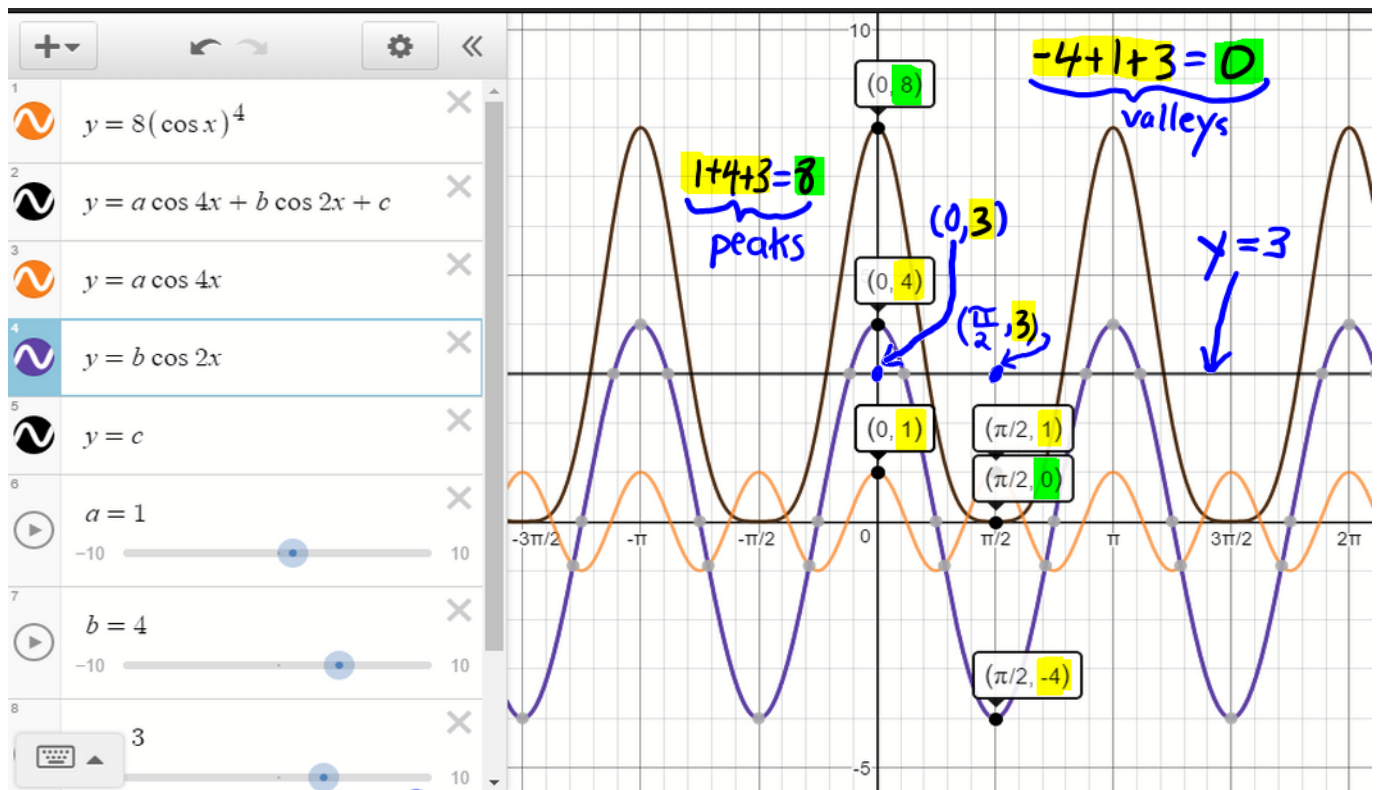


8. Express $8\cos^4 x$ in the form $a\cos 4x + b\cos 2x + c$. State the values of the constants a , b and c .



There are two cycles of $y = \cos 4x$ for every cycle of $y = 4\cos 2x$
 Peaks coincide at $0, \pm\pi, \pm 2\pi \dots \rightarrow$ curves "reinforce" each other
 Valleys of $y = 4\cos 2x$ coincide with peaks of $y = \cos 4x$ at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$

$$\begin{aligned}\cos^4 x &= \cos(2(2x)) \\ &= 2\cos^2(2x) - 1 \\ &= 2(2\cos^2 x - 1)^2 - 1 \\ &= 2(4\cos^4 x - 4\cos^2 x + 1) - 1 \\ &= 8\cos^4 x - 8\cos^2 x + 1\end{aligned}$$

$$\begin{aligned}\cos 2x &= 2\cos^2 x - 1\end{aligned} \rightarrow \text{curves "cancel" each other}$$

$$\begin{aligned}1\cos 4x + 4\cos 2x &= 8\cos^4 x - 8\cos^2 x + 1 + 8\cos^2 x - 4 \\ &= 8\cos^4 x - 3 \\ \therefore a &= 1, b = 4, c = 3\end{aligned}$$