MCR 3U9 INSIGHT-BUILDING PROJECT

Purpose

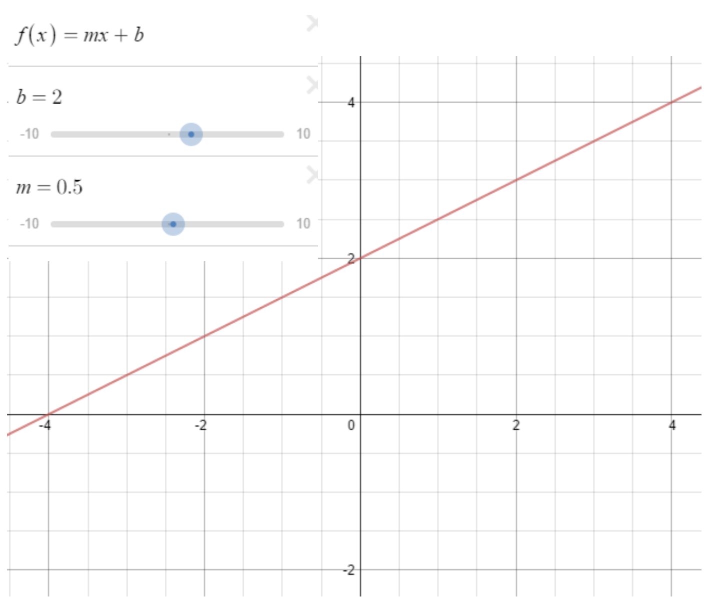
The purpose of this project is to give students in MCR3U9 additional opportunities to deepen their understanding and appreciation of mathematics.

Method

Students will write individualized responses to the questions given below. Each question will take the following form:

- Select an ***example*** of the concept in question.
- Explain in simple terms ***what*** the concept ***means***. Of course, diagrams should be included.
- Explain ***how*** the concept can be used to solve problems.
- Explain ***why*** the concept makes sense.

Example

Concept	$f(x)=mx+b$
Example	The equation $C(n)=110n+700000$ can be used to model the following situation: The manufacturing of a new kind of sports bicycle costs \$110 per bike plus \$700,000 to set up the manufacturing equipment. ($C(n)$ represents the cost of manufacturing *n* bicycles.)
What does it ***mean***?	• $f(x)=mx+b$ is the general equation of all functions whose ***rate of change is constant***. That is, for a fixed change in *x*, the change in $f(x)$ is also fixed. (Another way of putting this is as follows: If Δx is constant, then Δy is also constant.) • The graph of $f(x)=mx+b$ is a ***straight line*** with a slope of *m* and a *y*-intercept of *b*. Due to the straight-line nature of their graphs, such functions are often called "linear" but are more properly known as "affine." Strictly speaking, an affine function *f* is called linear only if $f(0)=0$, that is, if *b* = 0. (See Affine Transformation for more information.)  • For the example $C(n)=110n+700000$ given above, the slope is 110 and the *y*-intercept is 700000. This means that the graph of *C* intersects the *y*-axis at 700000 and rises by 110 units for every increase of 1 in the independent variable *n*. • Another way of interpreting this is that the manufacturing cost begins at \$700,000 (the "fixed cost" or "initial value") and rises by \$110 for every bicycle manufactured (the "rate of change.")
How is it used to solve problems?	Every function that takes the form $f(x)=mx+b$ describes the mathematical relationship between two quantities that are linearly related (strictly speaking ***affinely*** related). Thus, such functions can be used to solve problems that involve quantities that change at a constant rate with respect to one another. For instance, once again consider the example given above. Suppose that manufacturing costs amounted to \$923,300. The number of bicycles manufactured can easily be determined by solving the equation $110n+700000=923300$.
Why does the concept make sense?	The function $f(x)=mx+b$ is a polynomial function of degree one, which means that it contains no terms of degree two or higher. This makes it is easy to see why the rate of change of *f* must be constant. The constant term does not change at all while the degree-one term changes proportionately with changes in the value of *x*. Formally, this can be seen by comparing the values of $f(x)$ and $f(x+c)$ for any constant *c*: $\Delta y = f(x+c)-f(x)=m(x+c)+b-(mx+b)=mc$ This shows that when the value of *x* is changed by a constant value *c* (i.e. $\Delta x=c$), the change in *y* is equal to the constant *mc* (i.e. $\Delta y=mc$).

Questions

For each of the concepts given below, create a table like the one shown in the example on the previous page.

Concepts		
1. $y = f(x)$	**2.** If $g(x) = af(b(x-h)) + k$, then the graph of g can be obtained by applying the following transformation to the graph of *f*: $(x, y) \to (b^{-1}x + h, ay + k)$.	**3.** Mathematical ideas can be best understood when they are viewed from a variety of different perspectives.
4. In general, $f(x+y) \neq f(x) + f(y)$ and $f(ax) \neq af(x)$. Transformations can help explain why this is the case.	**5.** Trigonometric functions are perfectly suited for modelling periodic processes.	**6.** The general behaviour of polynomial functions can be understood by considering end behaviours, intervals of increase/decrease, the number of zeros and the number of turning points. For a more detailed understanding, calculus is needed.

Evaluation Guide

	Level 4	**Level 3**	**Level 2**	**Level 1**
Example	The example covers ***all*** or ***most*** aspects of the concept.	The example covers ***many*** aspects of the concept.	The example covers ***some*** aspects of the concept.	The example covers ***few*** aspects of the concept.
What does it mean?	Explanation demonstrates a ***deep*** understanding of the meaning of the concept.	Explanation demonstrates a ***good*** understanding of the meaning of the concept.	Explanation demonstrates a ***moderate*** understanding of the meaning of the concept.	Explanation demonstrates a ***limited*** understanding of the meaning of the concept.
How is it used to solve problems?	Explanation demonstrates a ***deep*** understanding of how the concept is connected to problem solving.	Explanation demonstrates a ***good*** understanding of how the concept is connected to problem solving.	Explanation demonstrates a ***moderate*** understanding of how the concept is connected to problem solving.	Explanation demonstrates a ***limited*** understanding of how the concept is connected to problem solving.
Why does it make sense?	Explanation demonstrates a ***deep*** understanding of the reasoning that supports or justifies the concept.	Explanation demonstrates a ***good*** understanding of the reasoning that supports or justifies the concept.	Explanation demonstrates a ***moderate*** understanding of the reasoning that supports or justifies the concept.	Explanation demonstrates a ***limited*** understanding of the reasoning that supports or justifies the concept.