MCR 3U9 **Semester 2, 2016 - 2017**

Grade 11 Pre-AP Mathematics

Unit 2 – Exponential and Logarithmic Functions – Test #2

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Victim: Mr. Solutions

As usual, your solutions are thoughtful, insightful and elegant!

+3A

APP	TIPS	COM
21 /18	10 /10	10 /10

1. Opie lent Brian and Stewie an amount of money (unknown principal P) at a rate of 4.80% p.a. (per year), compounded ***quarterly*** (four times per year). Brian and Stewie finally repaid Opie five years later in one lump sum of $5,000. How much money did Brian and Stewie borrow from Opie?

(a) If *P* represents the amount borrowed, complete the following: **(4 APP)**

annual rate = 0.048

quarterly rate = 0.012

Time (quarters)	***Amount ($)***
0	*P*
1	$P(1.012)$
2	$P(1.012)^2$
3	$P(1.012)^3$
⋮	⋮
t	$P(1.012)^t$

(b) Now solve the problem! **(6 APP)**

$P = ?$ $V(t)$ = value of money after t quarters

5 years = 20 quarters $\therefore V(20) = 5000$

$V(t) = P(1.012)^t$

$\because V(20) = 5000,$

$\therefore P(1.012)^{20} = 5000$

$\therefore P = \dfrac{5000}{1.012^{20}} \doteq 3938.76$

Brian and Stewie borrowed $3938.76.

2. In ***living*** carbonaceous material, the ratio of number of ^{14}C atoms to number of ^{12}C atoms is about $1:10^{12}$. In a wooden artifact found in an excavation, the ratio of number of ^{14}C atoms to number of ^{12}C atoms is $1:1.9\times10^{12}$. Estimate the age of the wood used to make the artifact. (The half-life of ^{14}C is 5730 years.) **(8 APP)**

t (years)	$^{14}C : {}^{12}C$
0	$\frac{1}{10^{12}}$
5730	$\frac{1}{10^{12}}\left(\frac{1}{2}\right)$
11460	$\frac{1}{10^{12}}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
⋮	⋮
t	$\frac{1}{10^{12}}\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

Let $R(t)$ represent the ratio of ^{14}C atoms to ^{12}C atoms t years after the death of the organism.

$\therefore R(t) = \dfrac{1}{10^{12}}\left(\dfrac{1}{2}\right)^{\frac{t}{5730}}$

When ratio of ^{14}C to ^{12}C is $1:1.9\times10^{12}$,

$\dfrac{1}{10^{12}}\left(\dfrac{1}{2}\right)^{\frac{t}{5730}} = \dfrac{1}{1.9\times10^{12}}$

$\therefore \left(\dfrac{1}{2}\right)^{\frac{t}{5730}} = \dfrac{10^{12}}{1.9\times10^{12}}$

$\therefore \left(\dfrac{1}{2}\right)^{\frac{t}{5730}} = \dfrac{1}{1.9} \doteq 0.526316$

By trial and error, $\dfrac{t}{5730} \doteq 0.926$.

$\therefore t \doteq 5306$

The artifact is about 5300 years old.

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

3. After a drug is given to a patient, the amount of drug remaining in the bloodstream is measured twice. The table at the right shows the results. **(10 TIPS)**

Time (min)	*Amount of Drug Left (mg)*
90	85
120	81

(a) Assume that the amount of drug remaining in the bloodstream decreases exponentially. Write an equation of the form $A(t) = A_0 b^t$ that describes the amount of drug remaining, in mg, after t hours.

$A(1.5) = 85 \rightarrow 85 = A_0 b^{1.5}$ ✓ ①

$A(2) = 81 \rightarrow 81 = A_0 b^2$ ✓ ②

$\frac{②}{①}$, $\frac{A_0 b^2}{A_0 b^{1.5}} = \frac{81}{85}$

$\therefore b^{0.5} = \frac{81}{85}$

$\therefore b = \left(\frac{81}{85}\right)^2 = \frac{81^2}{85^2}$

$\therefore A(t) = A_0\left(\frac{81^2}{85^2}\right)^t$

$\because A(2) = 81$

$\therefore 81 = A_0\left(\frac{81^2}{85^2}\right)^2 = A_0\left(\frac{81^4}{85^4}\right)$

$\therefore A_0 = \frac{81(85^4)}{81^4} = \frac{85^4}{81^3}$

$\therefore A(t) = \frac{85^4}{81^3}\left(\frac{81^2}{85^2}\right)^t \doteq 98.225(0.9081)^t$ ✓ ✓

(b) What dose was given to the patient? Explain.

The initial dose was $A(0) = \frac{85^4}{81^3} \doteq 98.225$ mg ✓ ✓

(c) Write an equation for A^{-1}. Explain the ***meaning*** of the input and the output of this function.

Let $y = \frac{85^4}{81^3}\left(\frac{81^2}{85^2}\right)^t \doteq 98.225(0.9081)^t$

To form the inverse, apply $(t,y) \rightarrow (y,t)$

$t = \frac{85^4}{81^3}\left(\frac{81^2}{85^2}\right)^y \doteq 98.225(0.9081)^y$ ✓

$\therefore \frac{81^3}{85^4}t = \left(\frac{81^2}{85^2}\right)^y$ or $\frac{t}{98.225} \doteq 0.9081^y$

$\therefore \log_{\frac{81^2}{85^2}}\left(\frac{81^3}{85^4}t\right) = y$ or $\log_{10}\left(\frac{t}{98.225}\right) \doteq \log_{10}(0.9081^y)$

$\therefore y \doteq \frac{\log_{10}\left(\frac{t}{98.225}\right)}{\log_{10} 0.9081}$

$\therefore A^{-1}(t) = \log_{\frac{81^2}{85^2}}\left(\frac{81^3}{85^4}t\right) \doteq \frac{\log_{10}\left(\frac{t}{98.225}\right)}{\log_{10} 0.9081}$ ✓ ✓

input meaning: amount of drug in bloodstream (in mg) ✓

output meaning: time it would take for amount of drug in bloodstream to reach input level (in hours)

(d) How long will it take for the amount of drug remaining in the bloodstream to be 1% of the dose given to the patient?

1% of $\frac{85^4}{81^3}$

$\doteq 0.01(98.225)$

$= 0.98225$

$A^{-1}(0.98225) \doteq \frac{\log_{10}\left(\frac{0.98225}{98.225}\right)}{\log_{10} 0.9081} \doteq 47.8$

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It would take about 48 hours for the level of the drug in the bloodstream to fall to 1% of the dose given to the patient.

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0