MCR 3U9 **Semester 2, 2016 - 2017**

Grade 11 Pre-AP Mathematics

Unit 2 – Exponential and Logarithmic Functions – Test #1

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Victim: Mr. Solutions

Once again, your work is inspiring Mr. N.!!

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24/24	15/15

Modified True/False **(5 KU)** ✓ = ½ mark

State whether each statement is ***true or false***. If false, ***change*** the ***underlined part*** to make the statement true.

1. T/F F ✓ For $f(x)=a^x$, $\frac{f(x)}{f(y)} = \underline{f\left(\frac{x}{y}\right)}$. $= \frac{a^x}{a^y} = a^{x-y} = f(x-y)$ **Change:** $f(x-y)$ ✓

2. T/F F ✓ For $f(x)=a^x$, $f(-x) = \underline{-f(x)}$. $= a^{-x} = \frac{1}{a^x} = \frac{1}{f(x)}$ **Change:** $\frac{1}{f(x)} = [f(x)]^{-1}$ ✓

3. T/F F ✓ $a^y = x$ means that $\underline{x = \log_a y}$. **Change:** $y = \log_a x$ ✓

4. T/F F ✓ $\log_{81} 9 = \underline{2}$ $81^{\frac{1}{2}} = \sqrt{81} = 9$ **Change:** $\frac{1}{2}$ ✓

5. T/F F ✓ $\underline{\log_{10}(100+10)} = \log_{10}100 + \log_{10}10$ **Change:** $\log_{10}[100(10)]$ ✓

$= 2+1$
$= 3 = \log_{10}1000$
$= \log_{10}[100(10)]$

Exercises I meant this to be $0.5(x-6)$. This is why points didn't fit well on grid.

6. Suppose that $g(x)=1.5\left(2^{0.5(x+6)}\right)+5$. **(14 KU)**

(a) State the transformations required to obtain g from the base function $f(x)=2^x$. (4)

Horizontal	***Vertical***
1. Stretch by a factor of 2 ✓	1. Stretch by a factor of 1.5 ✓
2. Translate 6 units to the left ✓	2. Translate 5 units up. ✓

(b) Express the transformation in ***mapping notation***. (2)

$(x,y) \to (2x-6,\ 1.5y+5)$ ✓✓

(c) Now apply the transformation to a few key points on the graph of the base function $f(x)=2^x$. (1)

Pre-image Point on $y=f(x)$	***Image Point on*** $y=g(x)$
$(-1, 1/2)$	$(-8, 5.75)$
$(0,1)$	$(-6, 6.5)$
$(2,4)$	$(-2, 11)$
$(4,16)$	$(2, 29)$
$(5,32)$	$(4, 53)$

✓

(d) Apply the transformation to the asymptote of $y=f(x)$. (1)

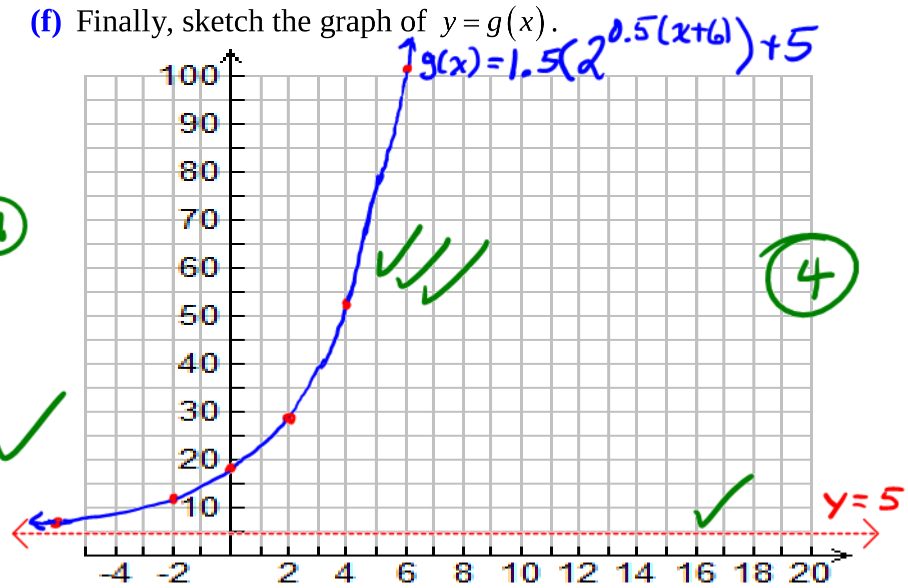
Pre-image Asymptote on $y=f(x)$	***Image Asymptote on*** $y=g(x)$
$y=0$	$1.5(0)+5=5$ $y=5$ ✓

(e) State the domain and range of g. (2)

domain $= \mathbb{R}$ ✓

range $= \{y \in \mathbb{R} \mid y>5\}$ ✓

(f) Finally, sketch the graph of $y=g(x)$. (4)



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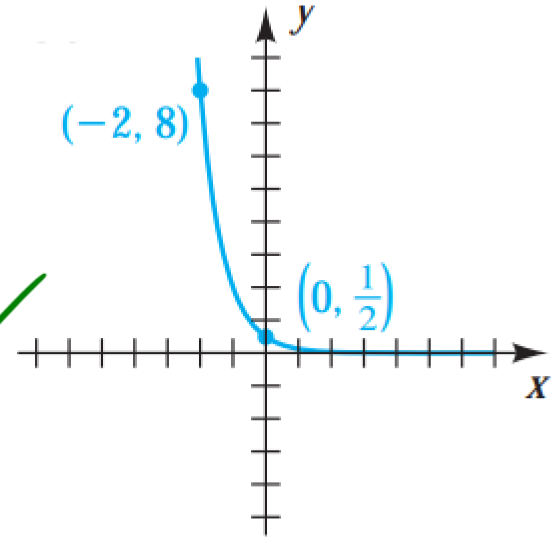
7. Find an exponential function of the form $f(x)=b(a^x)$ that has the given graph. **(5 KU)**

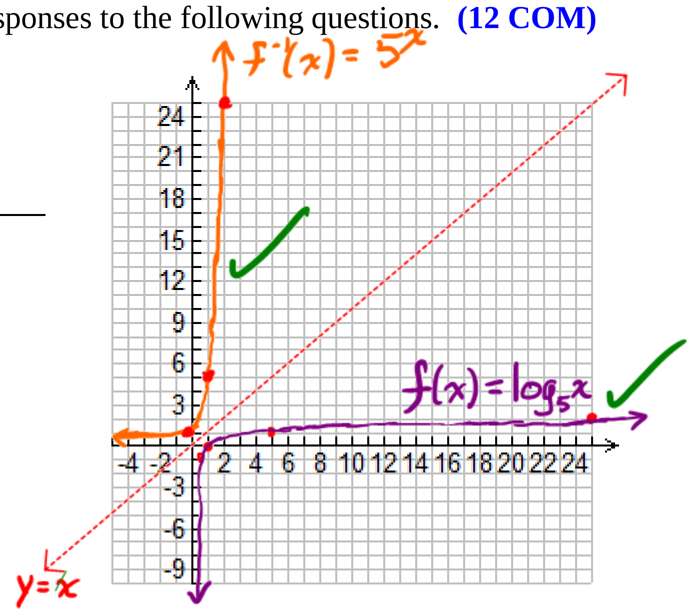


$\because f(0)=\frac{1}{2}$
$\therefore ba^0=\frac{1}{2}$
$\therefore b(1)=\frac{1}{2}$
$\therefore b=\frac{1}{2}$
$\therefore f(x)=\frac{1}{2}(a^x)$
$\because f(-2)=8$
$\therefore \frac{1}{2}(a^{-2})=8$
$\therefore a^{-2}=16$
Take reciprocal of B.S.

$a^2=\frac{1}{16}$
$\therefore a=\frac{1}{4}$

$\therefore f(x)=\frac{1}{2}\left(\frac{1}{4}\right)^x$

8. Use the provided grid as well as the provided space to write responses to the following questions. **(12 COM)**



(a) Sketch the graph of $f(x)=\log_5 x$ as well as the graph of $y=f^{-1}(x)$.

(b) State the equation of f^{-1}. $f^{-1}(x)=5^x$

(c) State the domain and range of f and f^{-1}.

Domain of f: $\{x\in\mathbb{R} \mid x>0\}$

Range of f: $\mathbb{R}$

Domain of f^{-1}: $\mathbb{R}$

Range of f^{-1}: $\{y\in\mathbb{R} \mid y>0\}$

(d) Explain why it is not necessary to impose any restrictions on the domain of f to form f^{-1}.

This is due to the fact that f is one-to-one.
The inverse of any function f is formed by interchanging the x and y-co-ordinates of every ordered pair belonging to f.
If f is one-to-one, then every "y-value" has a unique corresponding "x-value." Thus, when the inverse is formed, there is no possibility that there would be ordered pairs having the same "x-value" but different "y-values."

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(e) Suppose that $g(x)=\log_5|x|$. Would it be necessary to impose any restrictions on the domain of g to form g^{-1}? Explain.

The function g is NOT one-to-one:
For every $x\in\mathbb{R}$ such that $x\neq 0$, $g(x)=g(-x)$, meaning that g is symmetric in the y-axis.
Thus, the domain of g must be restricted to form g^{-1}.

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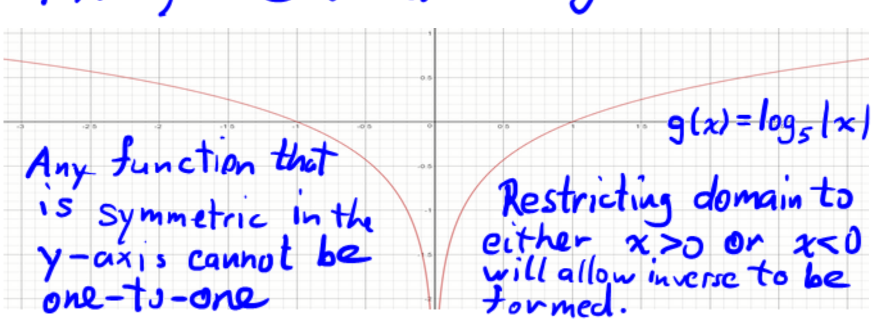

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