Grade 11 Pre-AP Mathematics
Trigonometric Identities, Solving Trigonometric Equations

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Victim: Mr. Solutions

Another supreme demonstration of your mathematical prowess Mr. S.!

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24/24	20/20	20/20	10/10

***Modified True/False* (5 KU)**

State whether each statement is ***true*** or ***false***. If false, ***change*** the ***underlined part*** to make the statement true.

1. T/F F The equation $\sin 3x = 1$ has $\underline{6}$ solutions in $[0, 2\pi]$. **Change:** 3

2. T/F F $\cot 3x$ is equivalent to $\underline{\dfrac{1}{3\cot x}}$. **Change:** $\dfrac{1}{\tan 3x}$

3. T/F F The equation $\boxed{\csc 3x} = \dfrac{1}{2}$ has $\underline{6}$ solutions in $[0, 2\pi]$. **Change:** 0

must be ≤ -1 or ≥ 1

4. T/F F If $\tan\theta = \dfrac{3}{4}$ then $\underline{\sin\theta = 3}$. **Change:** $\sin\theta = \frac{3}{5}$

$x^2 + y^2 = r^2$
$3^2 + 4^2 = 5^2$

5. T/F F For $f(x) = \sin x$, $f(x+y) = \underline{f(x) + f(y)}$. **Change:** $f(x)f(\frac{\pi}{2}-y) + f(\frac{\pi}{2}-x)f(y)$

$\sin x\cos y + \cos x \sin y$

Problems

6. Use the three methods indicated below to demonstrate that the equation $\cos(\theta + \frac{3\pi}{2}) = \sin\theta$ ***is*** an identity.

(a) Compound-angle identity **(3 KU)**

$\cos(\theta + \frac{3\pi}{2})$

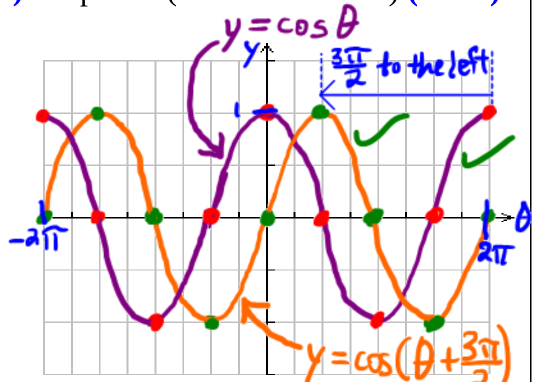
$= \cos\theta\cos\frac{3\pi}{2} - \sin\theta\sin\frac{3\pi}{2}$

$= \cos\theta(0) - \sin\theta(-1)$

$= 0 - (-\sin\theta)$

$= \sin\theta$

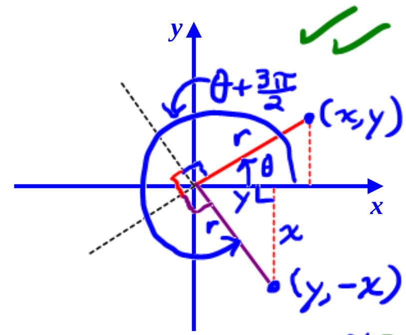
(b) Graphical (Transformations) **(3 KU)**



(graph of $y = \cos\theta$ translated $\frac{3\pi}{2}$ left)

The graph of $y = \cos(\theta + \frac{3\pi}{2})$ can be obtained by translating the graph of $y = \cos\theta$ $\frac{3\pi}{2}$ units to the left. When this is done, the graph of $y = \sin\theta$ is obtained!!

(c) Angles of Rotation **(3 KU)**



$\cos(\theta + \frac{3\pi}{2}) = \frac{y}{r}$

$\sin\theta = \frac{y}{r}$

$\therefore \cos(\theta + \frac{3\pi}{2}) = \sin\theta$ is an identity

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$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y} \qquad \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y} \qquad \cot(x+y)=\frac{\cot x\cot y-1}{\cot x+\cot y} \qquad \cot(x-y)=\frac{\cot x\cot y+1}{\cot y-\cot x}$$

7. Use a counterexample to demonstrate that the equation $\cot(x+y)=\cot x+\cot y$ ***is not*** an identity. Choose your counterexample in such a way that a calculator is not required to evaluate any of the trig ratios.

Hint: x and y can represent the same value. **(5 KU)**

Let $x=y=\frac{\pi}{4}$. Then,

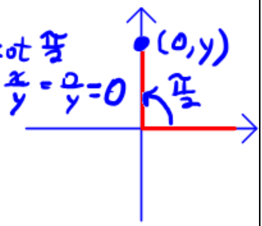
$L.S. = \cot(x+y)$

$= \cot\left(\frac{\pi}{4}+\frac{\pi}{4}\right)$

$= \cot\frac{\pi}{2}$

$= 0$

$\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{y} = 0$



$R.S. = \cot x + \cot y$

$= \cot\frac{\pi}{4} + \cot\frac{\pi}{4}$

$= 1 + 1 = 2$

$\therefore$ $L.S. \neq R.S.$, the given equation is NOT an identity.

8. Evaluate $\cot 285°$ without using a calculator. Exact values are required!

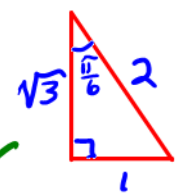
Hint: Use the identity for $\cot(x-y)$. **(5 KU)**

$\cot 285°$

$= \cot(315° - 30°)$

$= \cot\left(\frac{7\pi}{4} - \frac{\pi}{6}\right)$

$= \dfrac{\cot\frac{7\pi}{4}\cot\frac{\pi}{6} + 1}{\cot\frac{\pi}{6} - \cot\frac{7\pi}{4}}$



$= \dfrac{-1(\sqrt{3}) + 1}{\sqrt{3} - (-1)}$

$= \dfrac{1-\sqrt{3}}{\sqrt{3}+1}$ This is sufficient

$= \dfrac{(1-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \dfrac{\sqrt{3}-1-3+\sqrt{3}}{2}$

$= \dfrac{2\sqrt{3}-4}{2}$

$= \dfrac{2(\sqrt{3}-2)}{2}$

$= \sqrt{3}-2$

Rationalization of denominator is optional.

9. Write $\sin 3\theta$ entirely in terms of $\sin\theta$. **(10 APP)**

$\sin 3\theta = \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= (2\sin\theta\cos\theta)\cos\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

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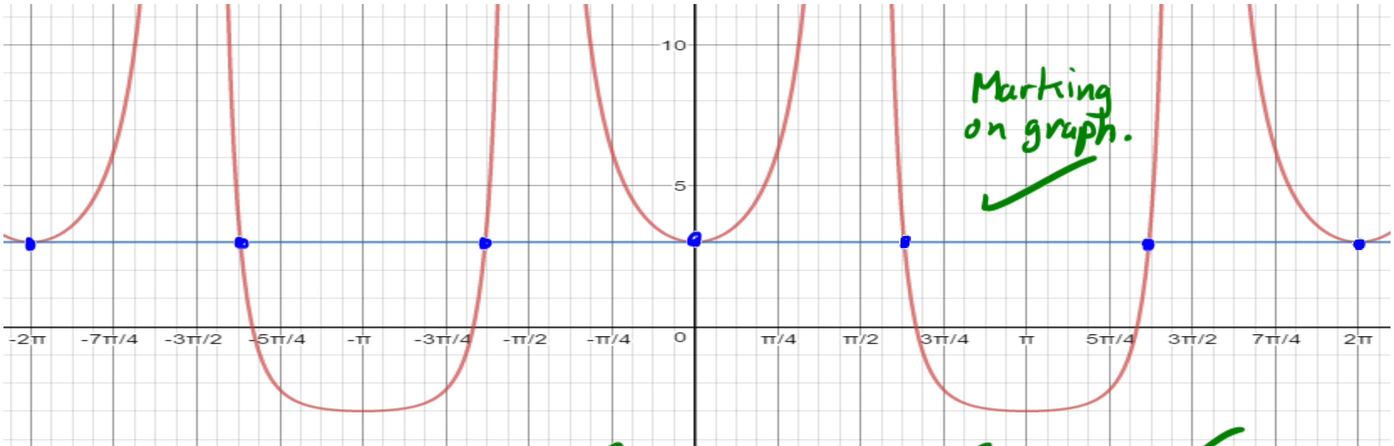
10. The following question deals with solving trigonometric equations both graphically and algebraically.

(a) Shown below are the graphs of $y=(\sec x+2)(2\sec x-1)$ and $y=3$. State ***approximate*** solutions to the equation $(\sec x+2)(2\sec x-1)=3$ for $x\in[-2\pi,2\pi]$. ***Mark*** the solutions on the graph. **(5 APP)**



Approximate Solutions: $x=0$, $x=\pm\frac{5\pi}{8}\doteq\pm1.963$, $x=\pm\frac{11\pi}{8}\doteq 4.320$, $x=\pm2\pi$

(b) Use an algebraic method to solve the equation $(\sec x+2)(2\sec x-1)=3$, where $x\in[-\pi,\pi]$. Verify that your solutions agree with those that you obtained in (a). **(5 APP)**

$(\sec x+2)(2\sec x-1)=3$

$\therefore 2\sec^2 x+3\sec x-2=3$

$\therefore 2\sec^2 x+3\sec x-5=0$

$\therefore (2\sec x+5)(\sec x-1)=0$

$\therefore 2\sec x+5=0$ or $\sec x-1=0$

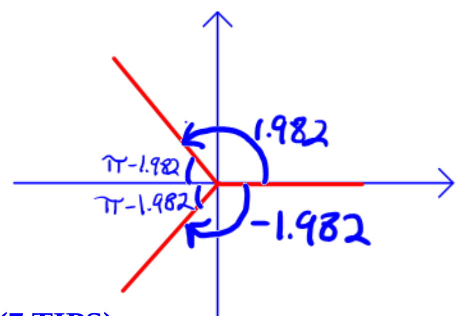
$\therefore \sec x=-\frac{5}{2}$ or $\sec x=1$

$\therefore \cos x=-\frac{2}{5}$ or $\cos x=1$

$\therefore x=\cos^{-1}\left(-\frac{2}{5}\right)$ or $x=\cos^{-1}(1)$

$\therefore x\doteq\pm1.982$ or $x=0$

These answers agree with the graphical estimates



11. Prove that the equation $\dfrac{\cos x}{1-\tan x}+\dfrac{\sin x}{1-\cot x}=\sin x+\cos x$ is an identity. **(7 TIPS)**

$$\text{L.S.}=\frac{\cos x}{1-\frac{\sin x}{\cos x}}+\frac{\sin x}{1-\frac{\cos x}{\sin x}}$$

$$=\frac{\cos x}{\left(\frac{\cos x-\sin x}{\cos x}\right)}+\frac{\sin x}{\left(\frac{\sin x-\cos x}{\sin x}\right)}$$

$$=\left(\frac{\cos x}{1}\right)\left(\frac{\cos x}{\cos x-\sin x}\right)+\left(\frac{\sin x}{1}\right)\left(\frac{\sin x}{\sin x-\cos x}\right)$$

$$=\frac{\cos^2 x}{-(\sin x-\cos x)}+\frac{\sin^2 x}{\sin x-\cos x}$$

$$=\frac{-\cos^2 x+\sin^2 x}{\sin x-\cos x}$$

$$=\frac{\sin^2 x-\cos^2 x}{\sin x-\cos x}$$

$$=\frac{(\sin x-\cos x)(\sin x+\cos x)}{\sin x-\cos x}$$

$$=\sin x+\cos x$$

$=\text{R.S.}$

$\therefore$ L.S. = R.S., the given equation is an identity.

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12. Consider the functions $f(x)=\dfrac{\cot^2 x}{\csc^2 x+\sin^2 x+\cos^2 x-1}$ and $g(x)=\cos^2 x$. **(13 TIPS)**

(a) Use the grid to sketch the graph of g. **Hint:** First sketch the graph of $y=\cos x$. Then "square" it.

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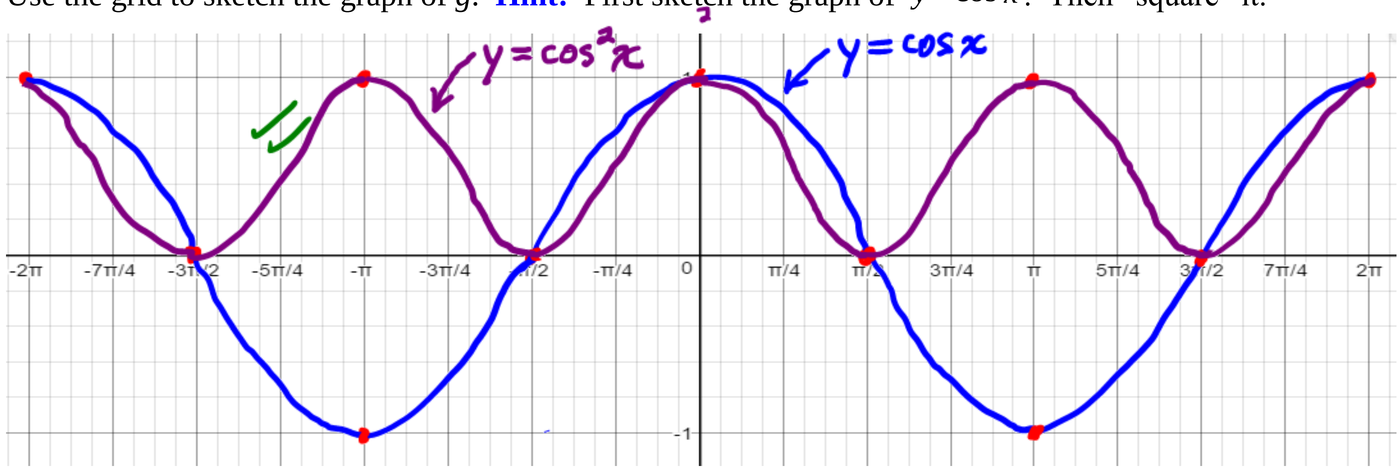
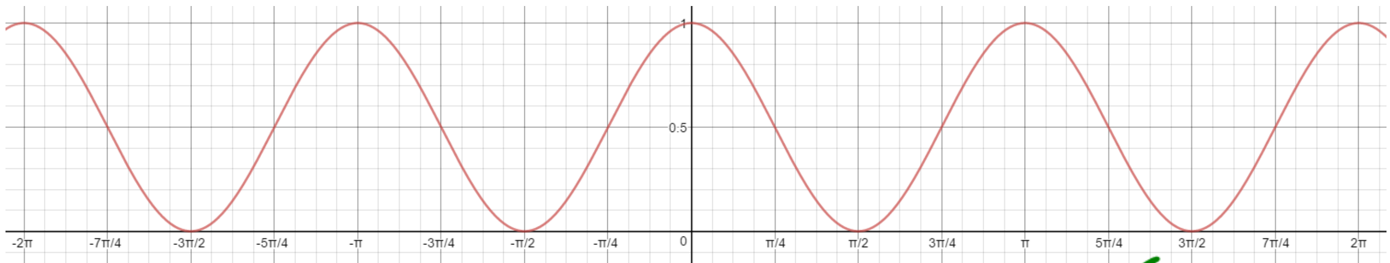


(b) Shown below is the graph of f. What identity is suggested by this graph? Explain.



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The graph of f looks identical to the graph of g. This suggests that the equation $\dfrac{\cot^2 x}{\csc^2 x+\sin^2 x+\cos^2 x-1}=\cos^2 x$ is an identity.

(c) Prove that the equation that you wrote in (b) is an identity.

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$$\text{L.S.} = \frac{\cot^2 x}{\csc^2 x+1-1}$$

$$= \frac{\cot^2 x}{\csc^2 x}$$

$$= \frac{\left(\frac{\cos^2 x}{\sin^2 x}\right)}{\left(\frac{1}{\sin^2 x}\right)}$$

$$= \frac{\cos^2 x}{\sin^2 x}\left(\frac{\sin^2 x}{1}\right)$$

$$= \cos^2 x = \text{R.S.}$$

$\therefore$ the equation in (b) is an identity.

(d) Finally, solve the equation $\dfrac{\cot^2 x}{\csc^2 x+\sin^2 x+\cos^2 x-1}=\sin^2 x$ for $0\le x\le 2\pi$.

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$\therefore \cos^2 x=\sin^2 x$ (because of identity in (c))

$\therefore 1=\dfrac{\sin^2 x}{\cos^2 x}$

$\therefore 1=\tan^2 x$

$\therefore \pm 1=\tan x$

$\therefore x=\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

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