MCR 3U9 **Semester 1, 2015 - 2016**

Grade 11 Pre-AP Functions
Diagnostic Test

Mr. N. Nolfi

***Victim*: Mr. Solutions**

It is very obvious that you are ready for MCR3U9 Mr. S.!!

$\frac{50}{50}$

1. Complete the following statements by filling in the blanks with logical answers that relate to what we have learned in the review unit of this course. **(6)**

(a) Many students find mathematics difficult because they see it as a massive collection of complicated, incomprehensible rules that are used to manipulate myriad meaningless symbols. Gladly, there are simple strategies that students can apply that will help them develop a mindset that makes mathematics much easier to understand. Mr. Nolfi described two of these strategies in Unit 0. List the two strategies in the space provided below.

✓ = $\frac{1}{2}$ mark

(i) Focus on important ideas rather than blindly memorized facts. ✓

(ii) View mathematical relationships from different perspectives. ✓

As examples of one of these strategies, Mr. Nolfi pointed out that for equations of lines, we only need to remember slope = slope ✓, for the midpoint of a line segment, we only need to remember the average of a and b is $\frac{a+b}{2}$ ✓ and for the length of a line segment, we only need to remember the Pythagorean theorem ✓.

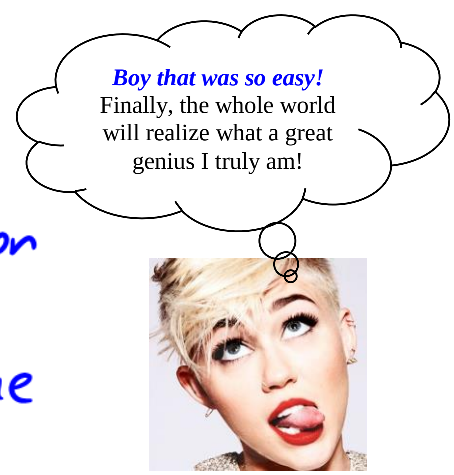
(b) ***Linear relationships*** model quantities that change at a constant rate ✓. For example, if a car moves with a constant ✓ velocity, the relationship between ***distance travelled*** and ***time elapsed*** is ***linear***. Given a ***table of values***, it is possible to spot linear relationships because the ***first differences*** are always linear ✓. ***Quadratic relationships***, on the other hand, model certain quantities that do not change at a constant rate ✓. For example, if a cannonball is fired vertically into the air, then the relationship between its height ✓ and ***time elapsed*** is ***quadratic***. Given a ***table of values***, it is possible to spot quadratic relationships because the ***first differences*** always change at a constant rate ✓ and the ***second differences*** are constant ✓.

2. When Miley C. was asked to ***factor*** the expression x^2+4x-5, she wrote the "solution" shown below. Is it correct? Explain. **(2)**

$x^2+4x-5=(x-1)(x+5)$

$\therefore x-1=0$ or $x+5=0$

$\therefore x=1$ or $x=-5$



Instead of factoring, Miley solved the equation $x^2-4x-5=0$. ✓

She should have stopped writing after the first step! ✓

3. Feeling that she was on a roll, Miley decided to tackle another question. Here is the "solution" that she offered. Is Miley's solution correct? If not, provide a ***correct solution*** along with a ***graph*** that clearly shows the ***roots*** of the ***equation***. **(6)**

$5x^2 - 10x - 120 = 0$

$\therefore 5(x^2 - 2x - 24) = 0$

$\therefore x^2 - 2x - 24 = 0$

$\cancel{\therefore x^2 - 2x = 24}$

$\cancel{\therefore x(x-2) = 24}$

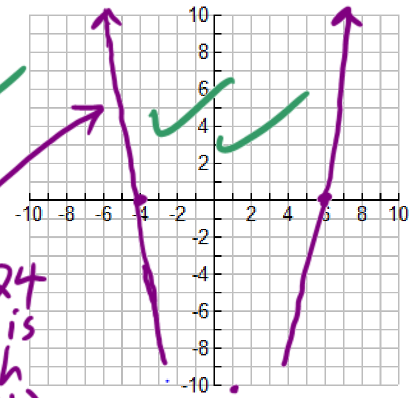
$\cancel{\therefore x = 24 \text{ or } x - 2 = 24}$

$\cancel{\therefore x = 24 \text{ or } x = 26}$

$\therefore (x-6)(x+4) = 0$ ✓

$\therefore x - 6 = 0$ or $x + 4 = 0$ ✓

$\therefore x = 6$ ✓ or $x = -4$ ✓



$y = x^2 - 2x - 24$

(this parabola is hard to sketch on given grid)

Vertex: $(1, -25)$

4. Solve. Show all steps.

(a) Solve the following linear equation. **(5)**

$$\frac{14}{1}\left[\frac{6}{7}(3x-1) + 3x\right] = \left[-4 - \frac{13}{14}x\right]\left(\frac{14}{1}\right) ✓$$

$\therefore 12(3x-1) + 42x = -56 - 13x$ ✓

$\therefore 36x - 12 + 42x = -56 - 13x$ ✓

$\therefore 78x + 13x = -56 + 12$ ✓

$\therefore 91x = -44$

$\therefore x = \frac{-44}{91}$ ✓

(b) Solve the following quadratic equation. **(7)**

$$2\left[2(3z-1)(z+1)\right] = \left[-\frac{3}{2}z(2z-5) + \frac{3}{2}\right](2) ✓$$

$\therefore 4(3z-1)(z+1) = -3z(2z-5) + 3$ ✓

$\therefore (12z-4)(z+1) = -6z^2 + 15z + 3$ ✓

$\therefore 12z^2 + 8z - 4 = -6z^2 + 15z + 3$ ✓

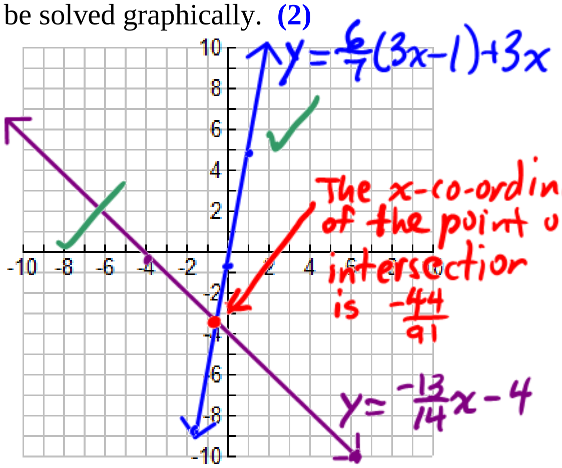
$\therefore 18z^2 - 7z - 7 = 0$ ✓

$\therefore z = \dfrac{7 \pm \sqrt{7^2 - 4(18)(-7)}}{2(18)}$ ✓

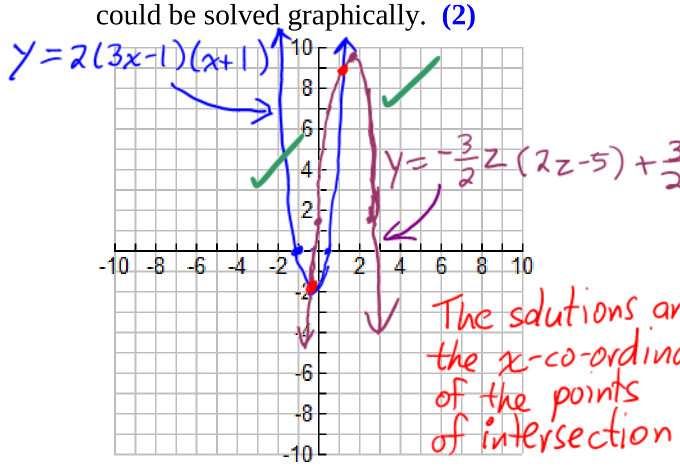
$= \dfrac{7 \pm \sqrt{553}}{36}$ ✓

Note that this quadratic cannot be factored. Its discriminant is 553, which is not a perfect square.

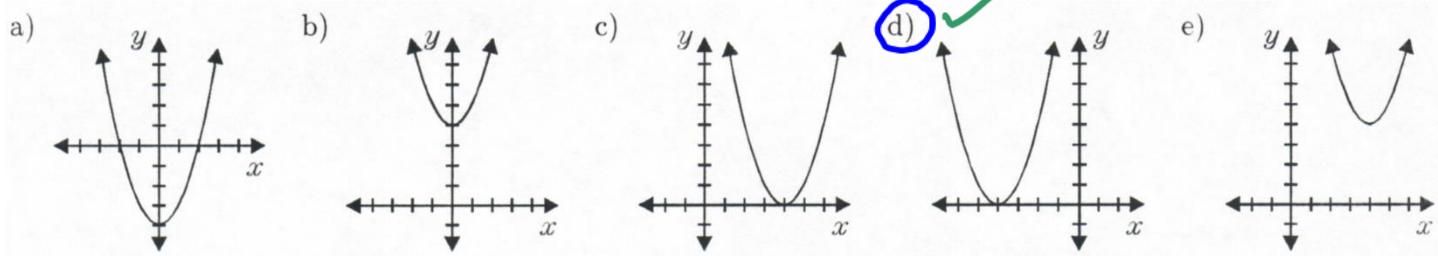
(c) Sketch a graph that shows how the equation in 4(a) could be solved graphically. **(2)**



(d) Sketch a graph that shows how the equation in 4(b) could be solved graphically. **(2)**

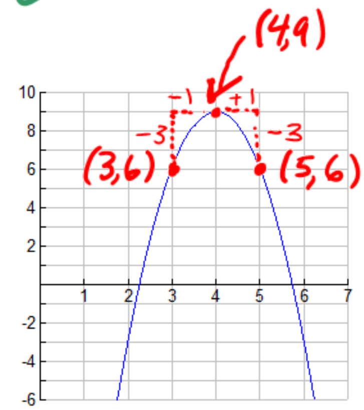


5. Which of the following could be the graph of $y=(x+4)^2$? Explain. **(2)**



The graph of $y=(x+4)^2$ can be obtained by translating the graph of $y=x^2$ four units to the LEFT ✓

6. State an equation of the graph shown at the right. Justify your answer. **(4)**



✓✓ Equation $y=-3(x-4)^2+9$ ← translated nine units upward

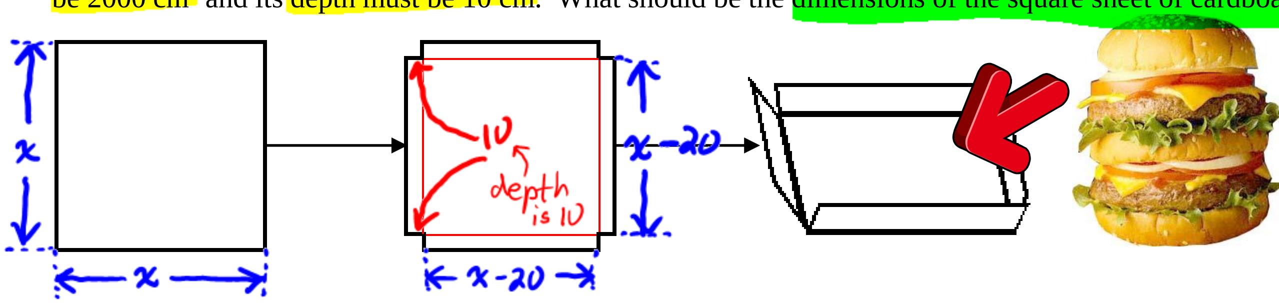
✓✓ Justification

opens downward

stretched vertically by a factor of 3

translated four units to the right

7. McDonald's Canada® has hired Foram P. to design a box, which is to be manufactured from a ***square*** sheet of cardboard, for their new ultra-sized burger (see diagram below). To contain the new massive hamburger, the volume of the box must be 2000 cm^3 and its depth must be 10 cm. What should be the dimensions of the square sheet of cardboard? **(6)**



Since volume = 2000,

$10(x-20)^2=2000$ ✓

$\therefore (x-20)^2=200$ ✓

$\therefore x-20=\pm\sqrt{200}$ ✓

$\therefore x=20\pm10\sqrt{2}$ ✓

Since all dimensions of the box must be positive,

$x=20+10\sqrt{2}$. ✓

(Otherwise, the bottom of the box would have a negative length and width.)

Therefore, the sheet of cardboard should be

$20+10\sqrt{2}$ cm by $20+10\sqrt{2}$ cm ✓

(or about 34 cm by 34 cm).

8. Let $f(x) = x^2 + bx + 6$ and $g(x) = -x^2 + 6x - 9$. For what values of b do the graphs of f and g intersect at …

(i) …two points?
(ii) …one point?
(iii) …no points? **(8)**

At the points of intersection,

$$y_{f(x)} = y_{g(x)}$$

$$\therefore x^2 + bx + 6 = -x^2 + 6x - 9$$

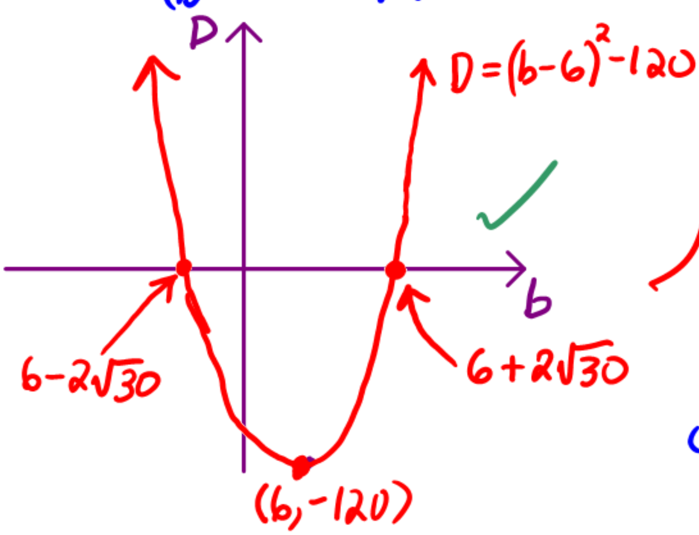
$$\therefore 2x^2 + bx - 6x + 6 + 9 = 0$$

$$\therefore 2x^2 + (b-6)x + 15 = 0$$

The nature of the roots of any quadratic equation $ax^2 + bx + c = 0$ is determined by the discriminant D.

For the above equation,

$$D = (b-6)^2 - 4(2)(15)$$
$$= (b-6)^2 - 120$$



From the graph of D vs. b we can see that

$D > 0$ if $b < 6 - 2\sqrt{30}$ or $b > 6 + 2\sqrt{30}$,

$D = 0$ if $b = 6 - 2\sqrt{30}$ or $b = 6 + 2\sqrt{30}$

AND

$D < 0$ if $6 - 2\sqrt{30} < b < 6 + 2\sqrt{30}$

Therefore,

(i) there are 2 roots and hence 2 points of intersection if $b < 6 - 2\sqrt{30}$ or $b > 6 + 2\sqrt{30}$

(ii) there is one root and hence one point of intersection if $b = 6 - 2\sqrt{30}$ or $b = 6 + 2\sqrt{30}$

(iii) there are no roots and hence no points of intersection if $6 - 2\sqrt{30} < b < 6 + 2\sqrt{30}$.

FUN PUZZLE: ATTEMPT ONLY IF YOU HAVE COMPLETED ALL THE OTHER QUESTIONS!

An eccentric old king wants to give his throne to one of his two sons. He decides that a horse race will be run and the son who owns the slower horse will become king. The sons, each fearing that the other will cheat by having his horse run less fast than it is capable, ask the court fool for his advice. With only two words, the fool tells them how to make sure that the race will be fair. What are the two words?

Switch horses!