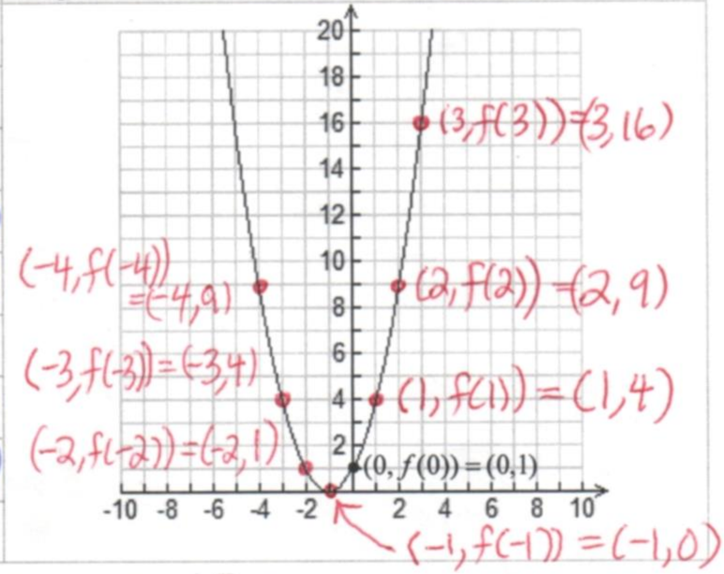
SUPER SKILLS REVIEW SOLUTIONS

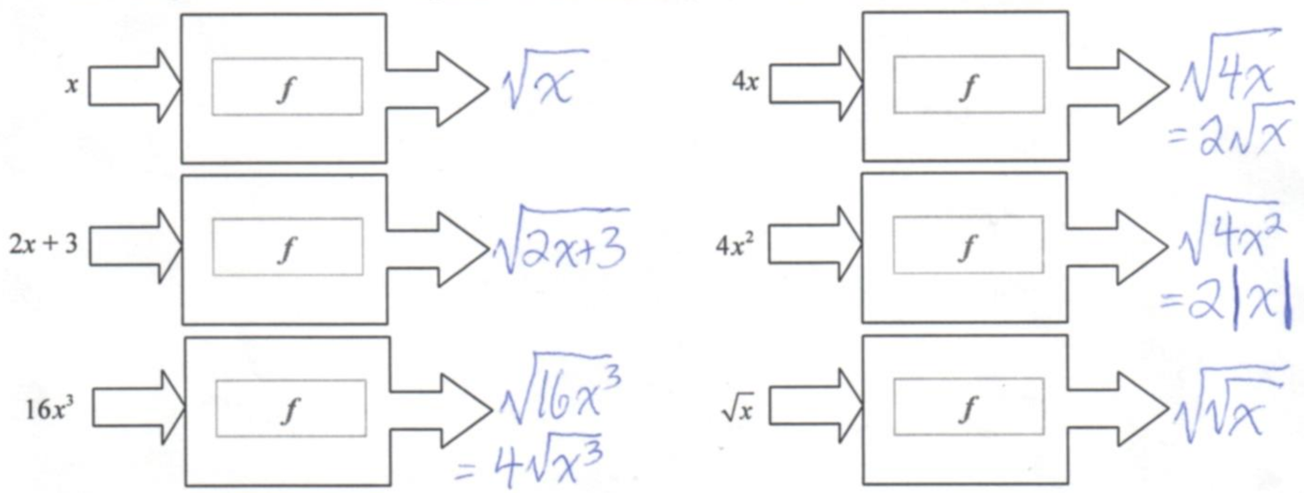
1. Complete the following table for $f(x)=(x+1)^2$. The first row is done for you.

Evaluate	*Ordered Pair* $(x, f(x))$
$f(0)=(0+1)^2=1$	$(0, f(0))=(0,1)$
$f(-1)=(-1+1)^2=0$	$(-1,f(-1))=(-1,0)$
$f(1)=(1+1)^2=4$	$(1,f(1))=(1,4)$
$f(-2)=(-2+1)^2=1$	$(-2,f(-2))=(-2,1)$
$f(2)=(2+1)^2=9$	$(2,f(2))=(2,9)$
$f(-3)=(-3+1)^2=4$	$(-3,f(-3))=(-3,4)$
$f(3)=(3+1)^2=16$	$(3,f(3))=(3,16)$
$f(-4)=(-4+1)^2=9$	$(-4,f(4))=(-4,9)$
$f(4)=(4+1)^2=25$	$(4,f(4))=(4,25)$

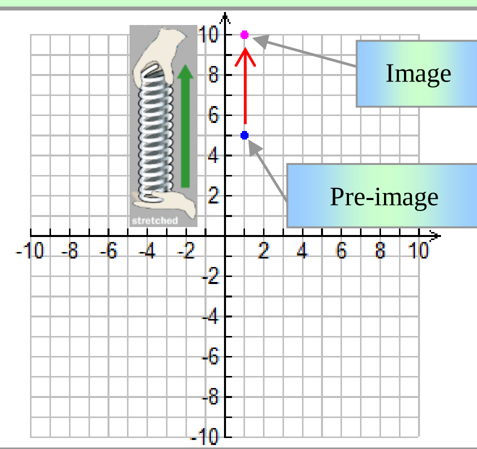
Graph – Mark the Ordered Pairs on the Graph

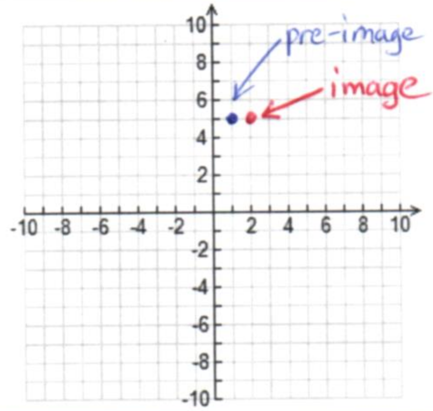
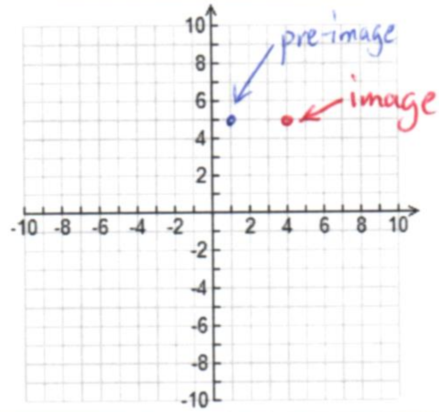
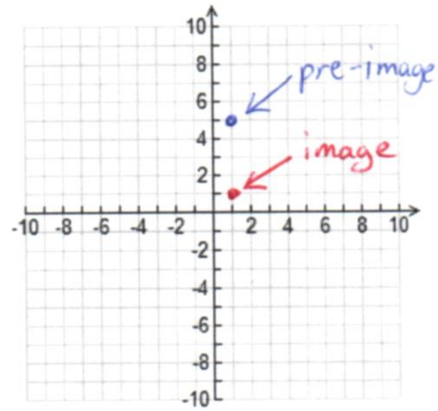
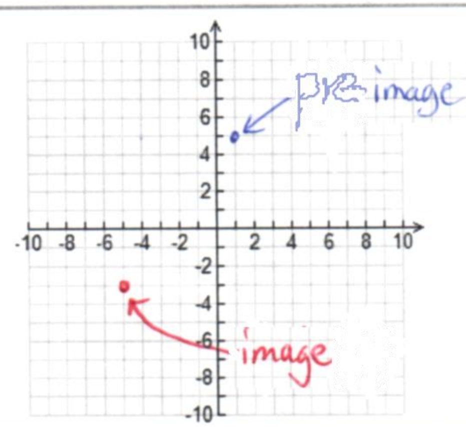


2. Complete the following function machine diagrams for the function $f(x)=\sqrt{x}$. Simplify if possible.

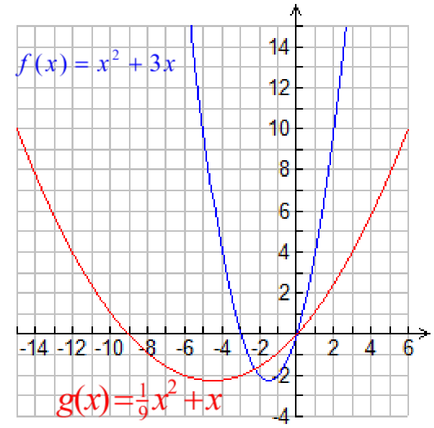
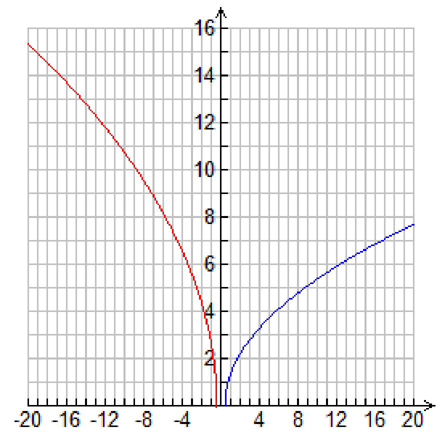
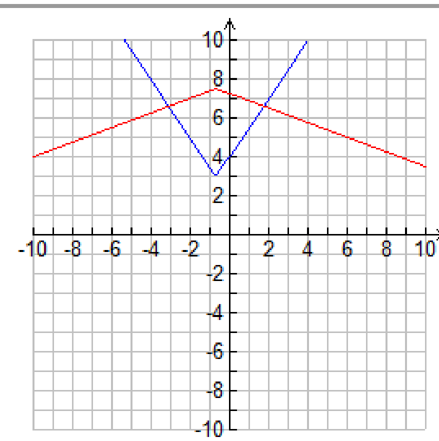
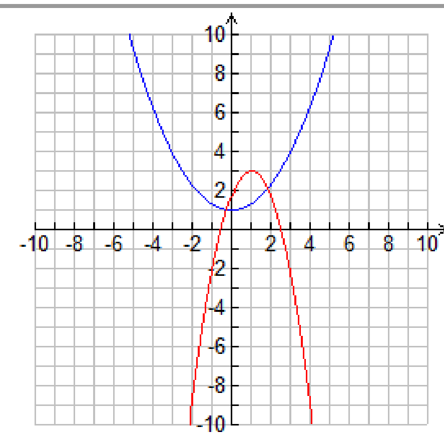


3. Complete the following table. The first row is done for you.

Pre-image	*Transformation*	*Transformation in Mapping Notation*	*Image*	*Graph*
$(1,5)$	Stretch vertically by a factor of 2.	$(x,y)\rightarrow(x,2y)$	$(1,10)$	

Pre-image	*Transformation*	*Transformation in Mapping Notation*	*Image*	*Graph*
(1,5)	Stretch horizontally by a factor of 2.	$(x,y)\to(2x,y)$	$(2,5)$	
(1,5)	Translate to the right 3 units.	$(x,y)\to(x+3,y)$	$(4,5)$	
(1,5)	Translate down 4 units.	$(x,y)\to(x,y-4)$	$(1,1)$	
(1,5)	*Horizontal* 1. Stretch by a factor of −2. 2. Translate 3 units left *Vertical* 1. Reflect in the *x*-axis. 2. Translate up 2 units.	$(x,y)\to$ $(-2x-3,-y+2)$	$(-5,-3)$	

4. Complete the following table. The first row is done for you.

Equation of Pre-image Function	***Transformation***	***Equation of Image Function***	***Graph***
$f(x)=x^2+3x$	***Verbal*** Stretch horizontally by a factor of 3. ***Function Notation*** $g(x)=f(\frac{1}{3}x)$ ***Mapping Notation*** $(x,y)\rightarrow(3x,y)$	$g(x)=f(\frac{1}{3}x)$ $=(\frac{1}{3}x)^2+3(\frac{1}{3}x)$ $=\frac{1}{9}x^2+x$	
$f(x)=\sqrt{3x-1}$	***Verbal*** Horizontal: Reflect in y-axis Vertical: Stretch by a factor of 2 ***Function Notation*** $g(x)=2f(-x)$ ***Mapping Notation*** $(x,y)\rightarrow(-x,2y)$	$g(x)=2f(-x)$ $=2\sqrt{3(-x)-1}$ $=2\sqrt{-3x-1}$	
$f(x)=\left\|\frac{3}{2}x+1\right\|+3$	***Verbal*** Compress vertically by a factor of $\frac{1}{2}$ and reflect in the x-axis, then shift up 9 units. Stretch horizontally by a factor of 2, reflect in y-axis, shift left 2 units ***Function Notation*** $g(x)=-\frac{1}{2}f(-\frac{1}{2}x-1)+9$ $=-\frac{1}{2}f(-\frac{1}{2}(x+2))+9$ ***Mapping Notation*** $(x,y)\rightarrow(-2x-2,-\frac{1}{2}y+9)$	$g(x)=-\frac{1}{2}f(-\frac{1}{2}x-1)+9$ $=-\frac{1}{2}\left(\left\|\frac{3}{2}(-\frac{1}{2}x-1)+1\right\|+3\right)+9$ $=-\frac{1}{2}\left\|-\frac{3}{4}x-\frac{3}{2}+1\right\|-\frac{3}{2}+9$ $=-\frac{1}{2}\left\|-\frac{3}{4}x-\frac{1}{2}\right\|+\frac{15}{2}$	
$f(x)=\frac{1}{3}x^2+1$	***Verbal*** Reflect in the x-axis, then shift up 4 units. Compress horizontally by a factor of 0.5, then shift right 1 unit. ***Function Notation*** $g(x)=-f(2(x-1))+4$ ***Mapping Notation*** $(x,y)\rightarrow(0.5x+1,-1y+4)$	$g(x)=-f(2(x-1))+4$ $=-\left[\frac{1}{3}[2(x-1)]^2+1\right]+4$ $=-\left[\frac{4}{3}(x-1)^2+1\right]+4$ $=-\frac{4}{3}(x-1)^2+3$	

5. Complete the following table. The first row is done for you.

Pre-image Function	***Image Function***	***Horizontal Stretch or Compression that produces the Image function***	***Vertical Stretch or Compression that produces the Image function***
$f(x)=x^2$	$g(x)=\frac{1}{3}x^2$	Horizontal stretch by a factor of $\sqrt{3}$. $g(x)=f\left(\frac{1}{\sqrt{3}}x\right)$ $(x,y)\to(\sqrt{3}x,y)$	Vertical compression by a factor of 1/3. $g(x)=\frac{1}{3}f(x)$ $(x,y)\to(x,\frac{1}{3}y)$
$f(x)=\lvert x\rvert$	$g(x)=\lvert 3x\rvert$	Horizontal Compression by a factor of $\frac{1}{3}$ $g(x)=f(3x)$ $(x,y)\to(\frac{1}{3}x,y)$	Vertical Stretch by a Factor of 3 $g(x)=\lvert 3x\rvert=3\lvert x\rvert=3f(x)$ $(x,y)\to(x,3y)$
$f(x)=x^3$	$g(x)=125x^3$	Horizontal compression by a factor of $\frac{1}{5}$ $g(x)=125x^3=(5x)^3=f(5x)$ $(x,y)\to(\frac{1}{5}x,y)$	Vertical stretch by a factor of 125 $g(x)=125f(x)$ $(x,y)\to(x,125y)$
$f(x)=\sqrt{x}$	$g(x)=\sqrt{10x}$	Horizontal compression by a factor of $\frac{1}{10}$ $g(x)=f(10x)$ $(x,y)\to(\frac{1}{10}x,y)$	Vertical stretch by a factor of $\sqrt{10}$ $g(x)=\sqrt{10x}=\sqrt{10}\sqrt{x}=\sqrt{10}f(x)$ $(x,y)\to(x,\sqrt{10}y)$

6. What conclusions can you draw from the table in question 5?

We can conclude that in each case considered, it was possible to produce the image function by using both vertical and horizontal stretches or compression. In addition, it appears that vertical stretches can also be viewed as horizontal compressions and vice versa

7. The following table lists the approximate accelerations due to gravity near the surface of the Earth, moon and sun.

Earth	***Moon***	***Sun***
9.87 m/s^2	1.62 m/s^2	2.74×10^2 m/s^2

The data in the above table lead to the following equations for the height of an object dropped near the surface of each of the celestial bodies given above. In each case, $h(t)$ represents the height, in metres, of an object above the surface of the body t seconds after it is dropped from an initial height h_0.

Earth	***Moon***	***Sun***
$h(t)=-4.94t^2+h_0$	$h(t)=-0.81t^2+h_0$	$h(t)=-1.37\times10^2t^2+h_0$

In questions (a) to (d), use an initial height of 100 m for the Earth, 25 m for the moon and 1000 m for the sun.

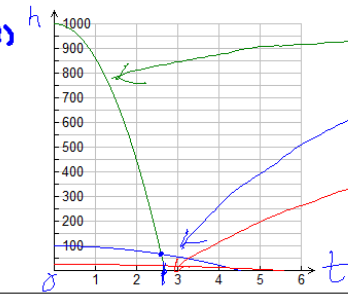
(a) On the same grid, sketch each function.

(b) Explain how the "moon function" can be transformed into the "Earth function."

(c) Consider the graphs for the Earth and the sun. Explain the ***physical meaning*** of the point(s) of intersection of the two graphs.

(d) State the domain and range of each function. Keep in mind that each function is used to ***model*** a physical situation, which means that the allowable values of t are highly restricted.

(a)



$h_1(t) = -1.37\times10^2 t^2 + 1000$

$h_2(t) = -4.94t^2 + 100$

$h_3(t) = -0.81t^2 + 25$

$\frac{-4.94}{-0.81} \doteq 6.1$

(b) $h_3 \longrightarrow h_2$

$a \doteq 6.1$

$k \doteq -52.5$

Conjecture (Guess)

Stretch h_3 by a factor of 6.1

$$6.1\,h_3(t) - 52.5 \doteq 6.1(-0.81t^2 + 25) - 52.5$$
$$= -4.94t^2 + 152.5 - 52.5$$
$$= -4.94t^2 + 100$$

$$h_2(t) \doteq 6.1\,h_3(t) - 52.5$$

(c) The point of intersection tells us the time at which both objects have the same height.

(d) Sun: $h(t) = -1.37\times10^2 t^2 + 1000$

$D \doteq \{t \in \mathbb{R} \mid 0 \le t \le 2.7\}$

$R = \{h \in \mathbb{R} \mid 0 \le h \le 1000\}$

For domain, we need to find t at which $h = 0$

$\therefore -1.37\times10^2 t^2 + 1000 = 0$

$\therefore -1.37\times10^2 t^2 = -1000$

$\therefore t^2 = \frac{+1000}{+1.37\times10^2}$

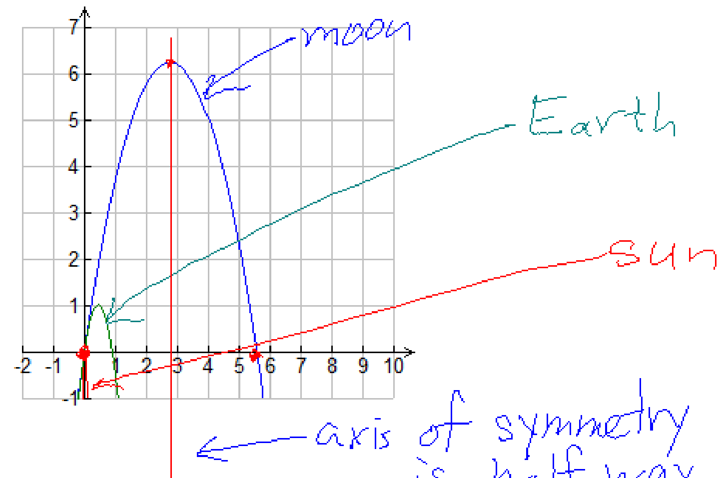
$\therefore t = \sqrt{\frac{+1000}{+1.37\times10^2}} \doteq 2.7$

(e) Let a represent acceleration due to gravity, v_0 represent initial velocity, t represent time and $h(t)$ represent height above the "ground" at time t. Then, the function $h(t) = -\frac{a}{2}t^2 + v_0 t$ can be used to describe the height above the "ground" of a person who jumps up from the surface of a body, at a time t after the initial jump. A typical human can jump vertically with an initial velocity of about 4.5 m/s, which on the Earth would result in a jump about 1 m high. How high would a typical human be able to jump on the moon? If the surface of the sun were solid, how high would a typical human be able to jump on the sun?

$$h_m(t) = -0.81t^2 + 4.5t$$

$$h_s(t) = -1.37\times 10^2 t^2 + 4.5t$$

m = moon
s = sun



← axis of symmetry is half way between roots

Moon

To find the max height reached

Method 1

Find vertex by completing the square

Method 2

(a) Find roots

(b) Calculate value half way between roots ("t-value" for axis of symmetry)

(c) Substitute this value into the equation