left and $\frac{3}{2}$ up.

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-0	-0	-0	-0

(e) State the domain and range of f^{-1}.

② Domain $= \left\{ x \in \mathbb{R} \mid x \geq -\frac{49}{4} \right\}$ ✓   Range $= \left\{ y \in \mathbb{R} \mid y \geq \frac{3}{2} \right\}$ ✓