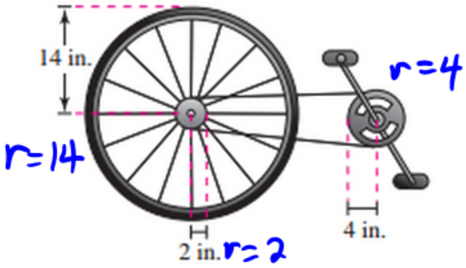
68. Speed of a Bicycle

The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.

(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

Ratio of Radii $\}$ $4:2:14 = 2:1:7$

Ratio of Circumferences $\}$ $8\pi : 4\pi : 28\pi = 2\pi : \pi : 7\pi = 2:1:7$

Same ratio! This happens because $\ell = r\theta$ is a linear function.

(a) one complete rotation of front sprocket
→ two complete rotations of back sprocket
→ two complete rotations of back wheel because sprocket is attached to the wheel

Since cyclist pedals at rate of 1 rev/s, distance travelled in one second
$= 2$ times circumference of wheel
$= 2[2\pi(14 \text{ inches})]$
$= 56\pi \text{ inches} = \frac{56\pi}{12} \text{ feet} = \frac{14\pi}{3} \text{ feet}$

1 mile = 5280 feet

1 hour = 3600 s

∴ speed in m.p.h.

$$= \frac{\frac{\overset{7}{\cancel{14}}\pi}{3} \text{ feet/s}}{\underset{\cancel{22}\ 11}{\cancel{5280}} \text{ ft/mile}} \left(\overset{15}{\cancel{3600}} \text{ s/h}\right) = \frac{105\pi}{33} \text{ miles/h} \doteq 10 \text{ miles/h}$$

(b) Distance travell d (in feet) = (# revolutions of front sprocket)(distance travelled per revolution)

$= n\left(\frac{14\pi}{3} \text{ feet/rev}\right)$

$\frac{14\pi}{3}$ feet
$= \frac{14\pi}{3} \div 5280$
$= \frac{\overset{7}{\cancel{14}}\pi}{3}\left(\frac{1}{\underset{2640}{\cancel{5280}}}\right)$ miles

$\therefore d(n) = \frac{14\pi n}{3} = \frac{14\pi}{3} n$ feet

$\therefore d(n) = \frac{7\pi}{7920} n$ miles

(c) Assumption: cyclist pedals at a rate of 1 rev/s

$d(t) = \frac{7\pi}{7920} t$ miles, t in seconds

Formula-Oriented Method

Angular Velocity

$$\omega = \frac{\theta}{t}$$

angle through which something rotates, divided by the time it takes

Linear Velocity

For rotational motion.

$$v = \frac{d}{t} = \frac{\text{arc length}}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = r\omega$$

Recall that θ MUST be in radians!

Given: $\omega_{FS} = 1$ rev/s, $r_{FS} = 4$ inches, $r_{BS} = 2$ inches, $r_{wheel} = 14$ inches
FS = front sprocket, BS = back sprocket

Solution

(a) Since the chain is attached to both sprockets,
velocity of a given point on chain = velocity of a given point on FS
= velocity " " " " BS

Let v represent this velocity.

$$\therefore\ v = r_{FS}\,\omega_{FS} = r_{BS}\,\omega_{BS}$$

$$\therefore\ (4 \text{ inches})(1 \text{ rev/s}) = (2 \text{ inches})\,\omega_{BS}$$

$$\therefore\ \omega_{BS} = \frac{(4 \text{ inches})(1 \text{ rev/s})}{2 \text{ inches}} = 2 \text{ rev/s}$$

Since the back sprocket is connected to the back wheel,

$$\omega_{wheel} = \omega_{BS} = 2 \text{ rev/s}$$

$$\begin{aligned}
\therefore\ v_{Bicycle} &= r_{wheel}\,\omega_{wheel} \\
&= (14 \text{ inches})(2 \text{ rev/s}) \\
&= (14 \text{ inches})[2(2\pi) \text{ rad/s}] \\
&= 56\pi \text{ inches/s} \\
&= \frac{105\pi}{33} \text{ miles/h} \quad \text{(as shown on first page)} \\
&\doteq 10 \text{ miles/h}
\end{aligned}$$