only one choice $2\sin\theta\cos\theta$

3 choices! $\cos^2\theta-\sin^2\theta$, $2\cos^2\theta-1$, $1-2\sin^2\theta$

Some Trigonometric Identities to Warm up your Neurons

Prove that each of the given equations are identities.

1. $\dfrac{\cos 2\theta}{1+\sin 2\theta}=\dfrac{\cot\theta-1}{\cot\theta+1}$

$$L.S.=\frac{\cos^2\theta-\sin^2\theta}{1+2\sin\theta\cos\theta}$$

$$=\frac{\dfrac{\cos^2\theta}{\sin^2\theta}-\dfrac{\sin^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin^2\theta}+\dfrac{2\sin\theta\cos\theta}{\sin^2\theta}}$$

Divide top & bottom by $\sin^2\theta$

$\cot^2\theta+1$

$$=\frac{\cot^2\theta-1}{\csc^2\theta+2\cot\theta}$$

$$=\frac{(\cot\theta+1)(\cot\theta-1)}{\cot^2\theta+2\cot\theta+1}$$

$$=\frac{(\cot\theta+1)(\cot\theta-1)}{(\cot\theta+1)^2}$$

$$=\frac{\cot\theta-1}{\cot\theta+1}=R.S.$$

$\therefore$ the equ'n is an identity.

2. $(\sin A)(1+\tan A)+(\cos A)(1+\cot A)=\sec A+\csc A$

$$L.S.=\sin A+\sin A\tan A+\cos A+\cos A\cot A$$

$$=\sin A+\sin A\left(\frac{\sin A}{\cos A}\right)+\cos A+\cos A\left(\frac{\cos A}{\sin A}\right)$$

$$=\cos A+\frac{\sin^2 A}{\cos A}+\sin A+\frac{\cos^2 A}{\sin A}$$

$$=\frac{\cos^2 A+\sin^2 A}{\cos A}+\frac{\sin^2 A+\cos^2 A}{\sin A}$$

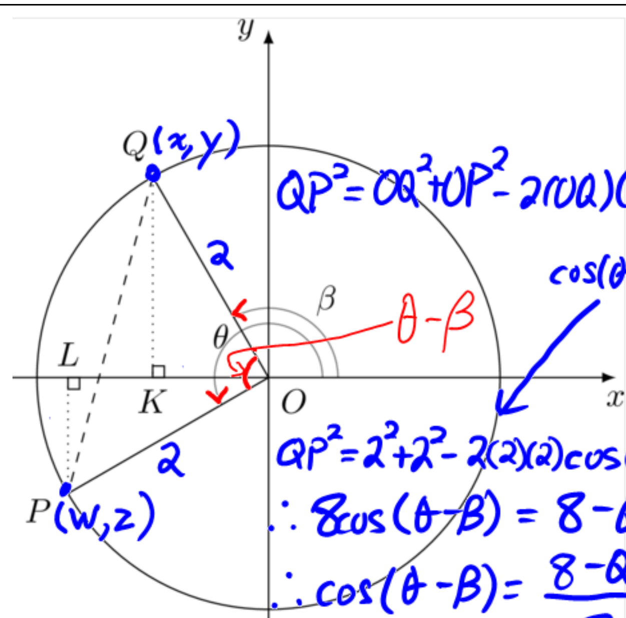
$$=\frac{1}{\cos A}+\frac{1}{\sin A}$$

$$=\sec A+\csc A$$

$= R.S.$

$\therefore$ the given equation is an identity.

3. In the diagram, points P and Q lie on a circle of radius 2 centred at the origin. Use this to prove that $\cos(\theta-\beta)=\cos\theta\cos\beta+\sin\theta\sin\beta$.



$QP^2=OQ^2+OP^2-2(OQ)(OP)\cos(\theta-\beta)$

$QP^2=2^2+2^2-2(2)(2)\cos(\theta-\beta)$

$\therefore 8\cos(\theta-\beta)=8-QP^2$

$\therefore \cos(\theta-\beta)=\dfrac{8-QP^2}{8}$

By the Pythagorean Theorem,

$$\left.\begin{aligned}x^2+y^2&=2^2=4\\ w^2+z^2&=2^2=4\end{aligned}\right\}\ (1)$$

Using the definitions of sin and cos,

$$\left.\begin{aligned}\cos\theta&=\frac{w}{2},\ \sin\theta=\frac{z}{2},\\ \cos\beta&=\frac{x}{2},\ \sin\beta=\frac{y}{2}\end{aligned}\right\}\ (2)$$

By the law of cosines,

$$\cos(\theta-\beta)=\frac{8-QP^2}{8}\quad (3)$$

By the distance formula (application of Pythagorean Theorem)

$$
\begin{aligned}
QP^2 &= (x-w)^2 + (y-z)^2 \\
&= x^2 - 2xw + w^2 + y^2 - 2yz + z^2 \\
&= x^2 + y^2 + w^2 + z^2 - 2xw - 2yz \\
&= 4 + 4 - 2xw - 2yz \quad \text{(see equations 1)} \\
&= 8 - 2xw - 2yz
\end{aligned}
$$

Substituting into equation 3,

$$
\begin{aligned}
\cos(\theta - \beta) &= \frac{8 - QP^2}{8} \\
&= \frac{8 - (8 - 2wx - 2zy)}{8} \\
&= \frac{2wx + 2zy}{8} \\
&= \frac{2wx}{8} + \frac{2zy}{8} \\
&= \frac{wx}{4} + \frac{zy}{4} \\
&= \frac{w}{2}\left(\frac{x}{2}\right) + \frac{z}{2}\left(\frac{y}{2}\right) \\
&= \cos\theta\cos\beta + \sin\theta\sin\beta \quad \text{(see equations 2)} \\
&= \text{R.S.}
\end{aligned}
$$

$\therefore$ $\cos(\theta - \beta) = \cos\theta\cos\beta + \sin\theta\sin\beta$ is an identity