67. Geometry A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(2, 1)$ (see figure). Write the area A of the triangle as a function of x, and determine the domain of the function.
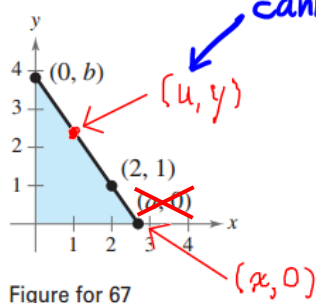

Figure for 67

cannot use x here

This question is somewhat misleading because it is not clear what x represents.

The solution given below assumes that x represents the x-co-ordinate of the x-intercept of the line passing through the point $(2, 1)$.

Let (u, y) represent the co-ordinates of any point on the line.

Let $(x, 0)$ represent the co-ordinates of the x-intercept of the line

① **Equation of Line**

$$\text{slope} = \text{slope}$$

$$\therefore \ \frac{y-1}{u-2} = \frac{0-1}{x-2}$$

$$\therefore \ y - 1 = (u-2)\left(\frac{-1}{x-2}\right)$$

$$\therefore \ y = \frac{2-u}{x-2} + 1$$

② **y-intercept as a function of x**

At the y-intercept, $u = 0$

$$\therefore \ y = \frac{2-0}{x-2} + 1$$

$$= \frac{2}{x-2} + 1$$

$$\therefore \ b = \frac{2}{x-2} + 1$$

③ **Area of the triangle as a function of x**

$$A = \tfrac{1}{2}(\text{base})(\text{height})$$

$$= \tfrac{1}{2} x b$$

$$= \tfrac{1}{2} x \left(\frac{2}{x-2} + 1\right)$$

$$\therefore \ A(x) = \frac{x}{2}\left(\frac{2 + x - 2}{x-2}\right)$$

$$= \frac{x}{2}\left(\frac{x}{x-2}\right)$$

$$= \frac{x^2}{2(x-2)}$$

$$\boxed{\therefore \ A(x) = \frac{x^2}{2(x-2)}, \quad x > 2}$$

Note: Why must $x > 2$?