MCR 3U9 **Semester 2, 2016 - 2017**

Grade 11 Pre-AP Functions

Unit 3 – Mid-unit Test (Radian Measure, Trig Ratios, Transformations, Modelling)

Mr. N. Nolfi

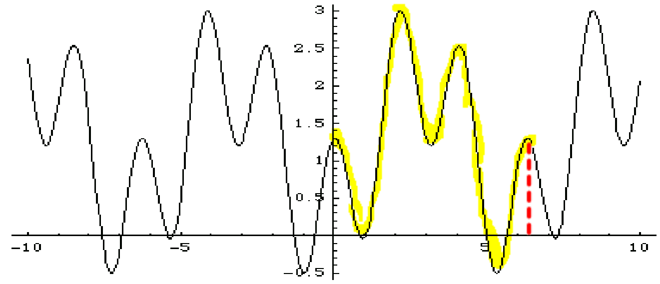
Another inspiring piece of work Mr. S.!!

Victim: Mr. Solutions

KU	APP	TIPS	COM
14/14	17/17	16/16	10/10

Part 1: Multiple Choice (9 KU)

Identify the choice that ***best*** answers the question.



1. c ✓ The graph of a periodic function is shown at the right. What is the approximate ***period*** of the function?

(a) 13 **(b)** 1.75 **(c)** 6.5 **(d)** 3.5

2. d ✓ Which of the following ***is most unlikely*** to produce a periodic graph?

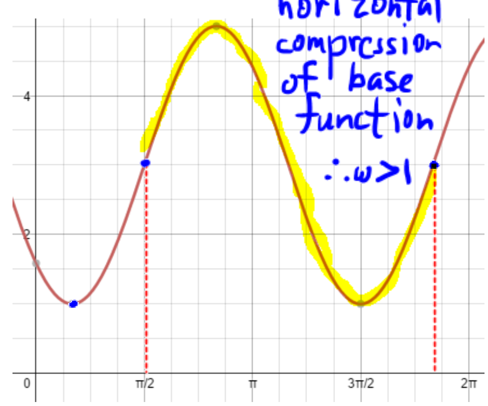
(a) Little Abinav's height above the floor as he jumps up and down in a playpen.

(b) The height above the floor of Ashutosh's (naughty former student) mother's hand as she "disciplines" him.

(c) Jenny's height above the ground as she rides an extremely fast Ferris wheel.

(d) The height above the ground of Chayanika's airplane as it descends toward a runway for a landing.

3. a ✓ Which of the following is a ***correct*** equation for the graph at the right?



horizontal compression of base function

$\therefore \omega > 1$

(a) $f(x)=2\sin\left(\frac{3}{2}\left(x-\frac{\pi}{2}\right)\right)+3$ ~~**(b)**~~ $f(x)=2\sin\left(\frac{3}{2}\left(x+\frac{\pi}{2}\right)\right)+3$

~~**(c)**~~ $f(x)=2\sin\left(\frac{2}{3}\left(x-\frac{\pi}{2}\right)\right)+3$ ~~**(d)**~~ $f(x)=2\sin\left(\frac{2}{3}\left(x+\frac{\pi}{2}\right)\right)+3$

4. a ✓ The function shown at the right has domain and range

(a) $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\le y\le 5\}$ ~~**(b)**~~ $D=\{x\in\mathbb{R}:1\ge x\ge 5\},\ R=\mathbb{R}$

~~**(c)**~~ $D=\mathbb{R},\ R=\{y\in\mathbb{R}:1\ge y\ge 5\}$ ~~**(d)**~~ $D=\{x\in\mathbb{R}:1\le x\le 5\},\ R=\mathbb{R}$

impossible

5. b ✓ Nancy is jumping up and down on a trampoline. Her height in metres above the ground after t seconds is given by the function $h(t)=0.5\sin(2\pi t)+1$. What does the "1" in the equation represent?

~~**(a)**~~ Nancy's maximum displacement from the average. **(b)** Nancy's average height above the ground.

~~**(c)**~~ Nancy's minimum height above the ground. ~~**(d)**~~ Nancy's maximum height above the ground.

6. b ✓ A sinusoidal function has an amplitude of 0.75 units, a period of 8π and a maximum at $(0,-3)$. Which of the following ***is not*** a possible equation of the function?

zero when $x=0$

sub. into each equation

(a) $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}(x+2\pi)\right)-\frac{15}{4}$ **(b)** $f(x)=\frac{3}{4}\sin\left(\frac{1}{4}x\right)-\frac{15}{4}$ $\rightarrow f(0)=-\frac{15}{4}\neq -3$

(c) $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}(x-8\pi)\right)-\frac{15}{4}$ **(d)** $f(x)=\frac{3}{4}\cos\left(\frac{1}{4}x\right)-\frac{15}{4}$

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7. __c__ Let $f(x)=\sin x$ and $g(x)=Af(\omega(x-p))+d$. Why must the period of g be $\frac{2\pi}{\omega}$?

~~(a)~~ This information is found in Mr. Nolfi's notes as well as the textbook. Everyone knows that neither source can ever be wrong. Textbooks and teachers are right about everything! Ridiculous!

~~(b)~~ It is true because period is calculated by dividing 2π by ω. Circular reasoning

(c) To obtain the graph of *g*, the graph of *f* must be stretched or compressed horizontally by the factor $\frac{1}{\omega}$, which means that the period of *f* is also stretched or compressed horizontally by the same factor.

~~(d)~~ To obtain the graph of *g*, the graph of *f* must be stretched or compressed horizontally by the factor ω, which means that the period of *f* is also stretched or compressed horizontally by the same factor.

8. __d__ 542° is equal to $\quad 271\,\cancel{542}\left(\frac{\pi}{\cancel{180}\,90}\right)$

(a) $\frac{90}{271}$ radians **(b)** $\frac{90\pi}{271}$ radians **(c)** $\frac{271}{90}$ radians **(d)** $\frac{271\pi}{90}$ radians

9. __d__ Consider two ***coterminal*** angles *x* and *y*, and their ***principal*** angle θ. If $2\pi < x < 4\pi$, $-4\pi < y < -2\pi$, $\boxed{\cos\theta=-\frac{1}{2} \text{ and } \sin\theta=\frac{\sqrt{3}}{2}}$, then $\rightarrow$ θ is in quadrant II $\rightarrow$ $\theta=\frac{2\pi}{3}$

2π

$x=\frac{2\pi}{3}+\frac{6\pi}{3}$

$y=\frac{2\pi}{3}-\frac{12\pi}{3}$

4π

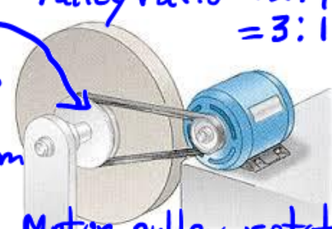
(a) $x=\frac{10\pi}{3}$, $y=\frac{-8\pi}{3}$ **(b)** $x=\frac{11\pi}{3}$, $y=\frac{-7\pi}{3}$ **(c)** $x=\frac{7\pi}{3}$, $y=\frac{-11\pi}{3}$ **(d)** $x=\frac{8\pi}{3}$, $y=\frac{-10\pi}{3}$

Part 2: Written Responses

10. An electric motor is used to turn a grinding wheel. The pulley on the motor has a radius of 4 cm, the pulley on the grinding wheel has a radius of 12 cm and the motor spins at a rate of 6000 RPM.

Pulley ratio 12:4 = 3:1

(a) If the grinding wheel has a radius of 25 cm, calculate the linear velocity, in cm/s, of a point on the circumference of the wheel. **(5 KU)**



$\omega = \frac{6000 \text{ rpm}}{3} = 2000 \text{ rpm}$

Motor pulley rotates 3 times for every rotation of grinding wheel pulley

$v=\frac{d}{t}=\frac{r\theta}{t}=r\left(\frac{\theta}{t}\right)=r\omega$ ← must be in rad/min to get cm/min

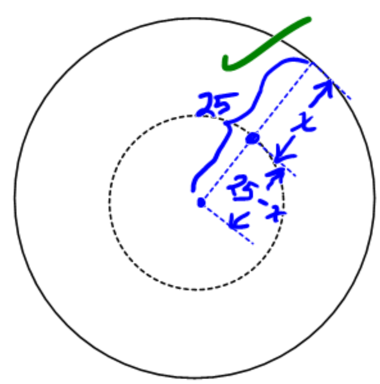
$\therefore v=(25\text{ cm})(2000\text{ rot/min})$

$=(25\text{ cm})[2000(2\pi)\text{ rad/min}]$

$=100000\pi\text{ cm/min}=\frac{100000\pi}{60}\text{ cm/s}=\frac{5000\pi}{3}\text{ cm/s}$

(b) Still assuming that the grinding wheel has a radius of 25 cm, write an equation of a function for the linear velocity, in cm/s, of a point on the grinding wheel *x* cm from the circumference. **(5 APP)**

Let $v(x)$ represent the linear velocity of a point on the wheel, x cm from the circumference



$\therefore v(x)=r\omega$

$=(25-x)(2000(2\pi)\text{ rad/min})$

$=4000\pi(25-x)\text{ cm/min}$

$=\frac{4000\pi(25-x)}{60}\text{ cm/s}$

$=\frac{200\pi(25-x)}{3}\text{ cm/s}$

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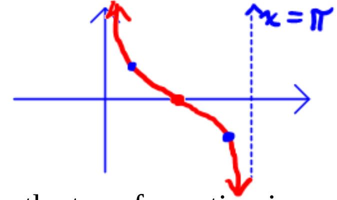
13. Suppose that $g(x) = -2\cot\left(\frac{1}{4}(x+\pi/4)\right)$. **(12 APP)**



(a) State the transformations required to obtain g from the base/parent/mother function $f(x) = \cot x$. 3

Horizontal	*Vertical*
1. Stretch by a factor of four ✓	1. Stretch by a factor of −2 (includes reflection in x-axis) ✓
2. Translate $\frac{\pi}{4}$ units to the left ✓	2. No vertical translation

(b) Express the transformation in ***mapping notation***.

$(x, y) \to (4x - \frac{\pi}{4}, -2y)$ ✓ ✓ 2

(c) Apply the transformation to a few key points on the graph of the base function $f(x) = \cot x$

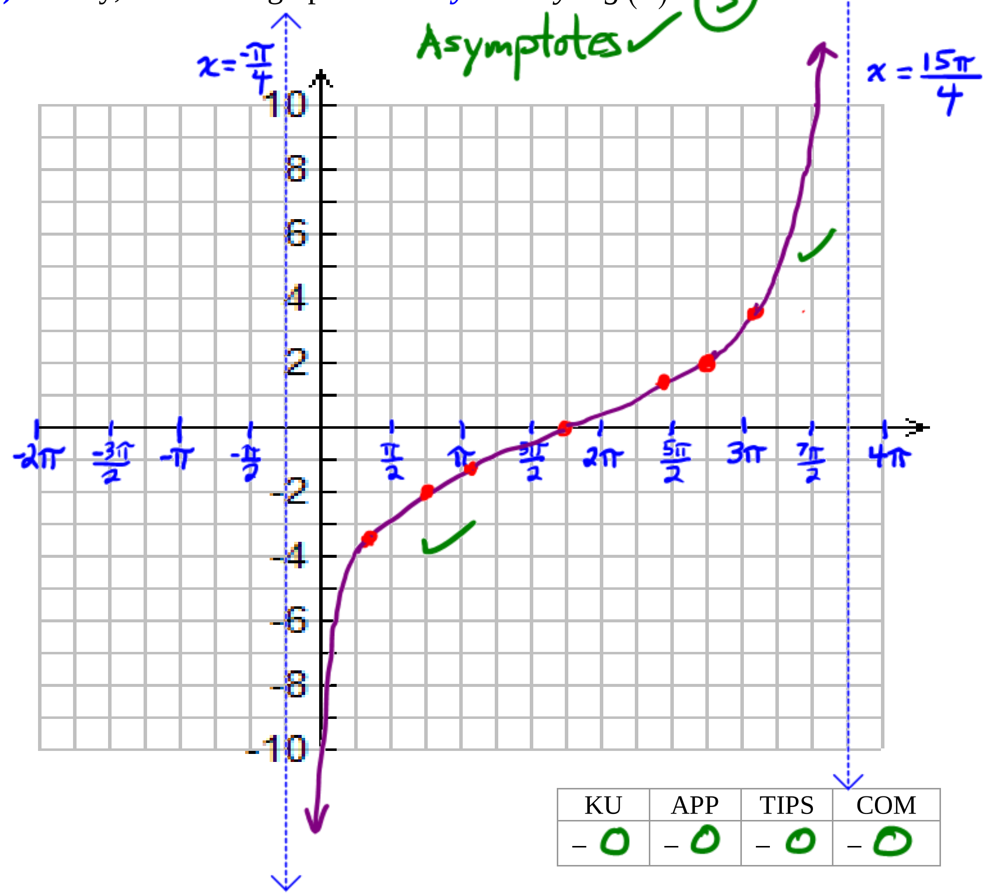
Pre-image Point on $y = f(x)$	*Image Point on $y = g(x)$*
$(\frac{\pi}{6}, \sqrt{3})$	$(\frac{5\pi}{12}, -2\sqrt{3})$
$(\frac{\pi}{4}, 1)$	$(\frac{3\pi}{4}, -2)$
$(\frac{\pi}{3}, \frac{1}{\sqrt{3}})$	$(\frac{13\pi}{12}, \frac{-2}{\sqrt{3}})$
$(\frac{\pi}{2}, 0)$	$(\frac{7\pi}{4}, 0)$
$(\frac{2\pi}{3}, -\frac{1}{\sqrt{3}})$	$(\frac{29\pi}{12}, \frac{2}{\sqrt{3}})$
$(\frac{3\pi}{4}, -1)$	$(\frac{11\pi}{4}, 2)$
$(\frac{5\pi}{6}, -\sqrt{3})$	$(\frac{37\pi}{12}, 2\sqrt{3})$

✓ 2

(d) Apply the transformation to the asymptotes of $f(x) = \cot x$ $x = 0$ and $x = \pi$ 2

Pre-image Asymptote of $y = f(x)$	*Image Asymptote of $y = g(x)$*
$x = 0$	$x = -\frac{\pi}{4}$ ✓
$x = \pi$	$x = \frac{15\pi}{4}$ ✓

(e) Finally, sketch the graph of ***one cycle*** of $y = g(x)$. 3



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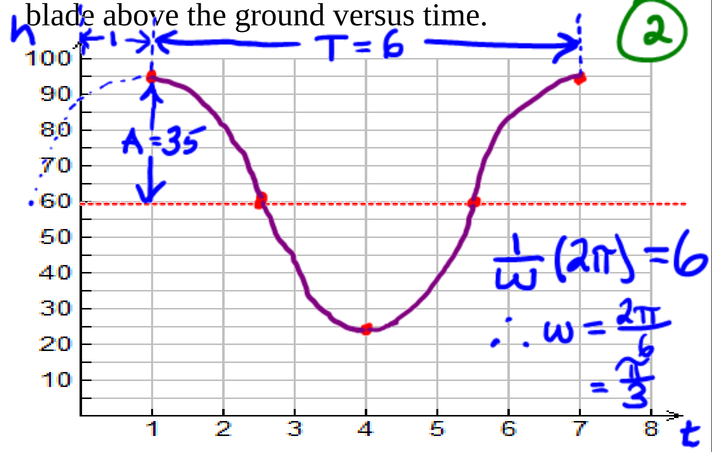
– 0	– 0	– 0	– 0

11. A wind turbine has three blades, coloured red, green and blue. An observer notices that at $t = 1$ s, the tip of the blue blade is 95 m above the ground. Then, over a period of 6 seconds, the tip of the blue blade moves from 95 m above the ground down to 25 m above the ground and back up to 95 m. **(16 TIPS)**

(a) Sketch the graph of height of the tip of the blue blade above the ground versus time.

2



$\frac{1}{\omega}(2\pi) = 6$

$\therefore \omega = \frac{2\pi}{6} = \frac{\pi}{3}$

(b) Write ***two different equations***, one using "cos" and the other using "sin," of a sinusoidal function that models the height of the tip of the blue blade above the ground versus time.

$$h(t) = 35\cos\left(\frac{\pi}{3}(t-1)\right) + 60$$

$$h(t) = 35\sin\left(\frac{\pi}{3}(t+0.5)\right) + 60$$

5

(c) What is the equation of the horizontal axis of this sinusoidal function? What does the horizontal axis represent in this context?

2

$h = 60$ (or $y = 60$)

The average height above the ground of the tip of the blue blade is 60 m.

(d) How high above the ground is the tip of the blue blade after 4 seconds?

$$h(4) = 35\cos\left(\frac{\pi}{3}(4-1)\right) + 60$$
$$= 35\cos\pi + 60$$
$$= 35(-1) + 60$$
$$= 25$$

1

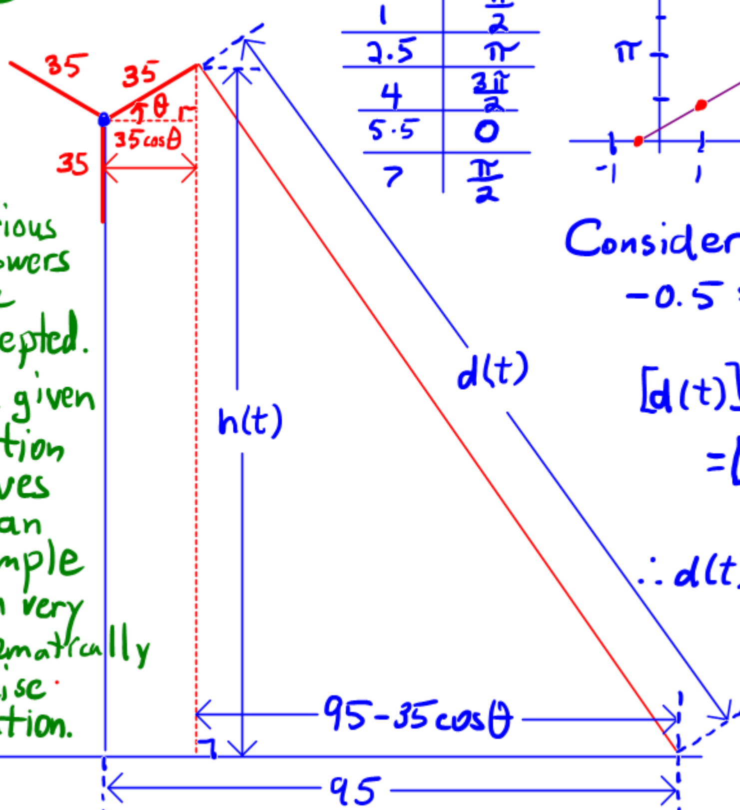
The tip of the blue blade is 25 m above the ground at 4 s (agrees with graph)

(e) The observer is standing at a point that is 95 m away from the centre of the base of the wind turbine. Would you expect the distance from the observer to the tip of one of the blades to change sinusoidally? Explain.

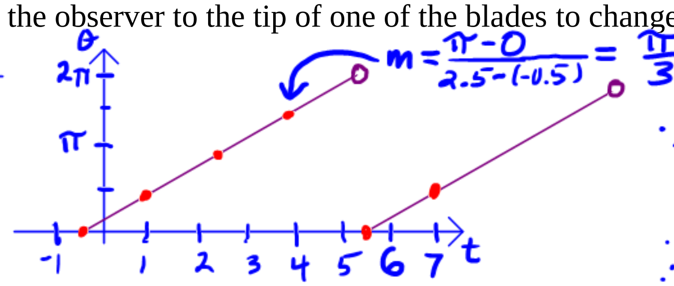
See next page for alternative solution

6

Various answers are accepted. The given solution serves as an example of a very mathematically precise solution.



t	$\theta(t)$
1	$\frac{\pi}{2}$
2.5	π
4	$\frac{3\pi}{2}$
5.5	0
7	$\frac{\pi}{2}$



$m = \frac{\pi - 0}{2.5-(-0.5)} = \frac{\pi}{3}$

$\theta(t) = \frac{\pi}{3}t + b$

$\because (-0.5, 0)$ lies on line, $\theta(-0.5) = 0$

$\therefore \frac{\pi}{3}(-0.5) + b = 0$

$\therefore b = \frac{0.5\pi}{3} = \frac{\pi}{6}$

$\therefore \theta(t) = \frac{\pi}{3}t + \frac{\pi}{6}$, $-0.5 \le t \le 5.5$

Consider only one cycle, $-0.5 \le t \le 5.5$

$$[d(t)]^2 = [h(t)]^2 + (95 - 35\cos\theta)^2$$
$$= \left[35\cos\left(\frac{\pi}{3}(t-1)\right)+60\right]^2 + \left[95 - 35\cos\left(\frac{\pi}{3}\left(t+\frac{1}{2}\right)\right)\right]^2$$

$$\therefore d(t) = \sqrt{\left[35\cos\left(\frac{\pi}{3}(t-1)\right)+60\right]^2 + \left[95-35\cos\left(\frac{\pi}{3}\left(t+\frac{1}{2}\right)\right)\right]^2}$$

$\because d(t)$ involves finding the square root of the sums of squares of sinusoidal functions, $d(t)$ cannot be sinusoidal.

KU	APP	TIPS	COM
– 0	– 0	– 0	– 0

Alternative Explanation

θ	t	d
0	-0.5	84.9
$\frac{\pi}{2}$	1	134.4
π	2.5	143.2
$\frac{3\pi}{2}$	4	98.2
2π	5.5	84.9

$+49.5$

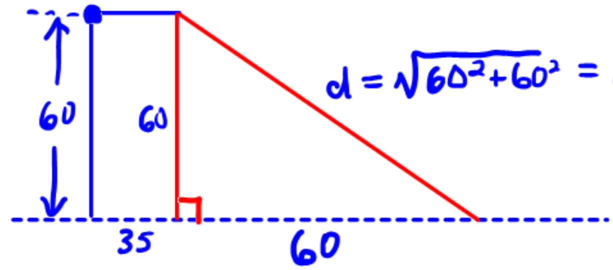
$+8.8$

-45

-13.3

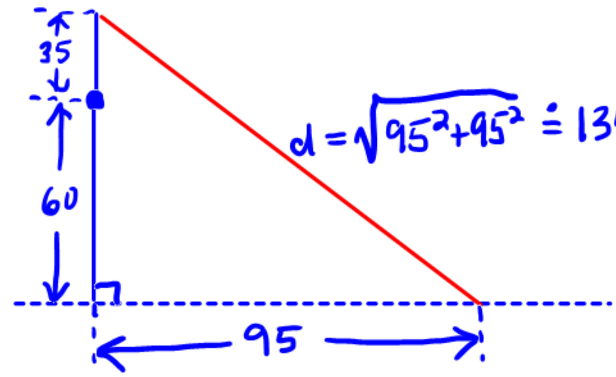
This lacks the symmetry of a sinusoidal function.

$\theta = 0$



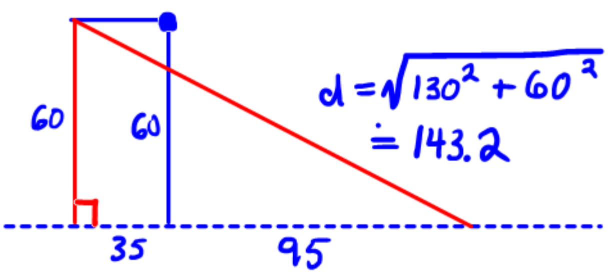
$d = \sqrt{60^2 + 60^2} = 84.9$

$\theta = \frac{\pi}{2}$



$d = \sqrt{95^2 + 95^2} \doteq 134.4$

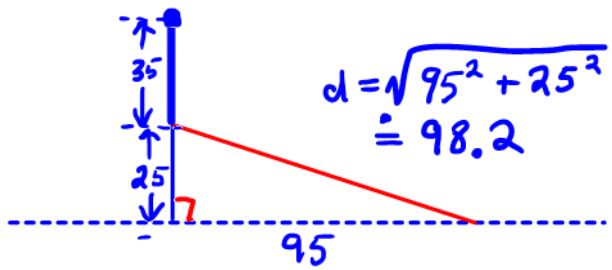
$\theta = \pi$



$d = \sqrt{130^2 + 60^2}$

$\doteq 143.2$

$\theta = \frac{3\pi}{2}$



$d = \sqrt{95^2 + 25^2}$

$\doteq 98.2$